Terrestrial Planets

Week 9

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Earth and Planetary Sciences
FD Adams 131C

Rotational dynamics of the planets and their satellites

We shall learn that the actual rotational dynamics of planets and our Moon can be quite complicated. The Earth wobbles as it rotates. Part of this wobbling is forced by the seasonal movement of mass over the Earth, the 12-month seasonal wobble.

Part of the wobbling is a so-called free wobble called the Chandler wobble which has a 14-month period. As well, the Earth's rotation axis precesses with a 26 000 year period.

The Moon librates as it orbits the Earth... It's all very complicated but we can learn quite a lot about the interior of planets and moons from these motions.

Periods and inclinations

Planet	Rotation period (hrs)	Axial inclination (degrees)
Mercury	1407.6	0
Venus	5832	178
Earth	23.934	23.4
Mars	24.623	25.2
Jupiter	9.842	3.1
Saturn	10.65	26.7
Uranus	17.2	97.9
Neptune	16.05	28.8
Pluto	153.36	119
Moon	655.73	6.7

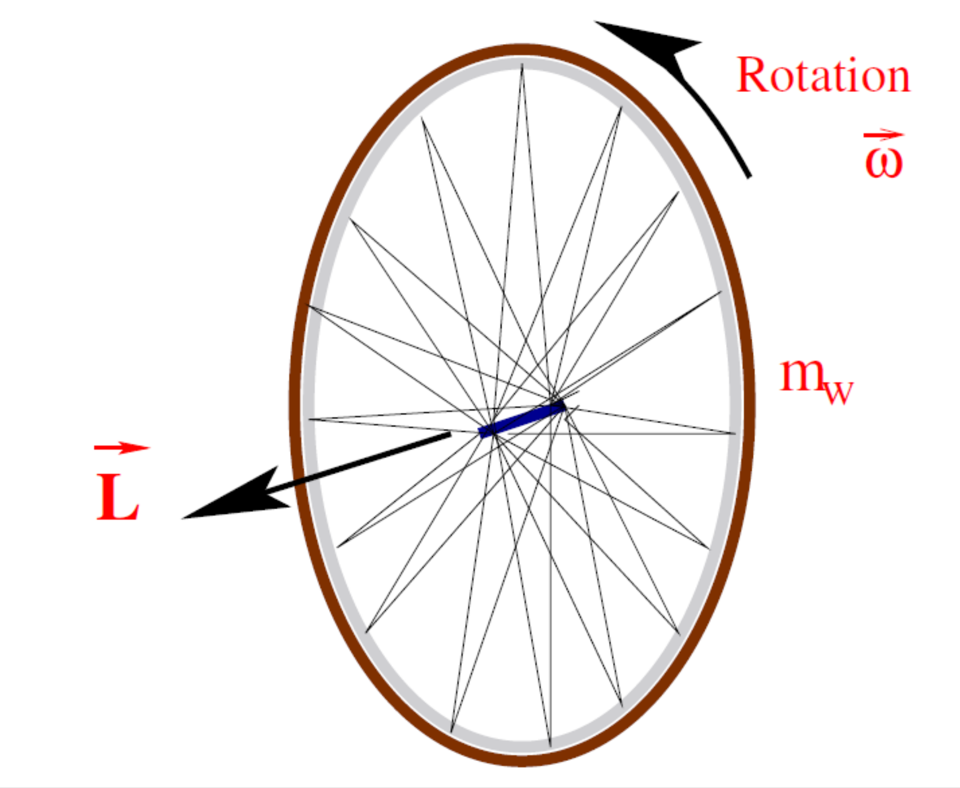
Rotation and Angular Momentum

Moments of inertia of the planets

A rotating body tends to stay in rotation unless acted upon by an external torque.

A spinning bicycle wheel continues to rotate unless braked.

Friction in the bearings of the wheel-axle does, of course, contribute to the slow braking of the spinning wheel.



About angular momentum

The quality and quantity which determines the continuing tendency to rotate is called ***angular momentum***; is usually designated by a vector form, $\vec{\boldsymbol{L}}$. The direction of the vector is established by a convention called the right-hand rule. If the fingers of the right hand are wrapped in the direction of rotation, the thumb points along the ***direction*** of the angular momentum vector.

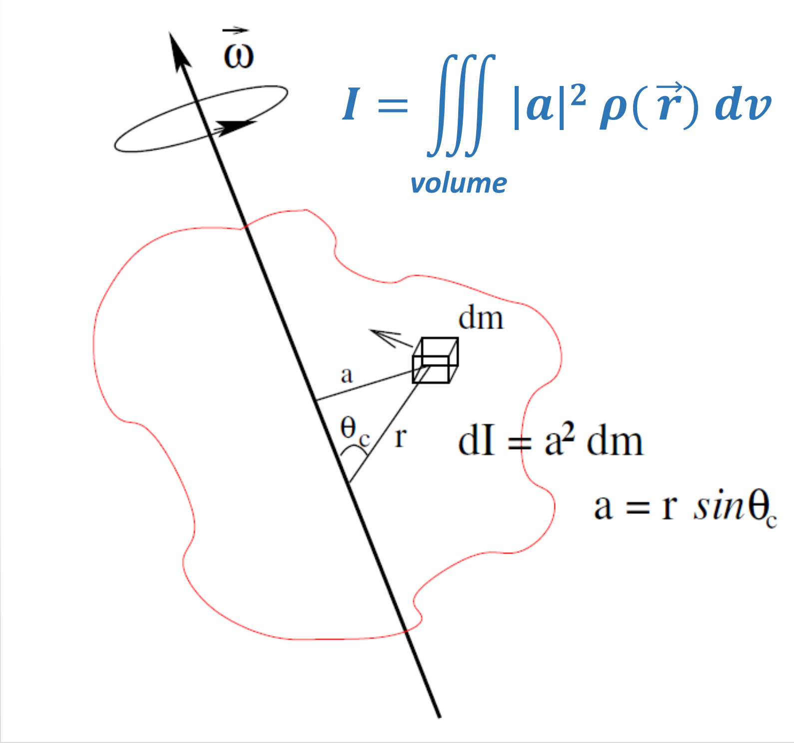
The rate of rotation, $\vec{\boldsymbol{\omega}}$ is determined as the number of turns – usually in radian measure – per unit time. It's ***direction*** is also determined by the right-hand rule. Rotation rates are usually described in units of $\boldsymbol{rad \cdot s^{-1}}$. There are $\boldsymbol{2\pi\, rad}$ (***radians***) in one full cycle of $\boldsymbol{360^{o}}$ of rotation and so $\boldsymbol{1\, rad = 57.2957\ldots^{o}}$.

For our bicycle wheel: $\vec{\boldsymbol{L}} = \boldsymbol{m_w a^2 \vec{\omega}}$, where $\boldsymbol{m_w}$ is the total mass of the wheel (tire and rim) and $\boldsymbol{a}$ is the radius of the wheel.

Other rotating bodies

Note that for our bicycle wheel with $\vec{L} = m_w a^2 \vec{\omega}$, the mass, m_w , and radius, a, are properties of the wheel and the rotation rate, $\vec{\omega}$, determines how fast the wheel is rotating.

We define the "***moment of inertia***" of the wheel as: $I = m_w a^2$. We can determine the moment of inertia of any rotating mass by integrating over all the pieces of the rotating mass referenced to the rotation axis.

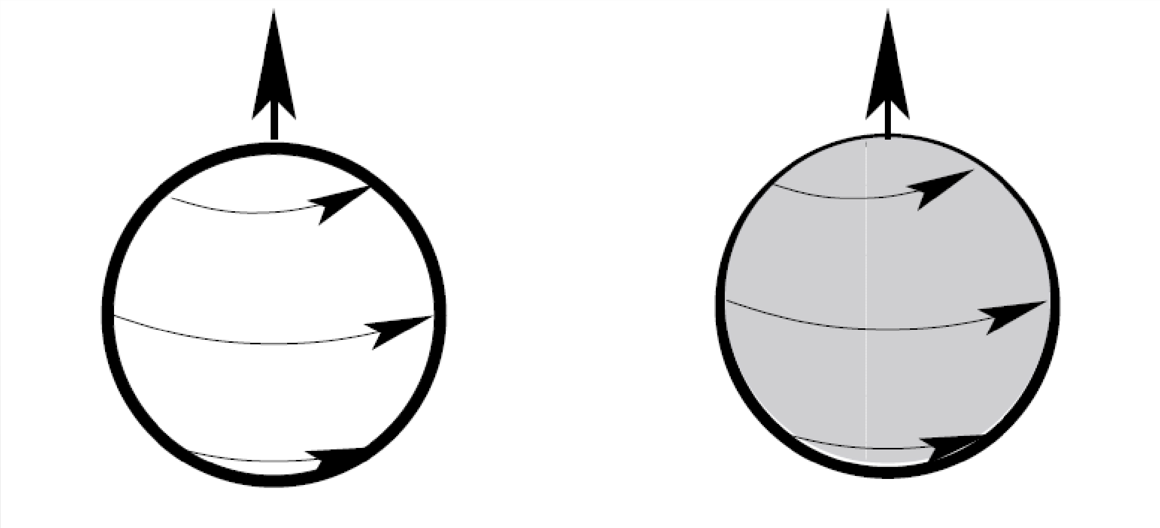


Spherical bodies?

Gravity tends to pull large masses into nearly spherical shapes like our ***planets*** and ***moons***. What of the moment of inertia of such bodies?

Empty shell of mass m_p

Sphere of uniform density of mass m_p



$$I = \frac{2}{3} m_p a^2$$

$$I = \frac{2}{5} m_p a^2$$

What do we learn?

We note that for spherical bodies with more mass assembled at depth and, so, closer to the rotation axis, the moment of inertia is lower than that for bodies with mass assembled at distance.

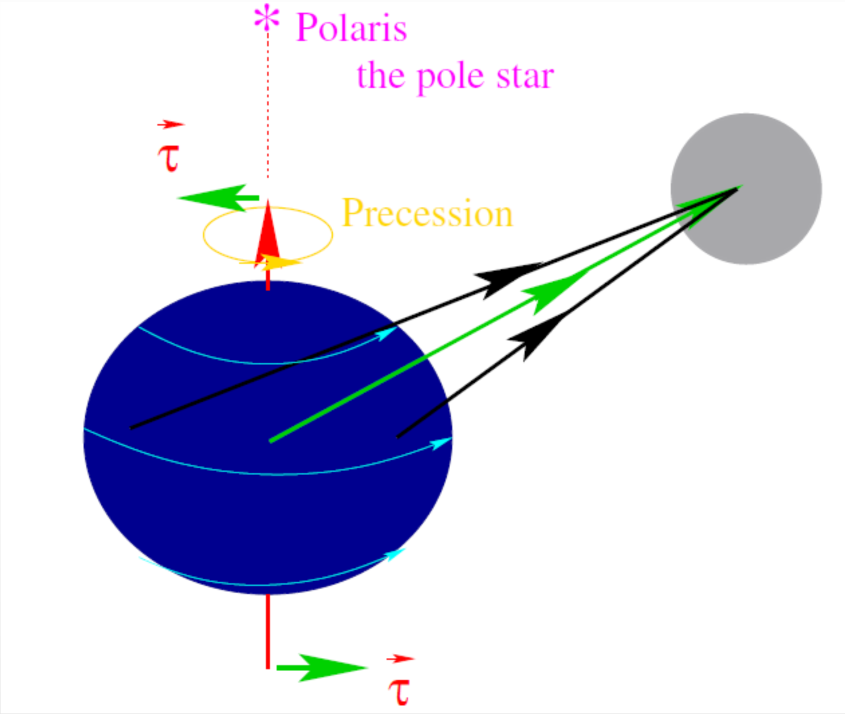
If we can determine the moment of inertia of a planet or moon, we can determine how mass within is distributed.

Astronometric measurement of I

Earth's shape is "flattened" by about 1 part in 300 due to its rapid rotation. The ***flattening*** measure $f = \frac{r_e - r_p}{r_e} \approx \frac{1}{300}$, where r_e is the equatorial radius (6,378 km) and r_p is the polar radius (6,357 km).

There is a differential gravitational attraction (due to the Earth's shape) that produces a twisting torque on Earth.

The twisting torque causes a "***precession***" of the rotation axis which now points toward Polaris, the pole star. The precession period is about 26000 years.



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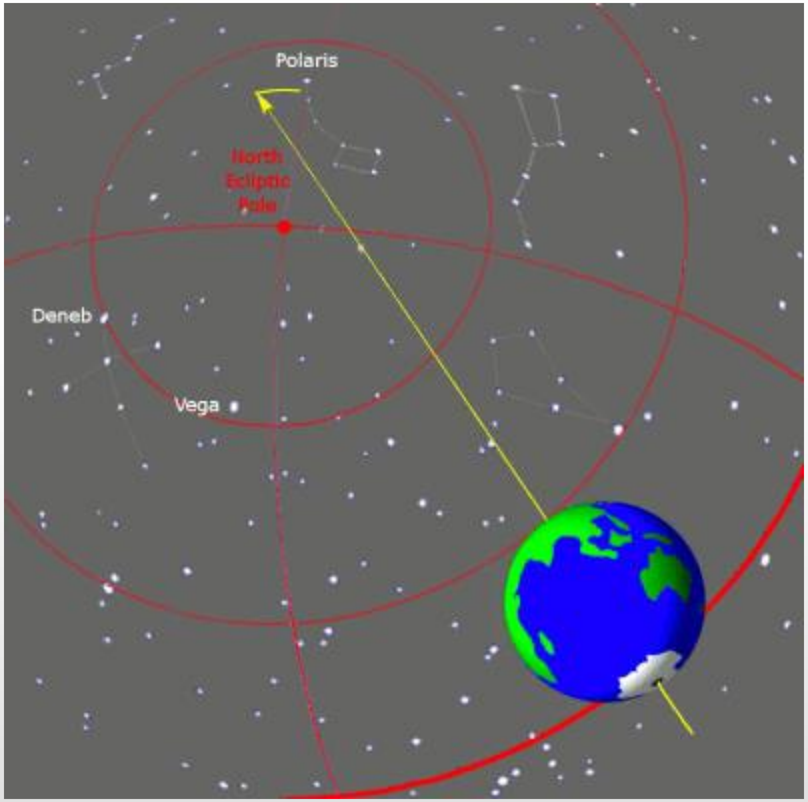
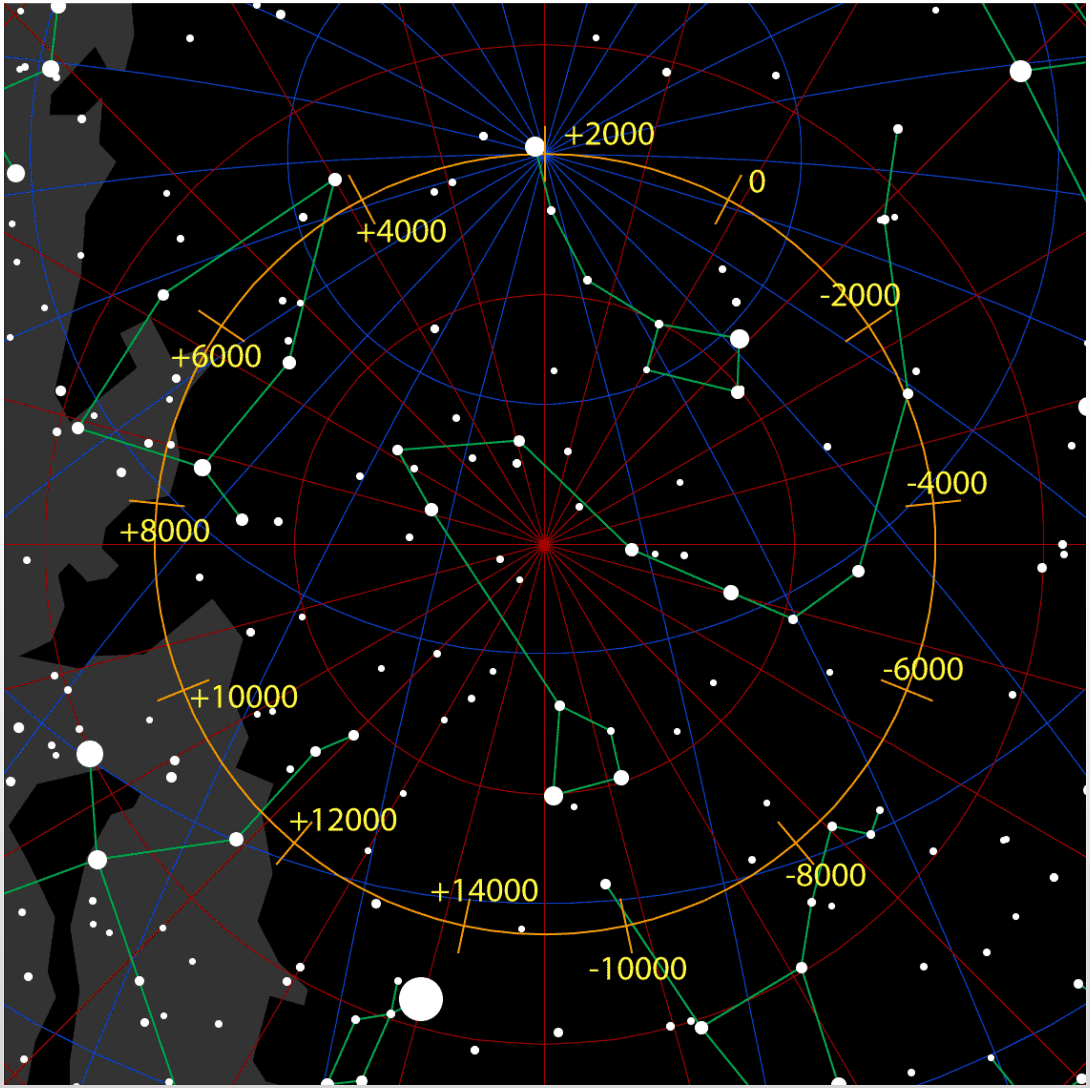


https://en.wikipedia.org/wiki/Precession

Precession of the Equinoxes



https://commons.wikimedia.org/wiki/File%3APrecession_N.gif

Density within planets and moons

Physical analysis of the gravitational-force-induced torques on the Earth due to the Moon and our observation of the period or rate of precession allowed us, already in the 1800s, to determine the moment of inertia $\boldsymbol{I_z}$ (about the rotation axis) for the Earth.

We know it to be *$\boldsymbol{I_z = 0.3307\ m_{\oplus} r^2}$. That tells us that the Earth's density rapidly increases with depth.

We were also able to determine the equivalent moment of inertia for Earth's Moon: $\boldsymbol{I_z = 0.393\ m_{c} r_{c}^{2}}$ indicating that Moon's density is much more nearly uniform.

Properly, the moment of inertia must be fully represented as a 9-element tensor. $\boldsymbol{I_z}$ is the trace of that tensor in a coordinate system with the rotation axis oriented in the $\boldsymbol{z}$ direction

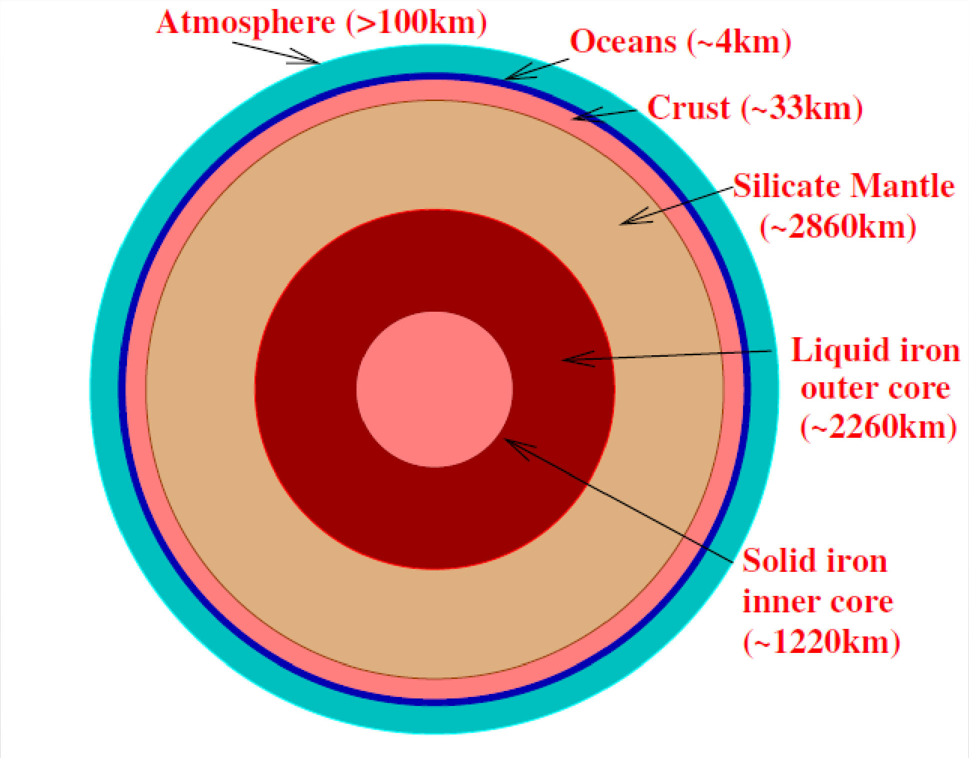
Other planets and moons

None of the other terrestrial planets have moons of substantial mass in orbit about them to provide any precession-causing torques.
… but with the advent of the space program, with satellites and probes orbiting around or near Mars, Venus and Mercury, we have been able to use perturbations in the probe orbits to estimate their representative moments of inertia factors:

Mars:	0.366 (and still poorly determined)
Venus:	not measured (perhaps ≈ 0.33)
Mercury:	0.346
Moon:	0.393
Earth:	0.3307

Planetary differentiation

We know from the rotational dynamics of Earth, Moon, Mercury and Mars that their internal density increases rapidly towards their centres. We infer the same for Venus though we really haven't obtained accurate measures of its moment of inertia.



With Earth (left) as model, we expect the terrestrial planets to be differentiated with:

- Iron and siderophile elements like **S**, **Ir**, **Ni**, **Co** at depth in a core.
- Silicates with a composition something like $\mathbf{Fe_xMg_{2-x}SiO_4}$ in a mantle overlying the core.
- A silicate crust rich in lighter metals: **Al**, **Na**, **K**.
- Volatiles including $\mathbf{H_20}$, $\mathbf{N_2}$, $\mathbf{CO_2}$, **Ar** and $\mathbf{O_2}$ forming oceans and atmosphere.

Geophysical differentiation

Properly, ***geophysics*** refers to the physics of the Earth, i.e. “geo-” and ***geochemistry*** to the chemistry of the Earth. Applied to the planets and Solar System and the Universe as a whole, the science of chemistry is sometimes better called ***cosmochemistry***. Presently we don’t use a similar term to describe a physics generalized to the description of processes on the planets. Some authors do use terms like, for example, ***selenophysics*** to describe the physics of the Moon.

Geochemical differentiation: Earlier, we noted how the elemental chemistry of the Earth is distributed with iron and siderophile elements carried to depth into the core leaving the lithophile elements to the mantle and crust.

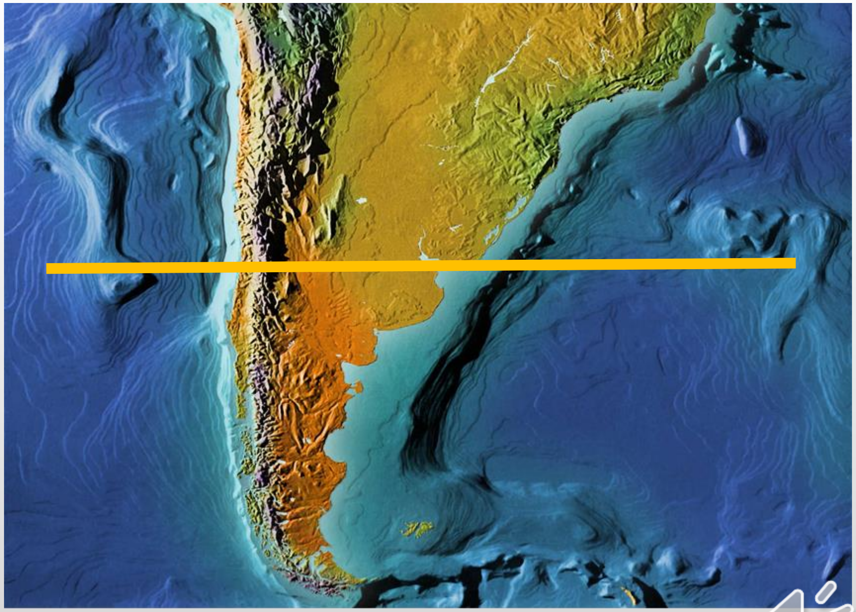
Geophysical differentiation depends on density of materials and their mobility in coming to an equilibrium depth-density profile.

Fluid Nature of the Mantle: Isostasy

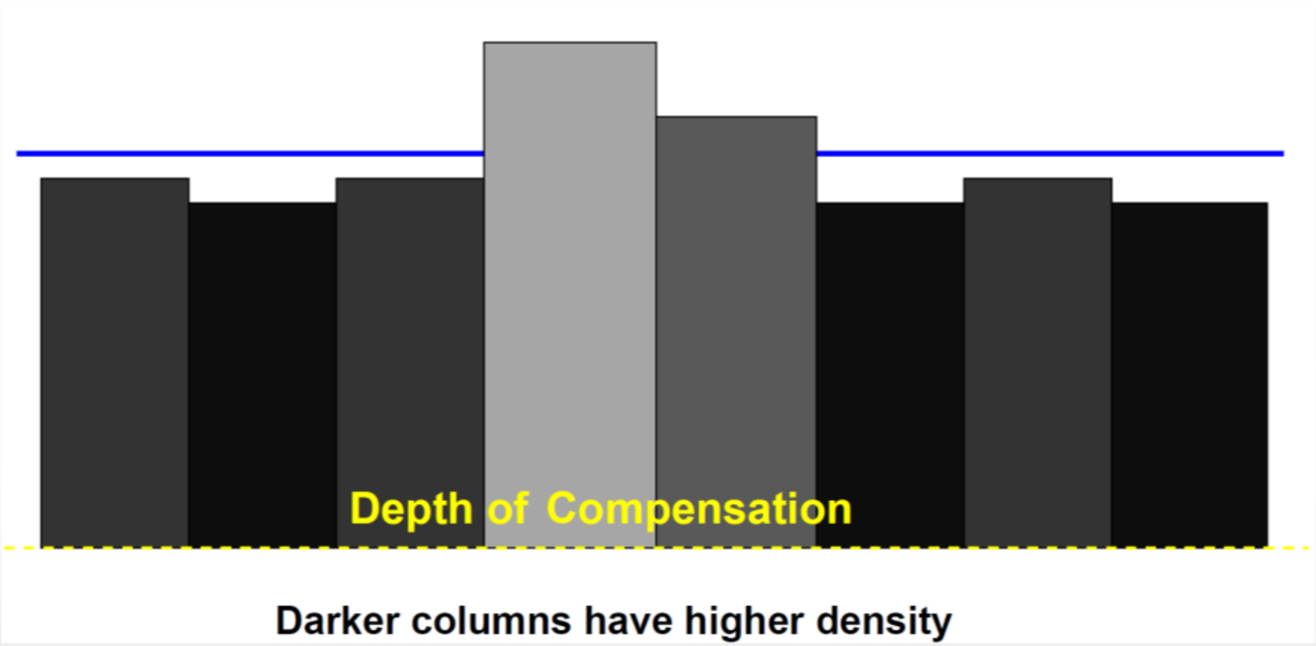
A question posed to members of the **Royal Society** (London) in 1855: Why are the mountains so high and the ocean basins so low?
Two members offered their answers:

- **J.H. Pratt** suggested that the reason was to be found in the density of crustal rocks.

- **G.B. Airy** proposed that high-standing regions are compensated by deeper roots but their densities are similar to those of low stands.

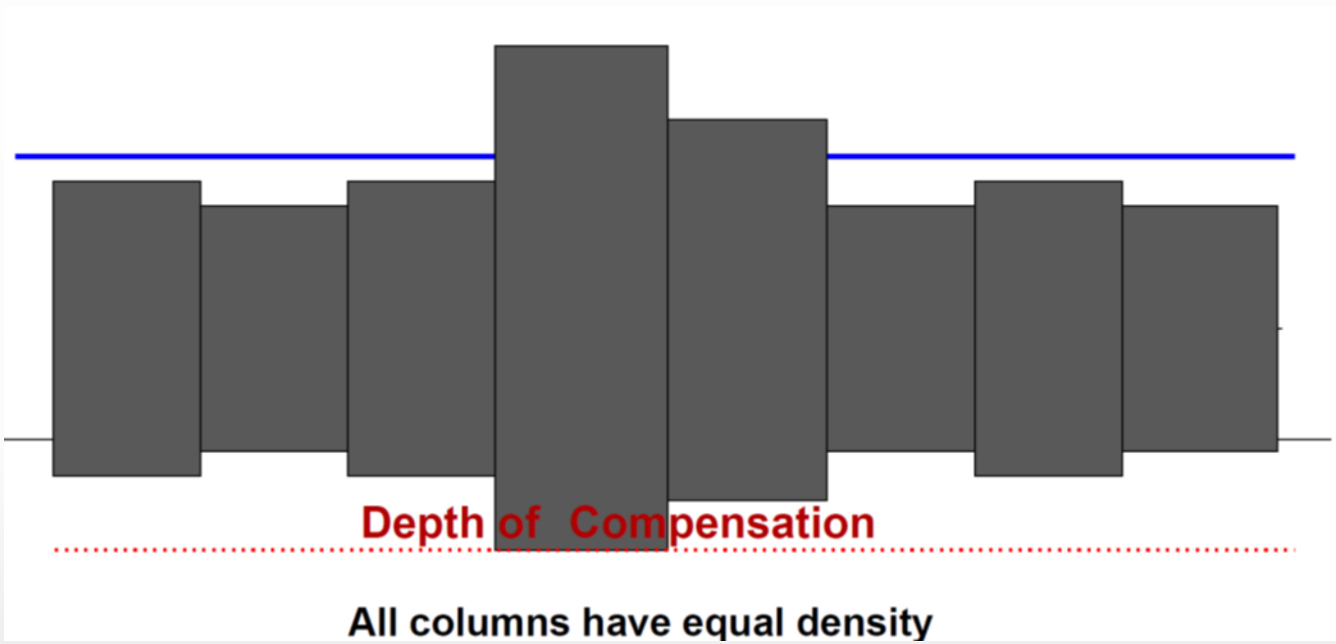


Pratt Model



Passing left to right through a **Pratt model**, traversing from the **east Pacific Ocean**, across southern South America and into the **west Atlantic**, the ocean basin rocks are dense and "***float***" low, the **Andes** are of low density and float high and the **Pampas** are of intermediate density.

Airy Model



Passing left to right through an **Airy model**, traversing from the **east Pacific Ocean**, across southern South America and into the **west Atlantic**, the ocean basin crust is thin and floats low, the **Andes** , with deep roots, float high and the **Pampas** are of intermediate crustal thickness. You might relate this to an “iceberg” model.

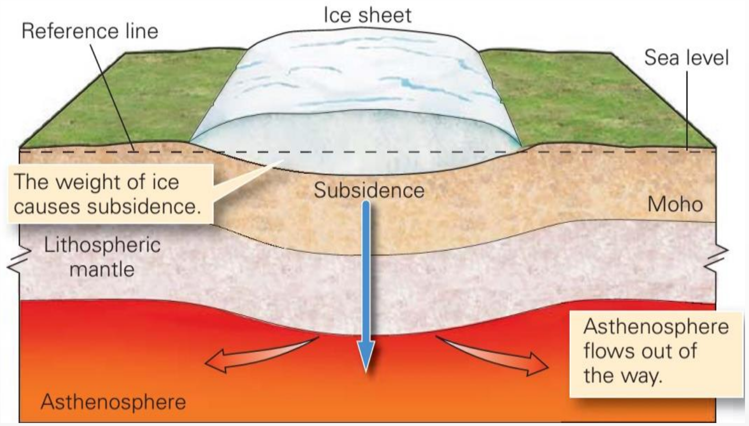
Iceberg?

Source of image:
https://www.pinterest.com/pin/198228821068014258/

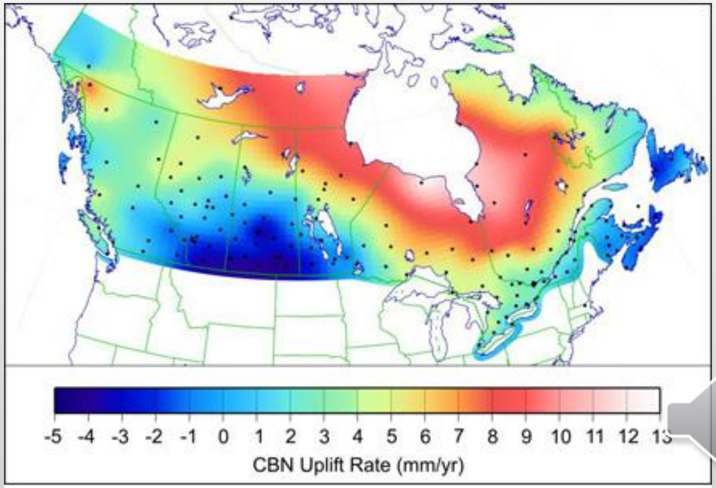
How “fluid” is Earth’s mantle?

We can use varying crustal loads to measure the mantle “viscosity”

- Ice sheets depress the lithosphere into the mantle.
- Slow crustal subsidence follows flow of asthenosphere.
- After ice melts, the depressed lithosphere rebounds.
- The last ice-age glacial rebound continues slowly today.



http://geologylearn.blogspot.ca/2016/03/consequences-of-continental-glaciation.html



https://isostacy.wikispaces.com/rebounding+glacial+area%27s

Viscosity of the fluid mantle

If the "***fluid***" upon which these depressed continental areas are floating could have moved very quickly, it would have infilled under the lowered load quickly. The fluid, though, flows extremely slowly; it has a very high ***viscosity***, usually represented by symbol, η, a measure of its self-stickiness.

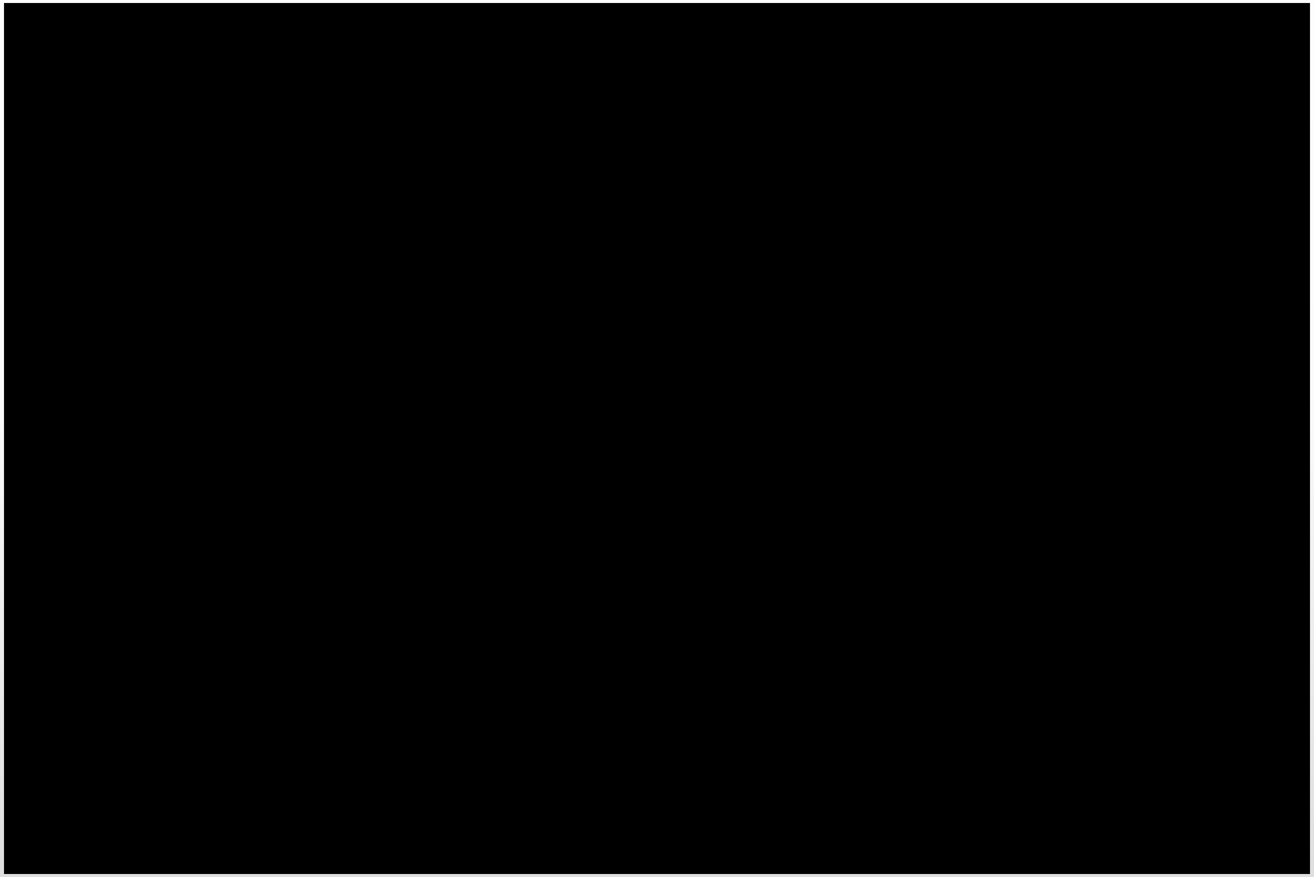
- Water has a viscosity of $\eta \approx 10^{-3}\ Pa\cdot s$, maple syrup, $\approx 2.5\ Pa\cdot s$ and red hot common glass, $\approx 10^{12}\ Pa\cdot s$.
- From the rate of rebound, we can calculate the viscosity of Earth's slow moving, underlying mantle fluid! $\eta \approx 10^{20} - 10^{23}\ Pa\cdot s$.
- It flows extremely slowly, so slowly in fact that we can't even recognize its fluid-like property over short times. At short time scales, we recognize Earth's mantle to be extremely rigid!
- Rigid on short time scale ($\lesssim 100\ yr$), fluid with very high viscosity on geological time scales.

Mantle convection

If the Earth's mantle is fluid, it can flow within itself and the crustal blocks on the surface could drift over it!

- **Abraham Ortelius** in 1596 and **Sir Francis Bacon** in about 1620 recognized that the coastline of North America could be nicely fit against the coastlines of Europe and northern Africa.
- **A. Snider**, in 1858, in France hypothesized that the coasts were once actually joined and have broken and drifted apart.
- **F. Taylor**, **H.D. Baker** and then **Alfred Wegener** described theories of Continental Drift.
- In 1963, **J. Tuzo Wilson**, a physicist at the University of Toronto, wrote a famous article entitled "**Continental Drift**" which was published in the April issue of Scientific American magazine.

A reconstructed past



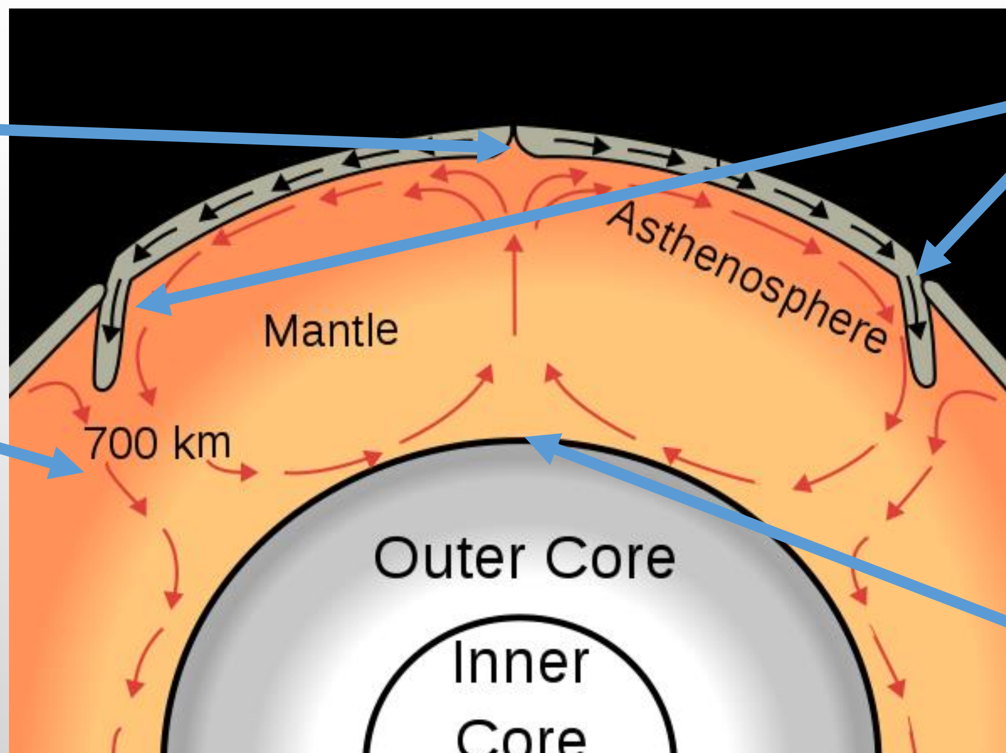
Animation through Phanerozoic: Plate tectonics in action

Mantle Convection

The movement of continents over the surface and the isostatic adjustment to loadings requires the mantle fluids to flow. As well as being "pushed" by near-surface **mechanical forces**, the mantle flow is driven by **convective heat transport** from depths to the surface.

Uplift at ridges produces a ***ridge push*** and ***gravitational sliding***.

Surface cooled materials sink due to high density: ***negative buoyancy***.



Subducting plates produce a mechanical ***slab-pull***.

Heating at core-mantle boundary lowers density: ***buoyancy***.

The adiabatic gradient and convection

The temperature boundary between internal buoyancy in the mantle and negative buoyancy is determined by the "***adiabatic gradient***". We know that when a material is squeezed under pressure and if no heat is allowed to enter or exit the material, its temperature rises. You might look at this as the heat being compressed into a smaller volume and so a temperature rise.

If we impose a small change in pressure, ΔP, on a material of density, ρ, having thermal expansivity at constant pressure of α_p and heat capacity at constant pressure of C_{Hp}, the pressure increase will produce a temperature increase:

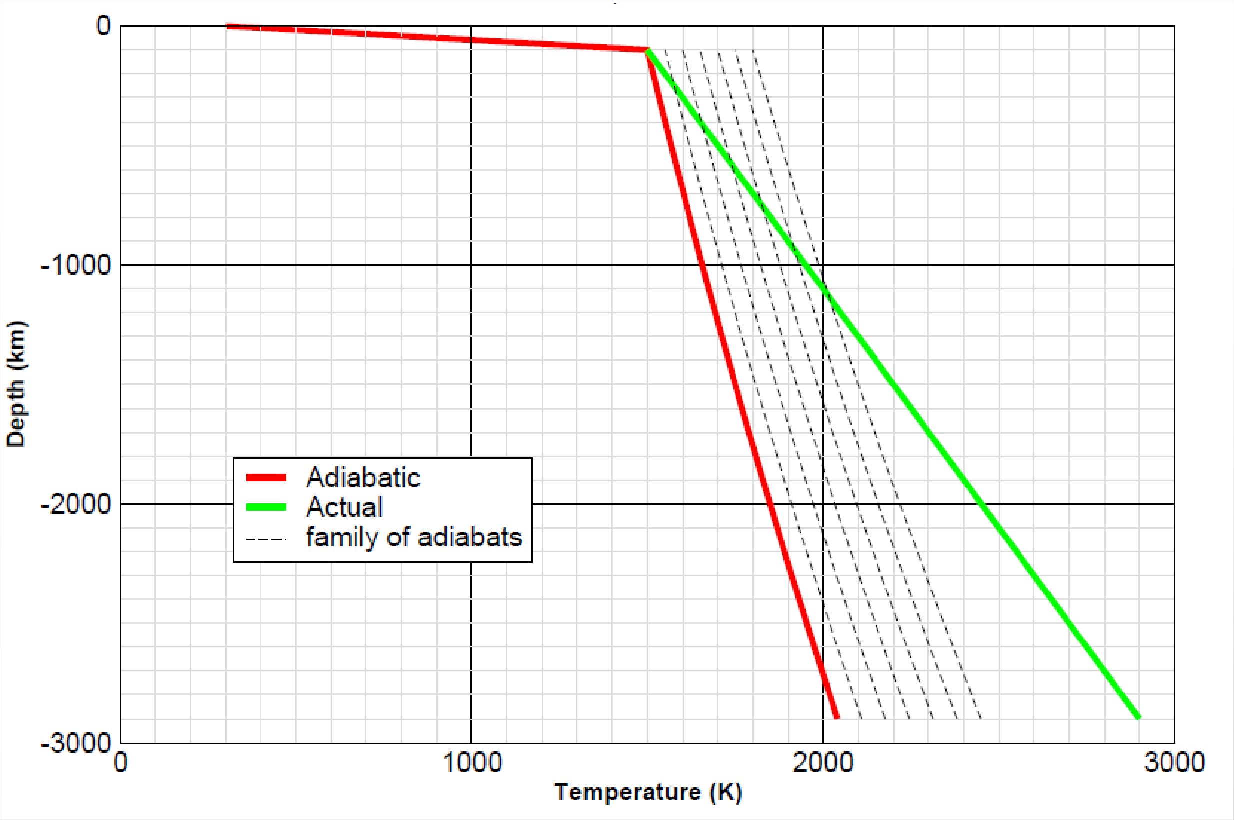
$$\Delta T = \frac{\alpha_p T}{C_{Hp}\, \rho} \Delta P$$

Within the mantle pressure increases with depth and so temperature increases with depth locally as:

$$\Delta T = \frac{g\, \alpha_p T}{C_{Hp}} \Delta z$$

The Temperature Gradient

$$\frac{\Delta T}{\Delta z} = \frac{g\,\alpha_p\,T}{C_{Hp}}$$



Rayleigh Number

The Rayleigh Number, usually designated $\boldsymbol{R}$, is a dimensionless scale determined as:

$$\boldsymbol{R} = \frac{\boldsymbol{Bouyancy\ forces}}{\boldsymbol{Viscous\ forces}}$$

$\boldsymbol{R}$ is a fluid-mechanical measure of the vigour of non-turbulent convection. For Earth, it is thought that $\boldsymbol{R > 2380}$ is required for full mantle convection.

The observed vigour of Earth's mantle convection suggests $\boldsymbol{R > 10^5}$.

Convection is cooling the Earth's interior, moving heat from depth to be released at the surface. Heat is being released at an average rate of about $45\ \mathrm{mW/m^2}$. This vigourous convection and release of heat should continue for billions of years.

Continents through time

Plate Tectonics (continental drift) is evidence of vigourous mantle convection suggesting "**super-adiabatic**" temperature gradient.

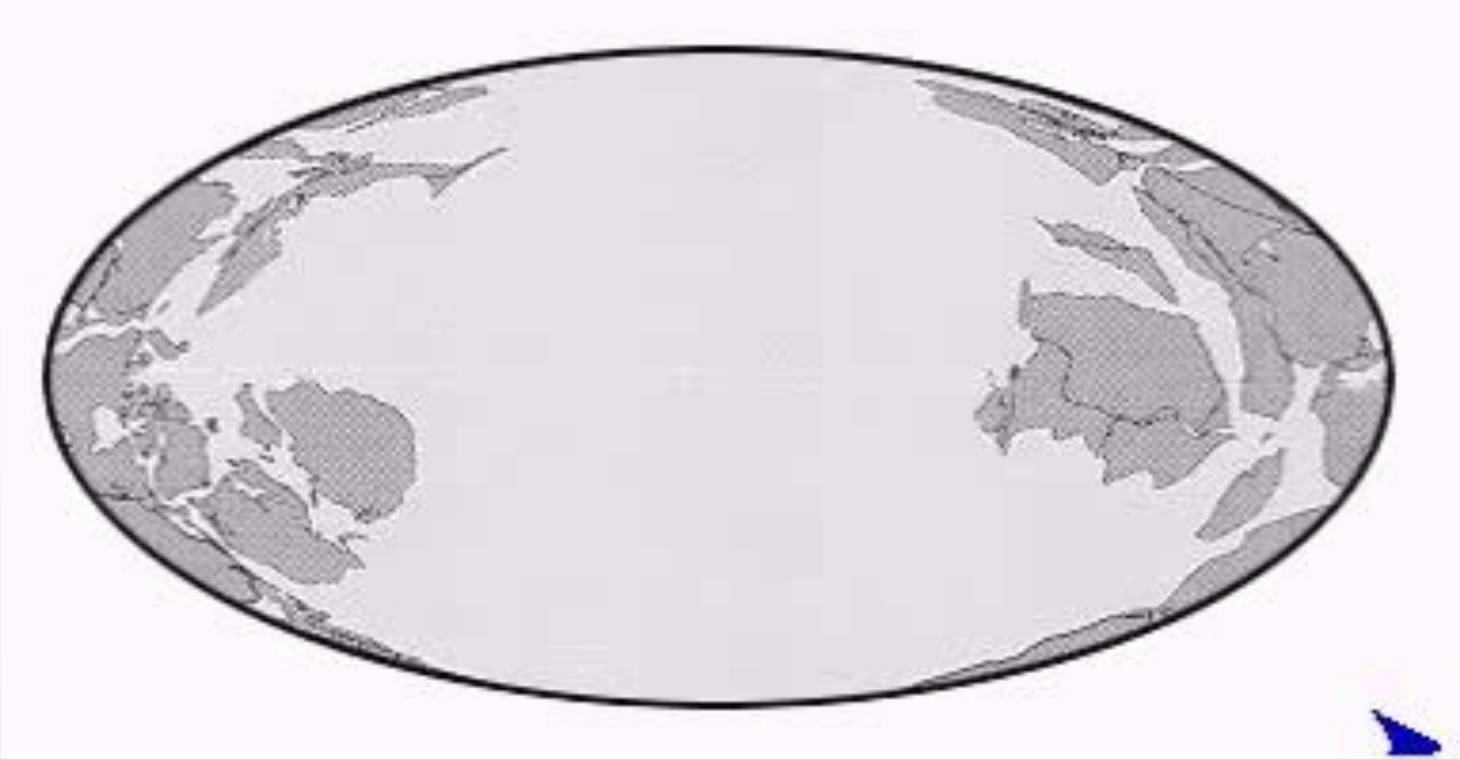


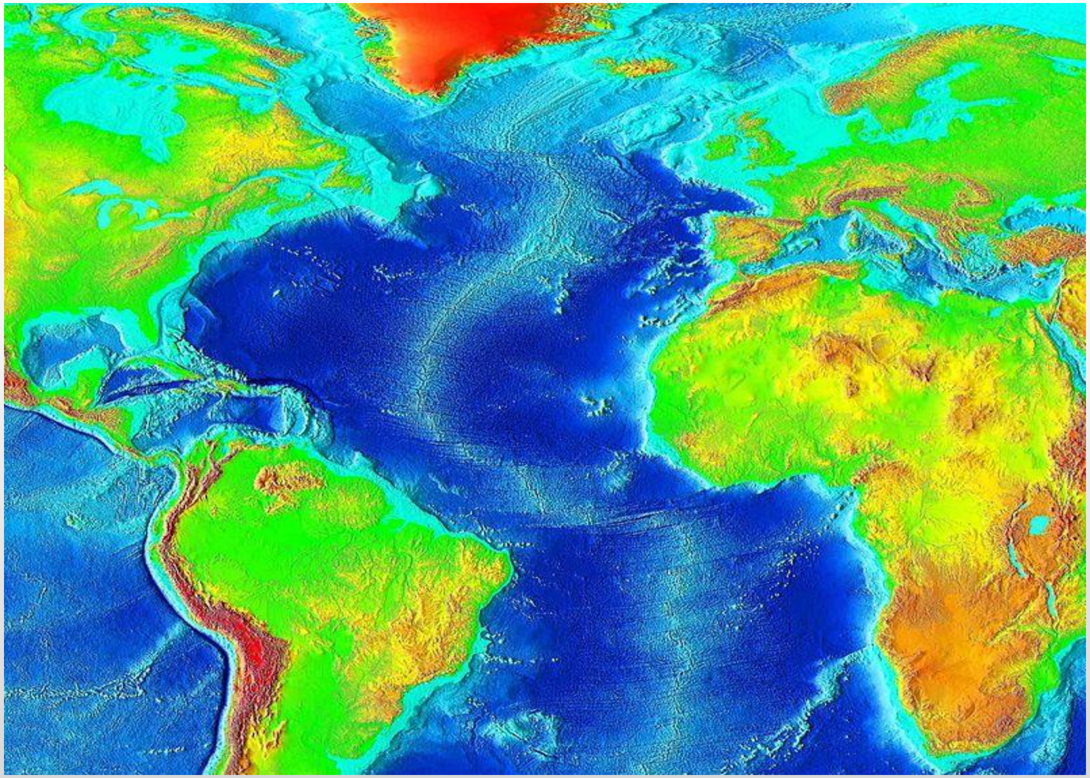
Plate Tectonics

A YouTube educational video: https://www.youtube.com/watch?v=ZTRu620bIsE

Plate tectonic topography

Our Earth, as a result of the mantle circulation and resulting plate tectonics has a bimodal distribution of surface (rock) topography.

Continents stand high; ocean basins lie low!



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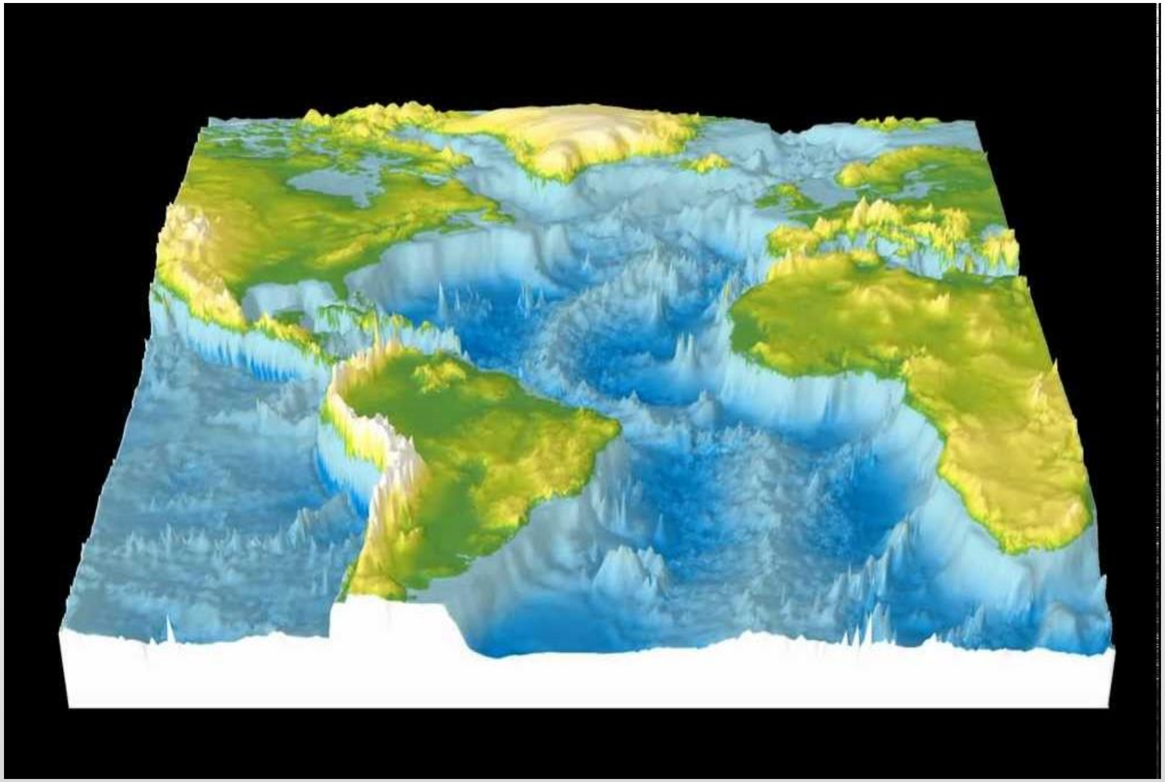


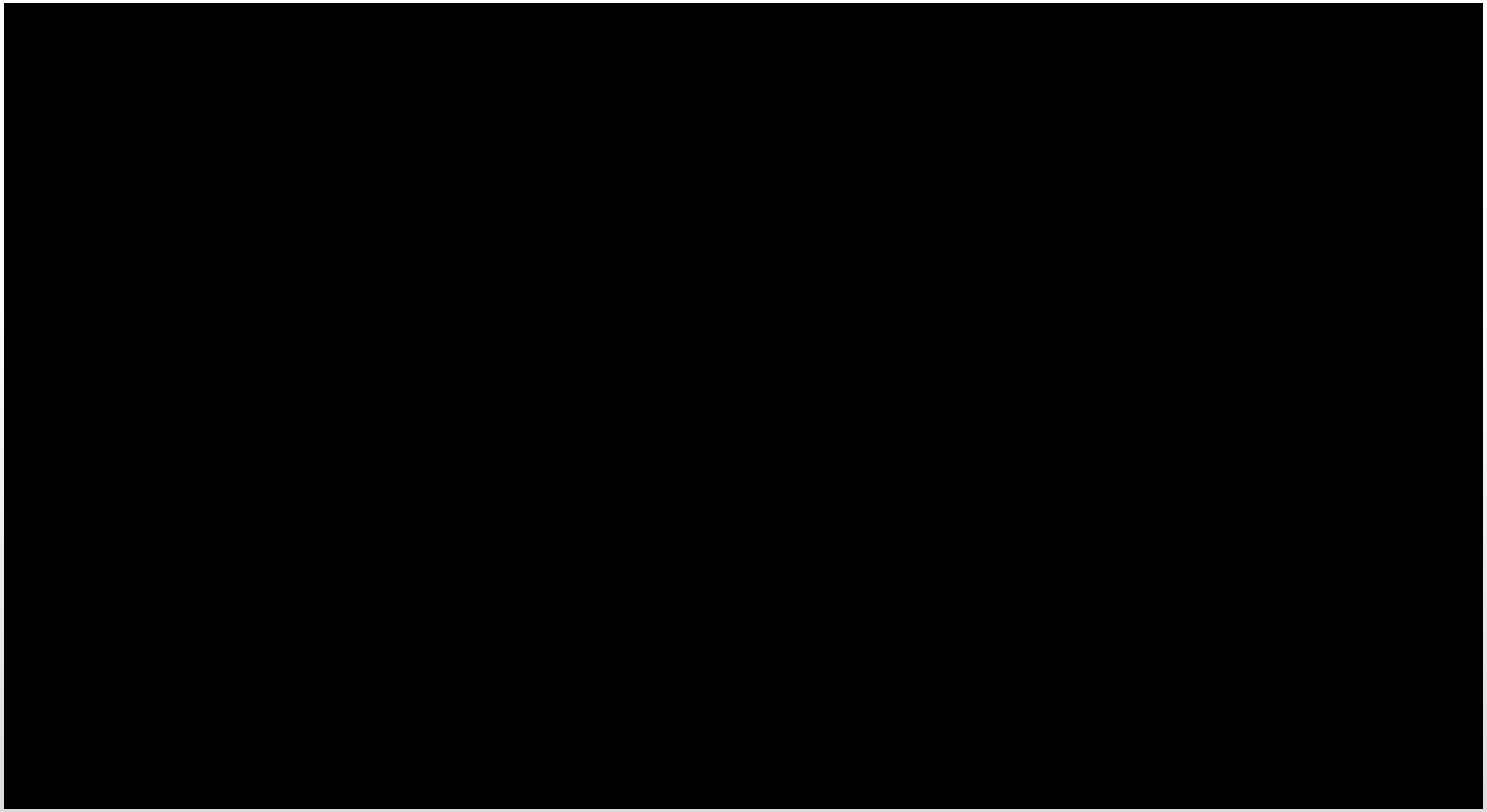
Other planets or moons?

How would we recognize a fluid mantle in convection on other planets and moons? Evidence?

Let us look for:

- **Plate tectonic topography (deep basins and high continents)**
- **Volcanism (active and ancient)**
- **Non-isostatic surface**
- **Faulting and seismic activity**
- **Thrust faulting and ridge spreading**
- **Magnetic imprinting of surface rocks**

Mercury

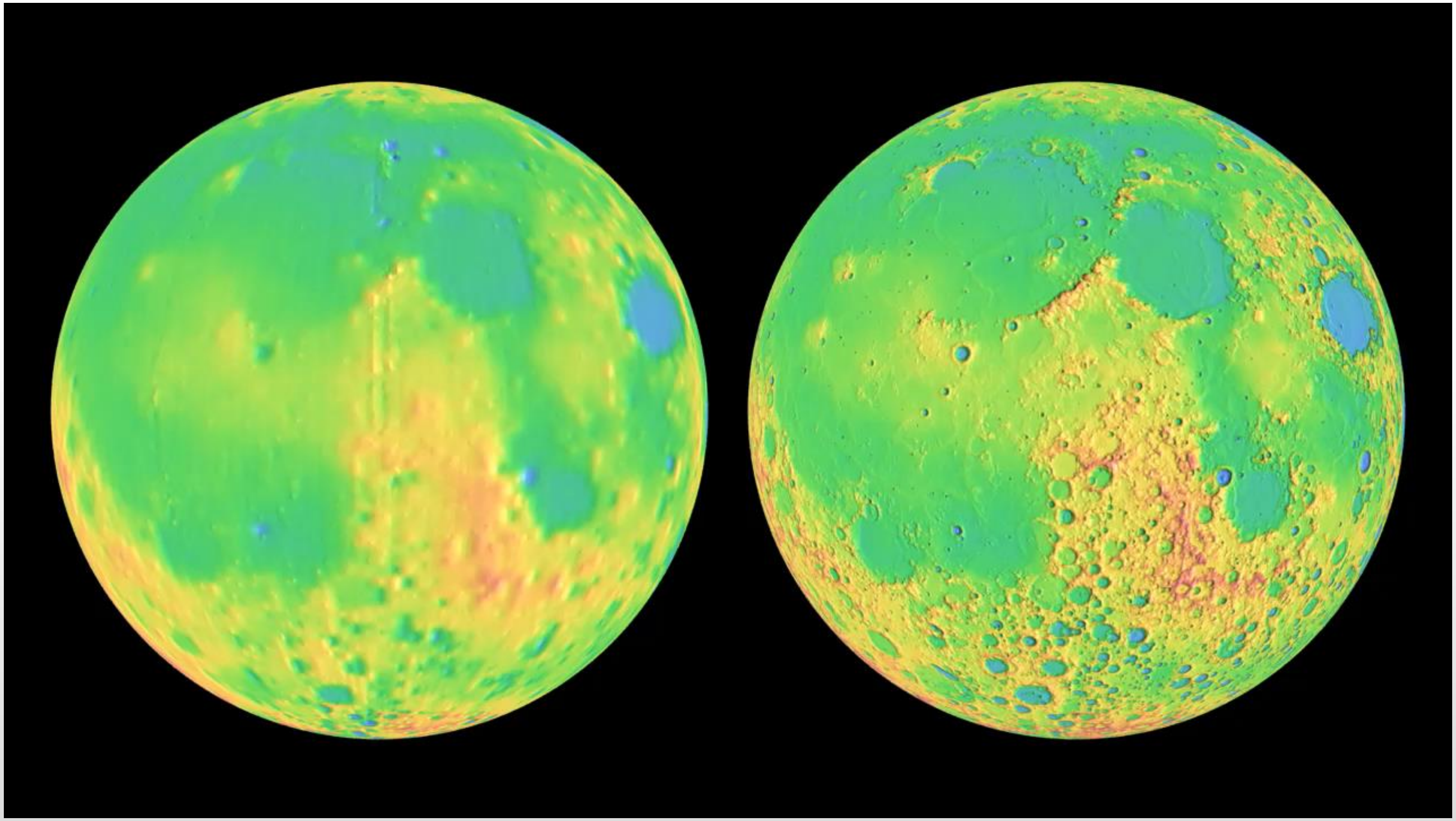


https://www.usgs.gov/news/first-global-topographic-map-mercury-released

Venus

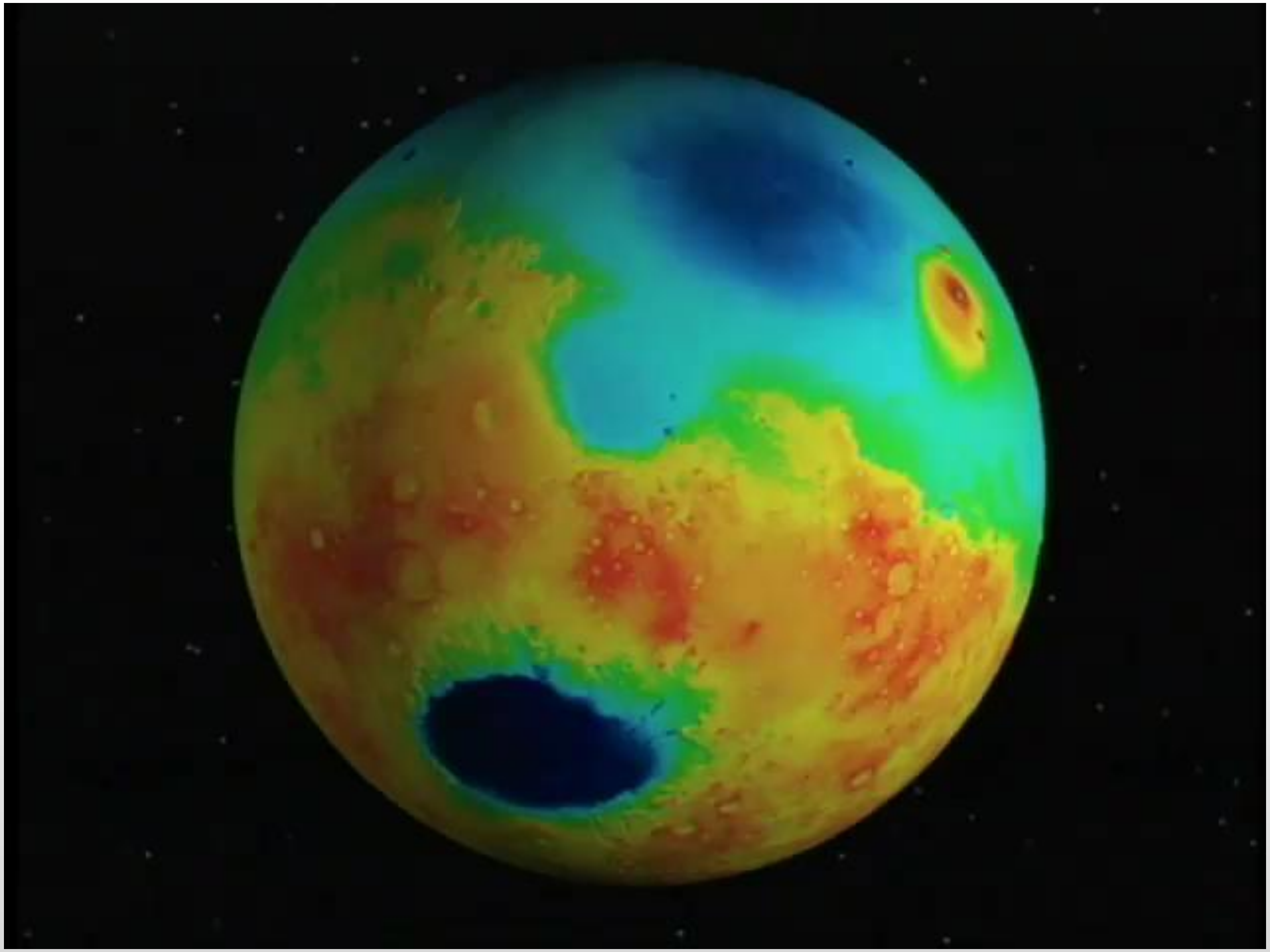
https://svs.gsfc.nasa.gov/vis/a000000/a003700/a003728/venus_topo_720p.mp4

Moon



https://svs.gsfc.nasa.gov/3727

Mars



https://svs.gsfc.nasa.gov/1090

Exploring our Solar System

BBC Horizon: Terra Firma