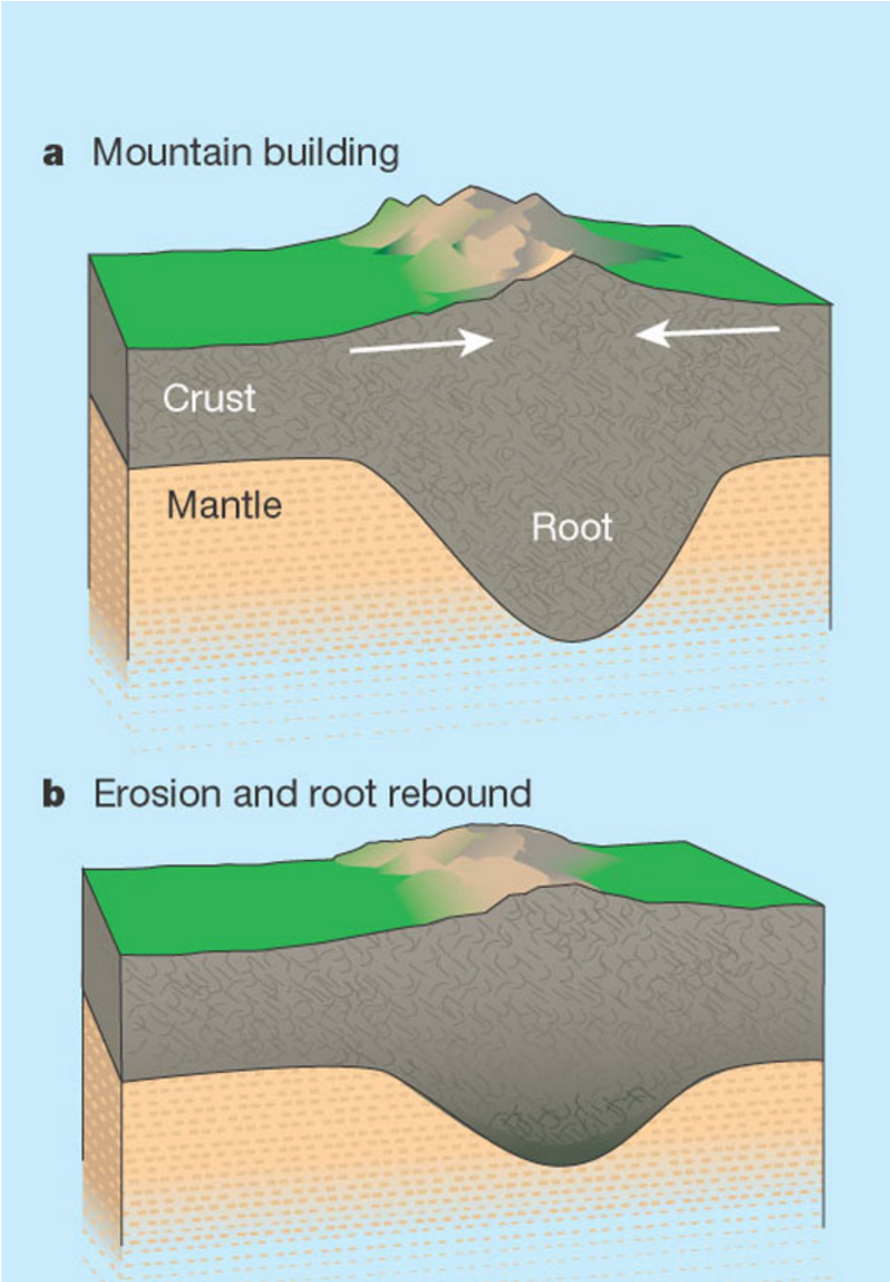
Course Business

- Today: isostasy and Earth rheology, paper discussion
- Next week: sea level and glacial isostatic adjustment
- Email – did you get my email today?
- Class notes, website
- Your presentations:
  - November 3rd and 10th
  - 30 minutes each, plus additional time for questions
  - Individual or in pairs (say what each contributed if in pairs)
  - 40% of final grade

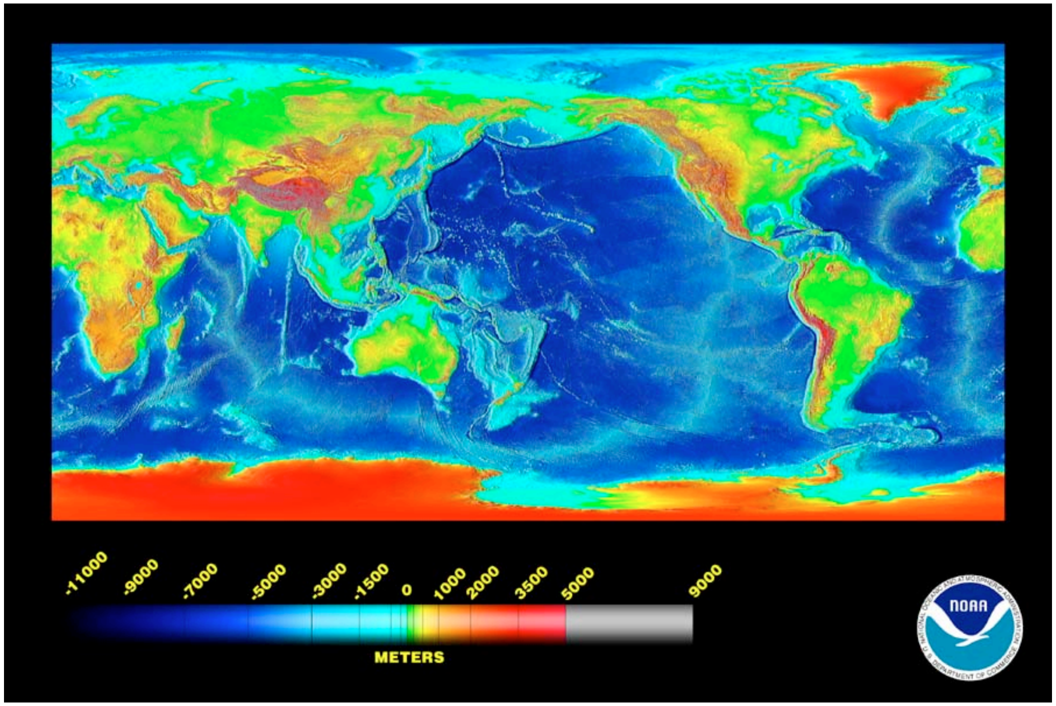
Presentations

- Required elements (90% of grade):
  - Reference list, including foundational and most recent work (the references you would include in the introduction section to a paper that will be reviewed by experts in the field)
  - Discussion of relevant data, models, physical processes at play, and methods of analysis
  - Slides (turn in a PDF and any auxilliary files)
  - Optional: your own analysis
- Feedback (10% of grade): You will be asked to give written positive and constructive feedback for each presentation. We will collect your feedback forms, grade the quality of the feedback, and then pass it on to the presenters along with our own feedback. (Giving useful feedback is an essential life skill – I encourage you to read up on how to do it well before the presentations start)
- You will be assessed on the content and quality (clarity, structure, use of visuals, engagement of the audience, etc.) of your presentatio

Isostasy

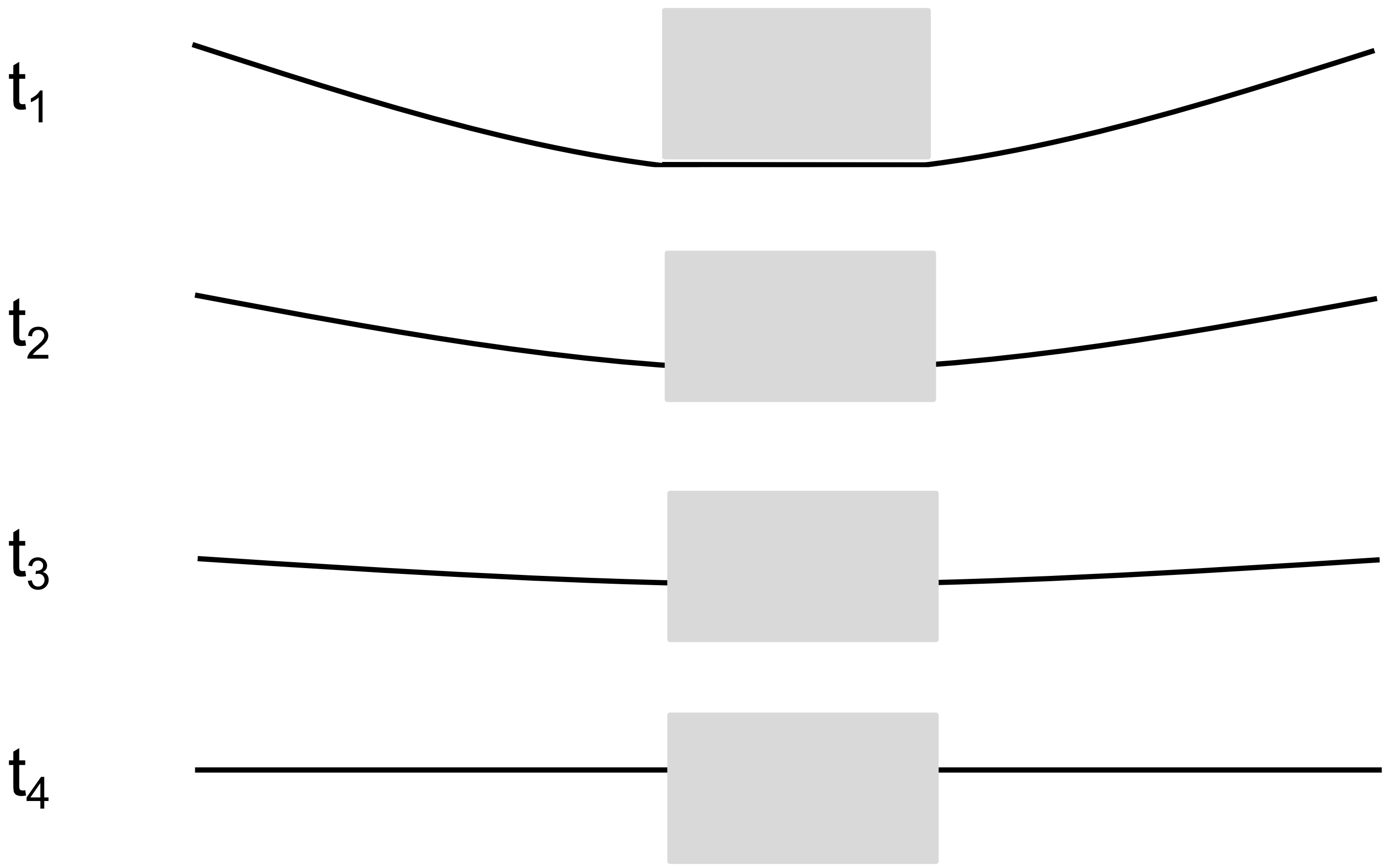


e.g. mountain roots
ocean bathymetry
"eustatic" long term sea level changes



Isostasy

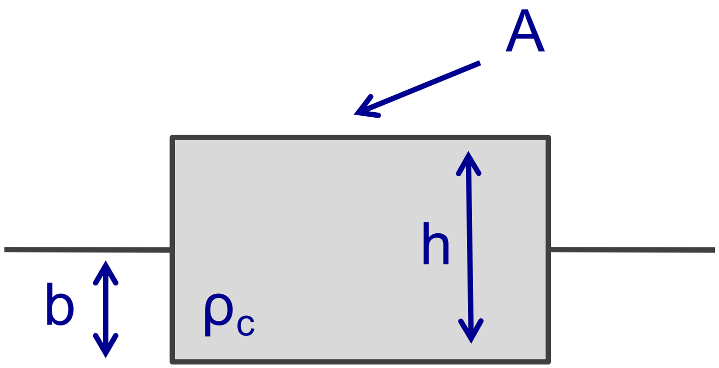
How can we say a continent floats?



On very long time scales, one can consider that continental crust is embedded in a fluid mantle

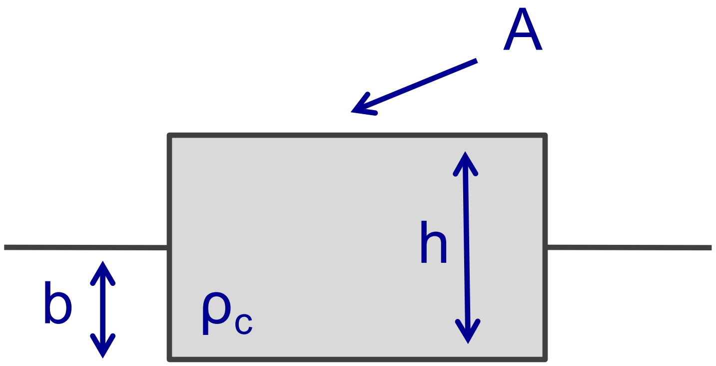
Isostasy

Now, treat the continent as a block of wood floating on a sea of mantle rock:



What is the equilibrium position of the continent?

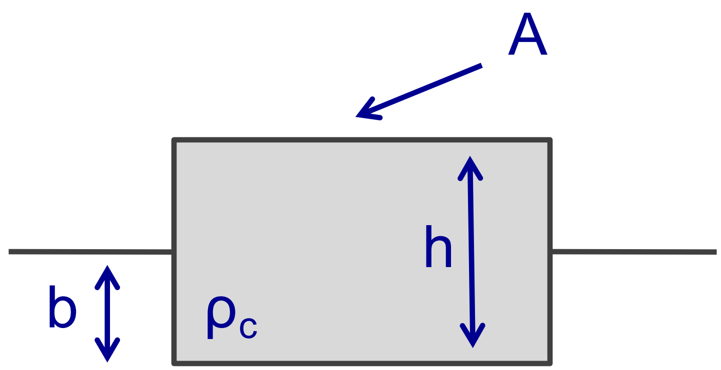
Isostasy



1. Apply Archimedes Principle

What's that?

Isostasy



1. Apply Archimedes Principle

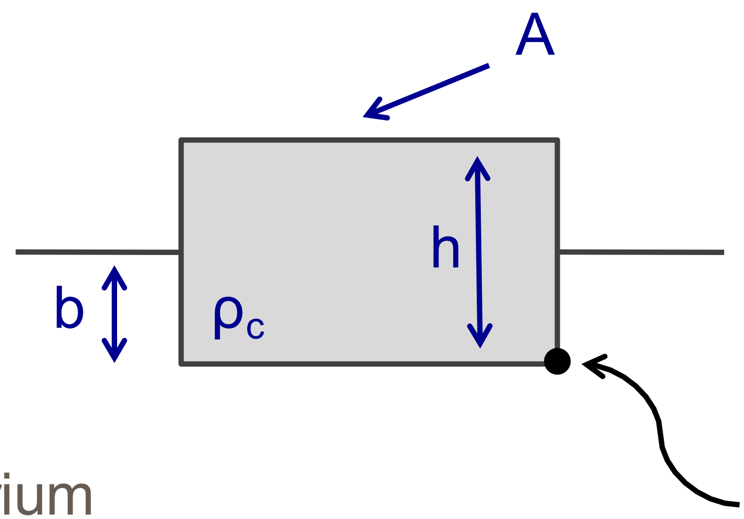
Buoyancy force on material = weight of material it displaces

$$\rho_c h A g = \rho_m b A g$$

$$\boxed{\frac{b}{h} = \frac{\rho_c}{\rho_m}}$$

$$\boxed{h - b = h\left[1 - \frac{\rho_c}{\rho_m}\right]}$$

Isostasy



What is the stress here?

2. Hydrostatic Equilibrium

Must be equal on both sides for the system to be in equilibrium!

CONTINENT SIDE $\rho_c h g = \rho_m b g$ MANTLE SIDE

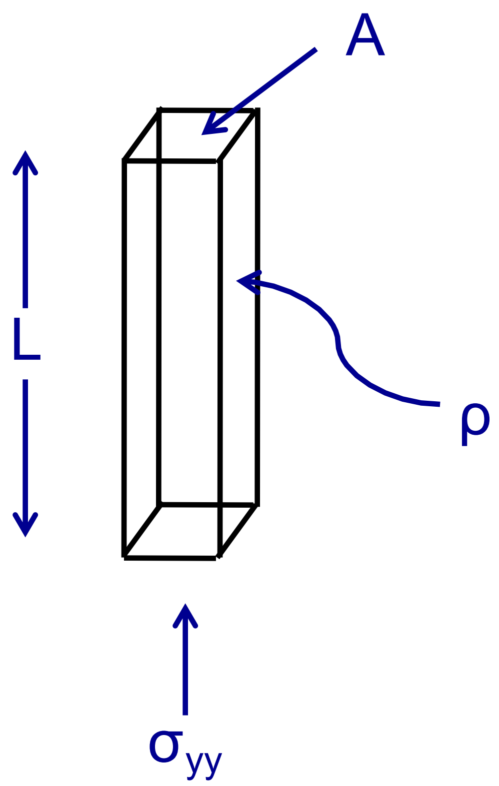
$$\frac{b}{h} = \frac{\rho_c}{\rho_m}$$

Archimedes Principle → simple restatement of hydrostatic equilibrium

Isostasy → application of hydrostatic equilibrium to continental crust!

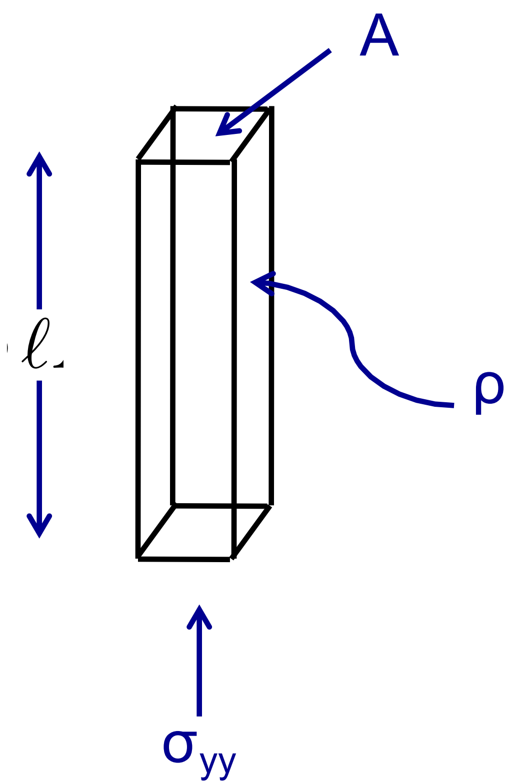
Isostasy

What is the stress (force per unit area) acting on a column of material in equilibrium?



Isostasy Applied...

What is the stress (force per unit area) acting on a column of material in equilibrium?



Weight of column = $\rho \ell A g$

In equilibrium forces balance:

$$\sigma_{yy} A = \rho \ell A g$$

$$\boxed{\sigma_{yy} = \rho \ell g}$$

LITHOSTATIC STRESS OR PRESSURE

What is the lithostatic stress at the bottom of 35km continental crust?

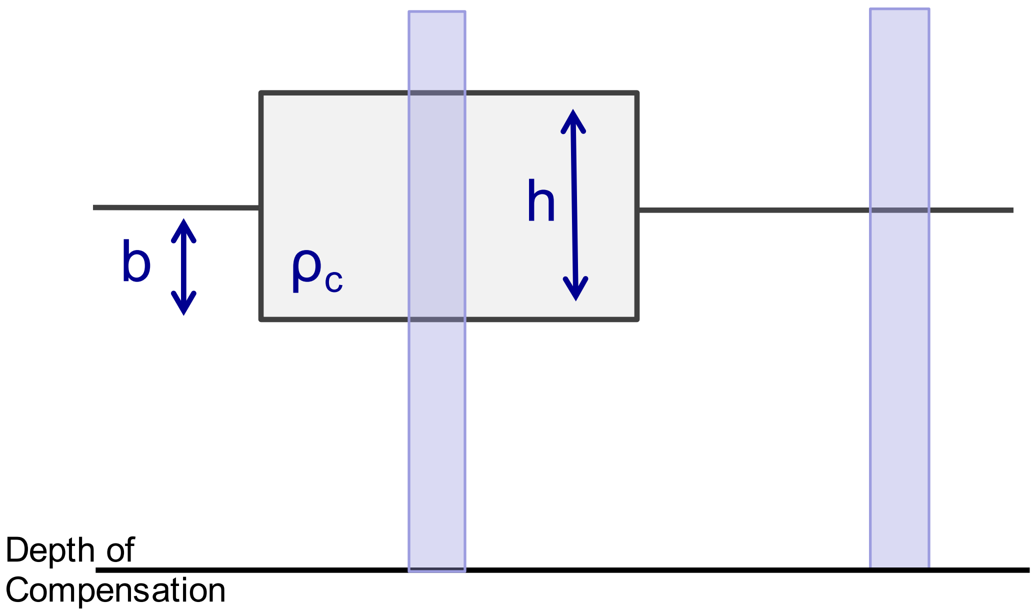
σ_{yy} = 2750 kg/m^3 × 3.5 × 10^4 m × 10 m/s^2

σ_{yy} = 9.6 × 10^8 Pa = 960 MPa = 9.6 kbars

Isostasy Applied...

3. Isostasy in Geophysics

Isostasy: The total mass in any vertical column through the crust must be equal no matter where the column is (when in isostatic equilibrium)



Column 1 Weight:

$$\rho_c h A g + \rho_m (D - b) A g$$

Column 2 Weight:

$$\rho_m D A g$$

Equate:

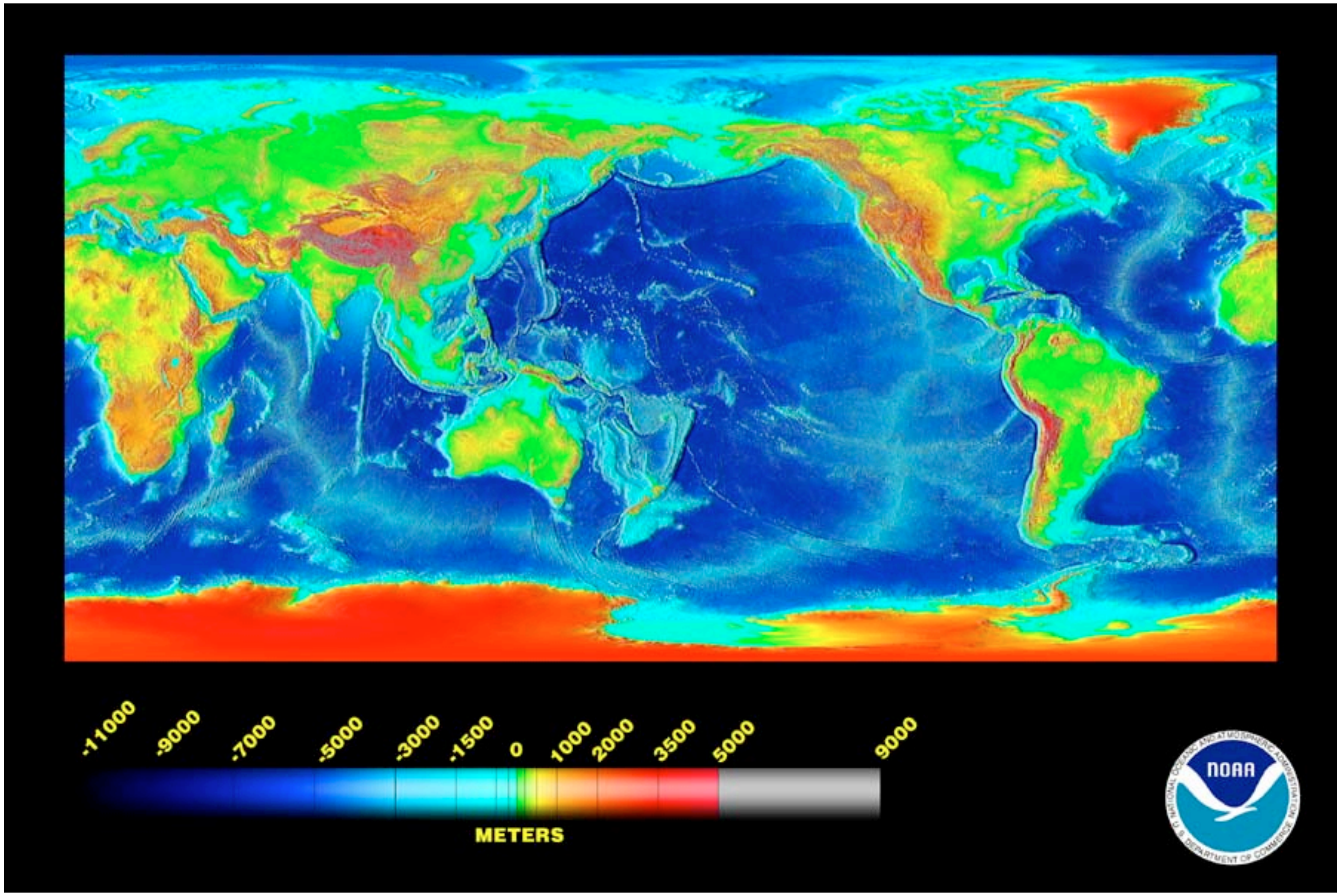
$$\rho_c h A g + \rho_m (D - b) A g = \rho_m D A g$$

$$\rho_c h + \rho_m (D - b) = \rho_m D$$

$$\boxed{\frac{b}{h} = \frac{\rho_c}{\rho_m}}$$

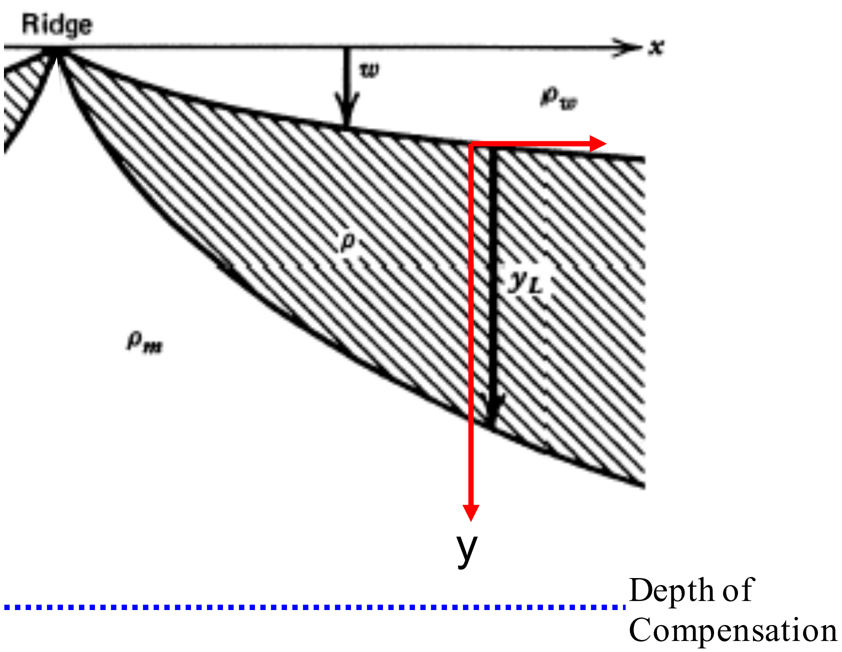
Isostasy Applied...

Application to Cooling of the Oceanic Lithosphere and Ocean Bathymetry



Isostasy Applied...

Application to Cooling of the Oceanic Lithosphere and Ocean Bathymetry



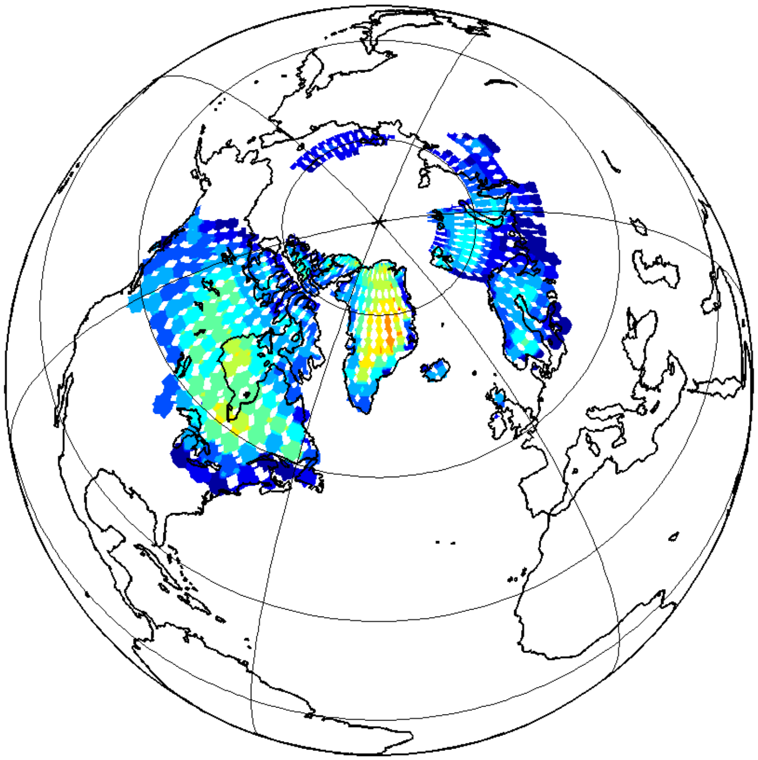
For next week:
Apply isostasy (and some other concepts) to come up with an expression for the depth of water as a function of distance from the ridge, w(x).

Hint: Here are some useful keywords…
Thermal contraction
Coefficient of thermal expansion
Error Function
Square root age law

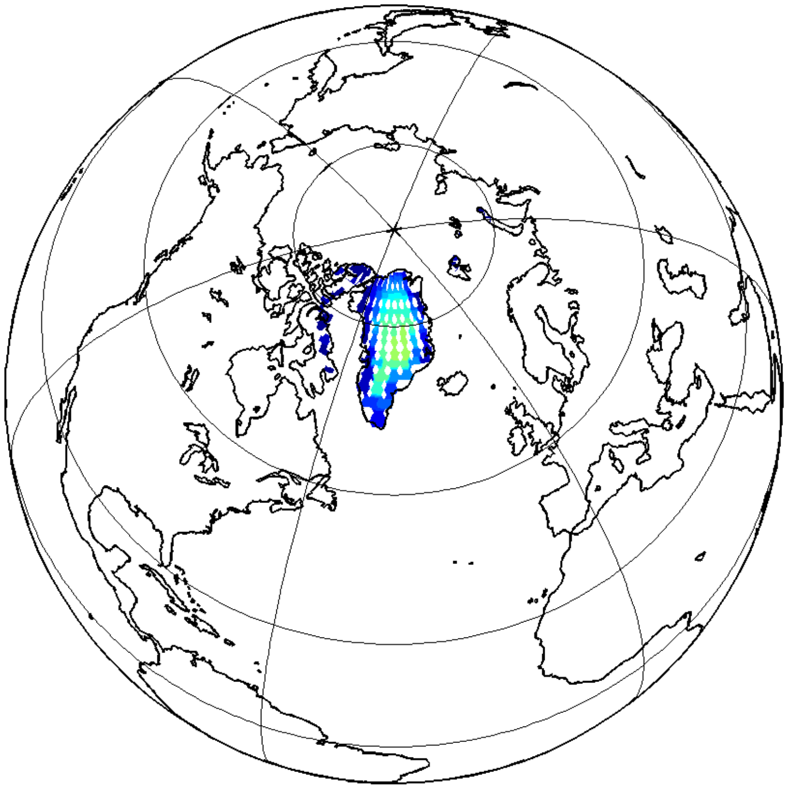
Isostasy Applied...

At the Last Glacial Maximum (LGM), ~ 20 ky ago, ~3 km of ice covered North America. Assuming the continent is close to isostatic equilibrium today, how much lower was Montreal at the LGM?

LGM

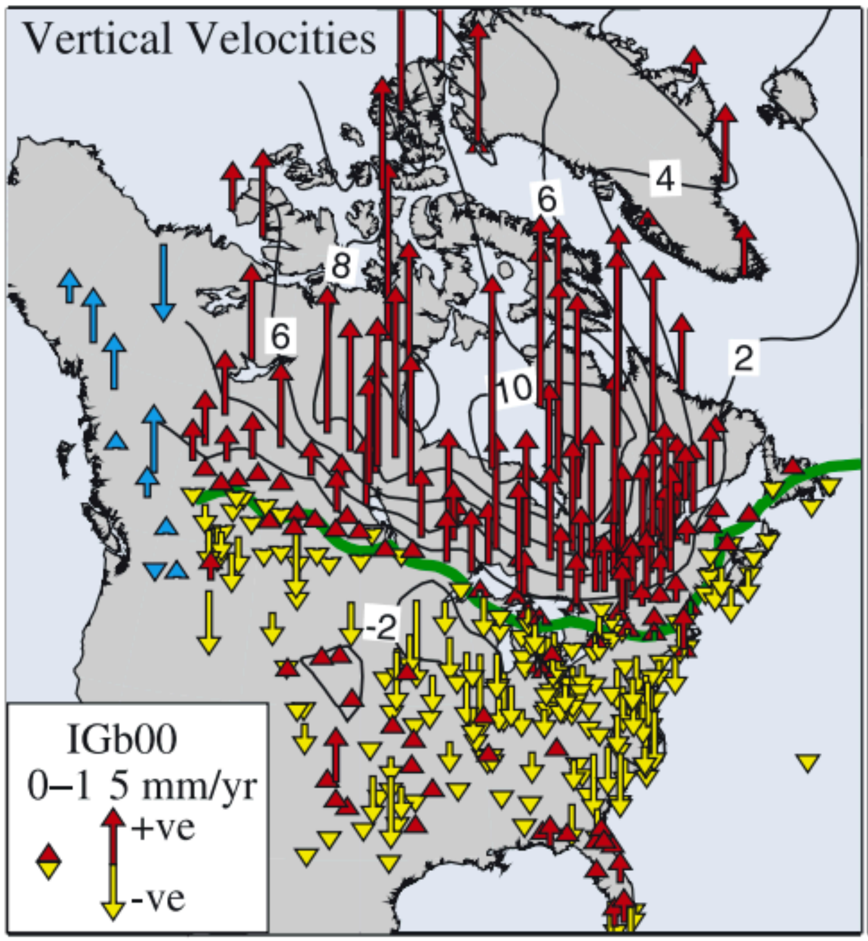


Modern



Isostasy Applied...

But! North America is not in isostatic equilibrium! [still O(100 m) of rebound to go]

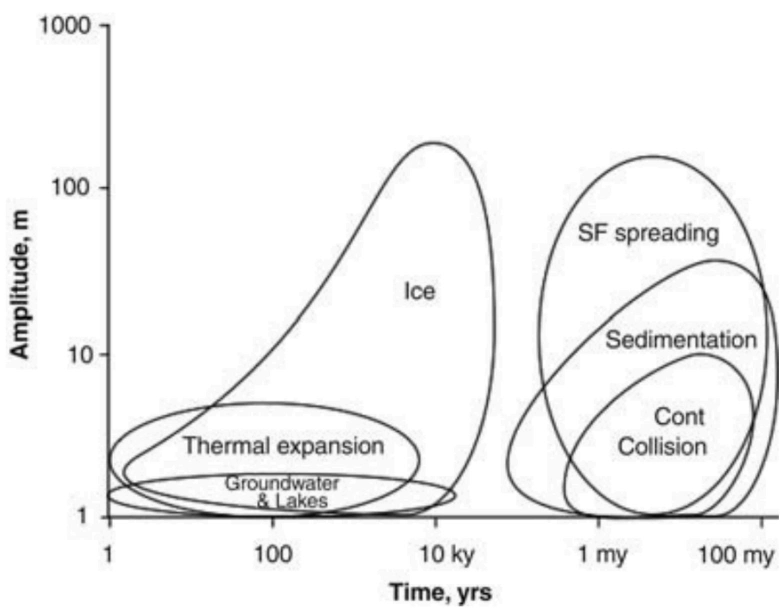


GPS vertical velocities from Sella et al. (2007)

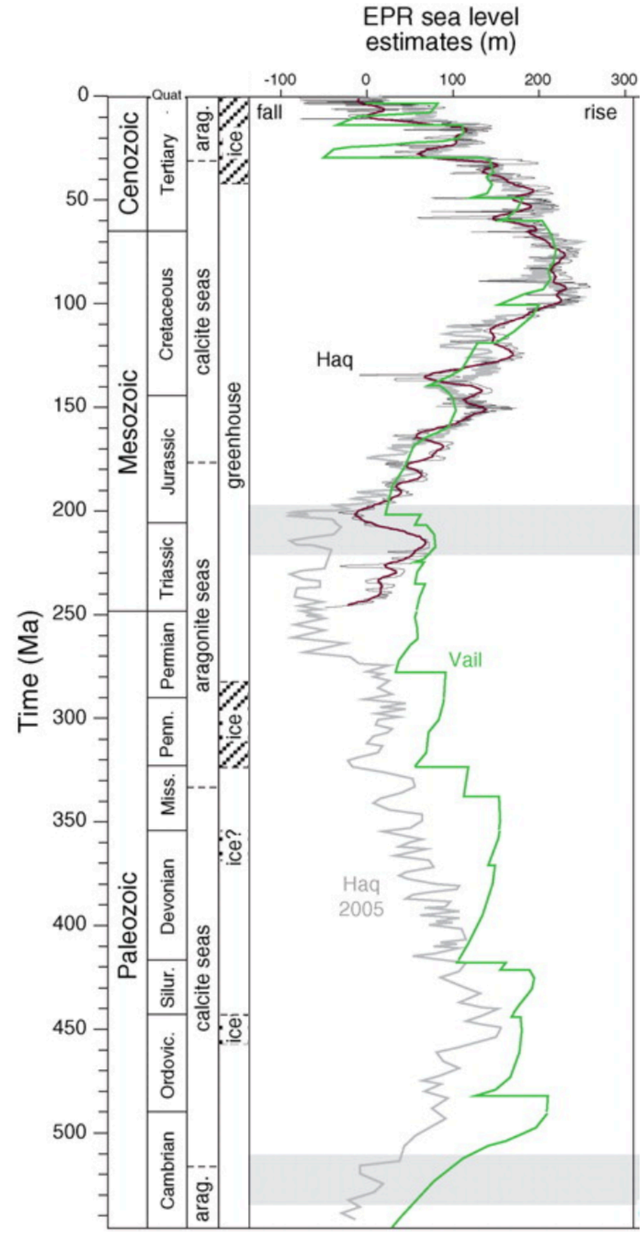
Isostasy Applied...

We may come back to isostasy in the context of discussing long term sea level over timescales of millions of years and longer. Topics here include:

- cautionary tales in sequence stratigraphy
- interpreting the Exxon Vail Curves as "eustatic"
- changes in sea level caused by changes in tectonic plate spreading rates, continental collisions, dynamic topography, …



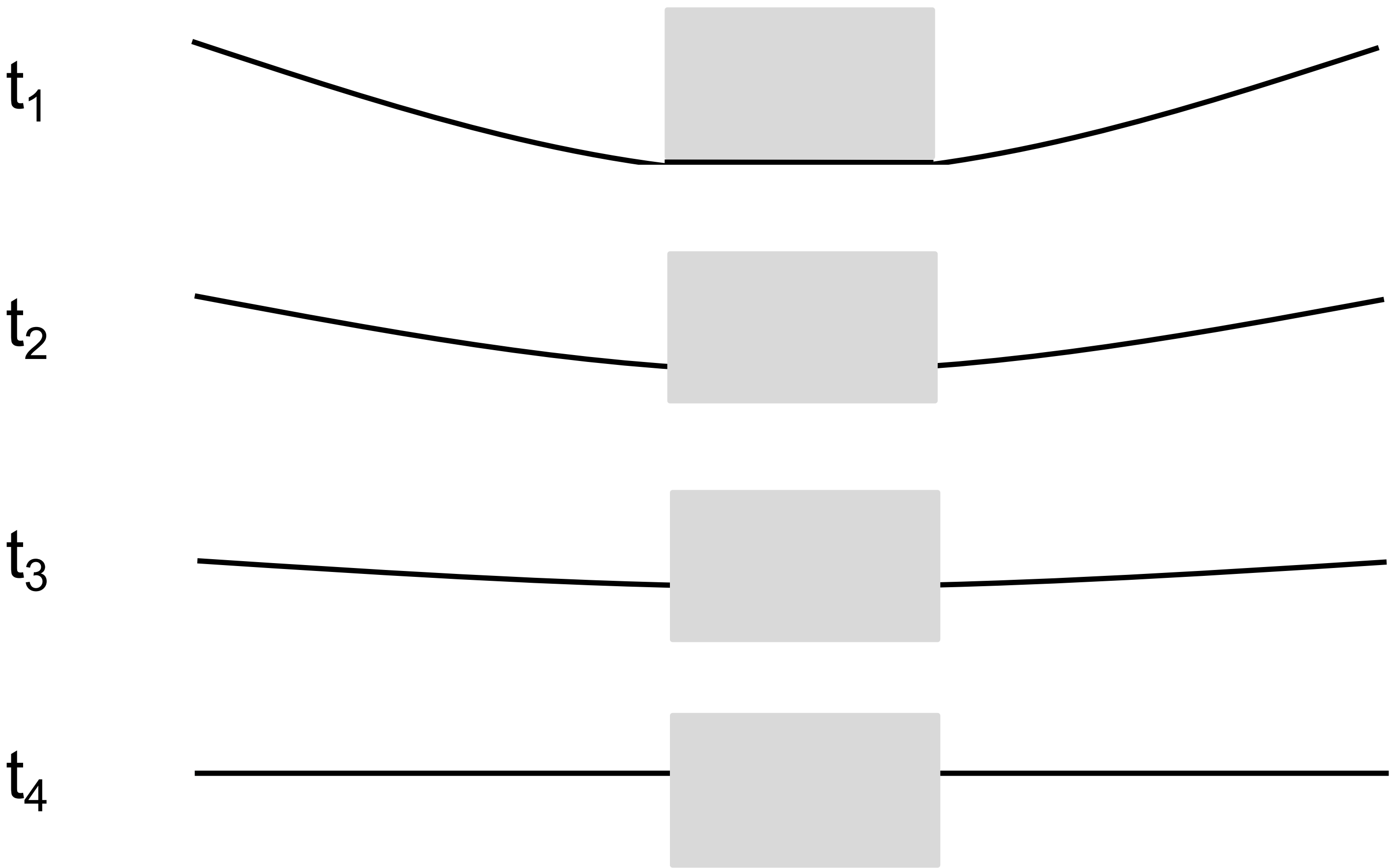
But for now, on to Earth rheology on shorter timescales.



Miller et al. [*Science*, 2005]

Isostasy → Rheology

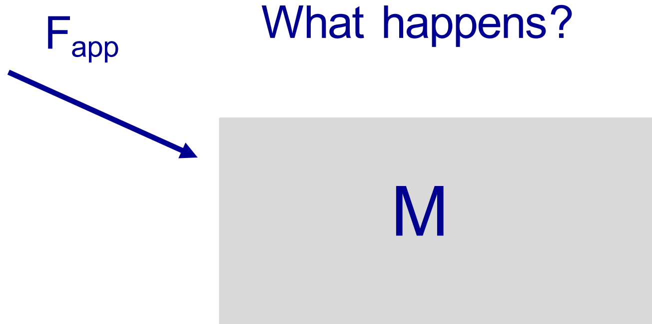
"T'ain't what you do it, it's the way that you do it."
Written by Melvin Oliver and James Young (1939),
first sung on recording by Ella Fitzgerald



Earth Rheology

Rheology: The macroscopic response of a material to stress (applied forcing)

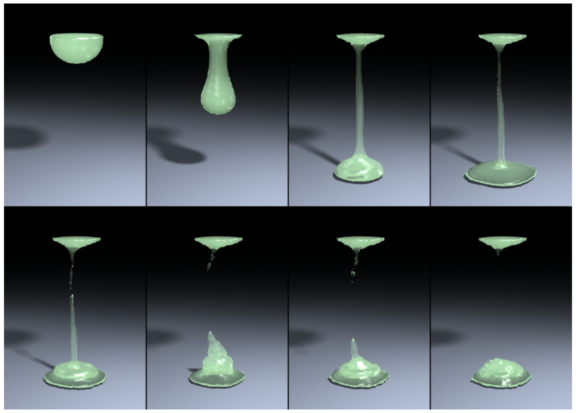
What happens?



Brittle

Viscous

Elastic



Viscoelastic

What is the Earth’s rheology?

Why does it matter?

Earth Rheology

RHEOLOGICAL CLASSIFICATIONS

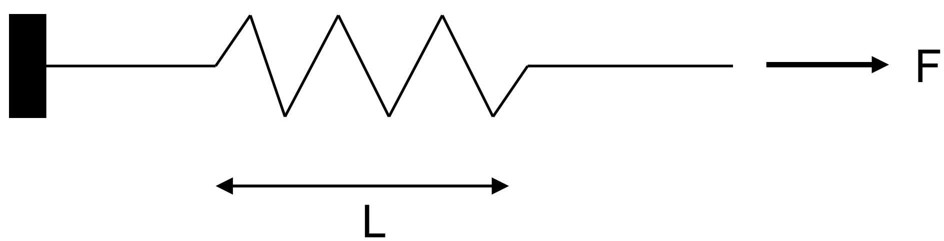
	BRITTLE	ELASTIC	VISCOELASTIC	VISCOUS
Application of Force				
Instantaneous Response:	fracture	Instant displacement (strain)	initial displacement	no instantaneous response
Long-Time Response:	no subsequent change in shape of fragments	no subsequent change in shape	gradual change in shape	gradual change in shape
Removal Of Force	pieces stop moving	oscillations & eventually back to original shape	oscillations & eventually PARTIAL return to original shape	displacement ceases & NO return to original shape

Earth Rheology

1-D MODELS OF RHEOLOGY

Stress:	force/unit area	F
Strain:	δlength/length	e

1) Elastic spring:



Hooke's Law states: $$F = k\frac{\Delta L}{L} = ke$$

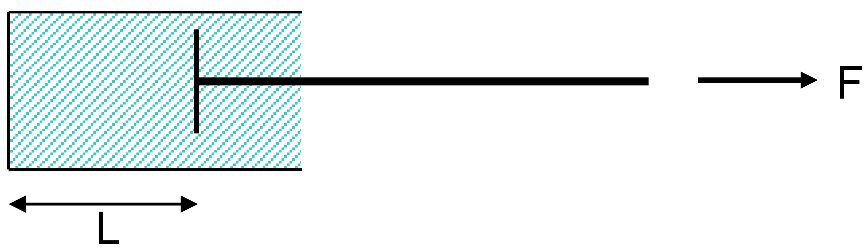
Stress is proportional to strain & constant of proportionality
'k' is spring constant (called "Young's modulus" in materials)

Earth Rheology

1-D MODELS OF RHEOLOGY

2) Viscous Dashpot

Pot filled with viscous fluid & piston



Newtonian fluid:

$$F = \eta \frac{d}{dt}\left(\frac{\Delta L}{L}\right) = \eta \frac{de}{dt}$$

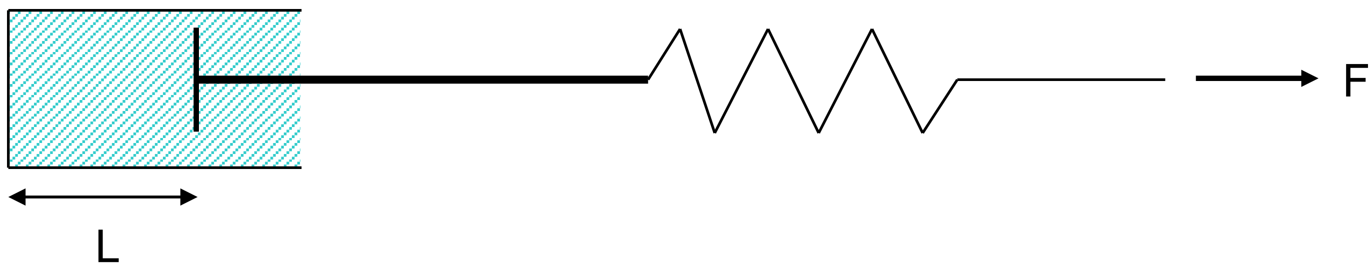
Stress is proportional to strain rate & constant of proportionality 'η' is coefficient of viscosity (units: Pa s = N m^{-2}s)

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

- many ways to define such a material
- assume the following 1-D linear response: “Maxwell Body”



Physics?

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

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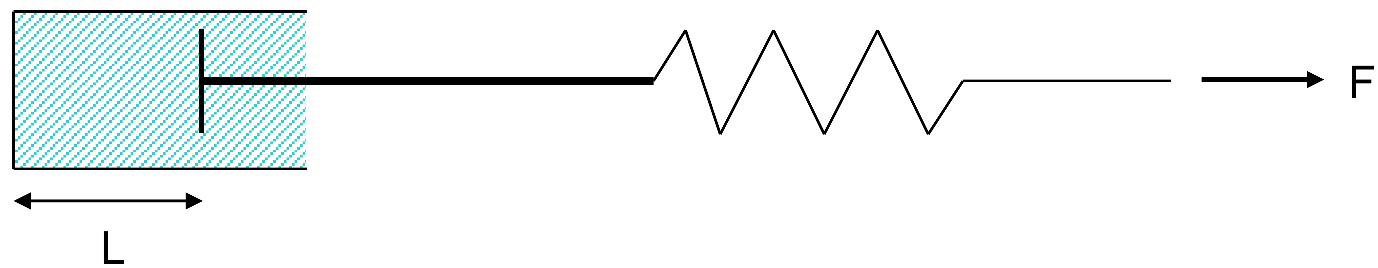

- application of stress yields immediate elastic (E) response and a subsequent viscous (v) response

How are stresses on each element related?

What is the total strain?

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

- many ways to define such a material
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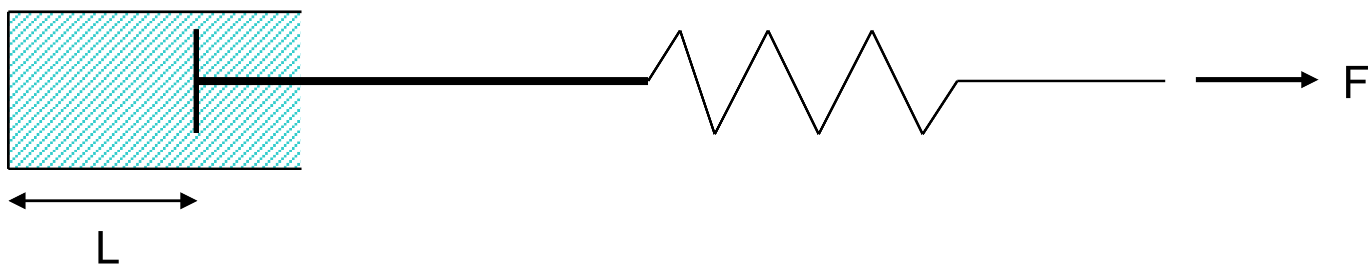


- application of stress yields immediate elastic (E) response and a subsequent viscous (v) response:

$$\frac{\Delta L}{L} = \left(\frac{\Delta L}{L}\right)^{E} + \left(\frac{\Delta L}{L}\right)^{v} \Rightarrow$$

$$\frac{d}{dt}\left(\frac{\Delta L}{L}\right) = \frac{d}{dt}\left(\frac{\Delta L}{L}\right)^{E} + \frac{d}{dt}\left(\frac{\Delta L}{L}\right)^{v} \Rightarrow \frac{d}{dt}e = \frac{d}{dt}\left(\frac{F}{k}\right) + \frac{F}{\eta}$$

Constitutive equation (i.e. equation relating stress & strain) for a Maxwell viscoelastic 1-D body

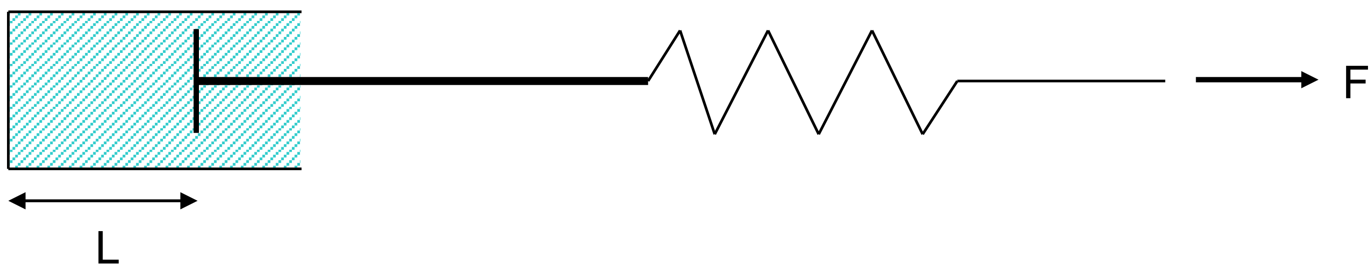
$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta}F$$

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

- many ways to define such a material
- assume the following 1-D linear response: “Maxwell Body”



- constitutive equation:

$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta}F$$

End members:

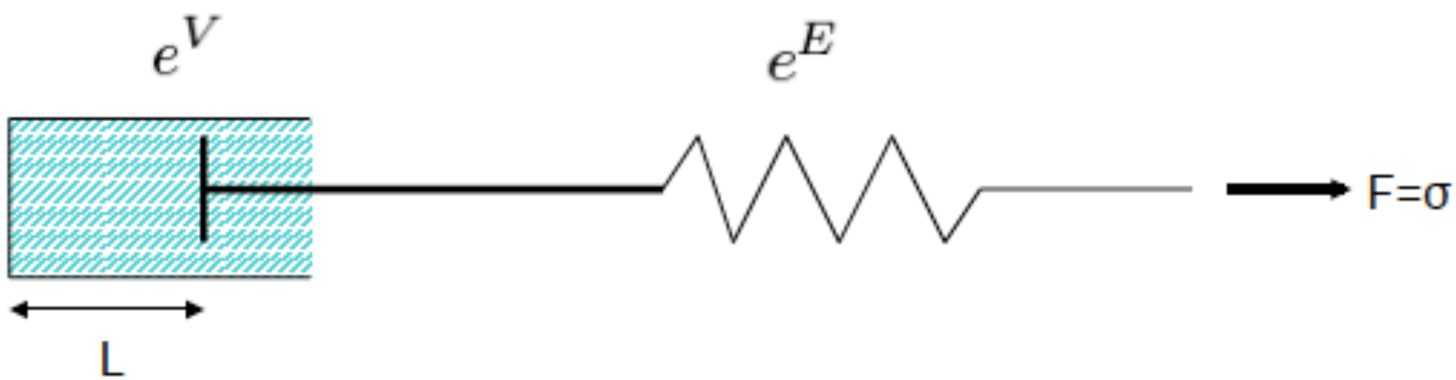
End members:	$k \rightarrow \infty$	Viscous body
	$\eta \rightarrow \infty$	Elastic body

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

- What happens if we apply a constant stress of σ?



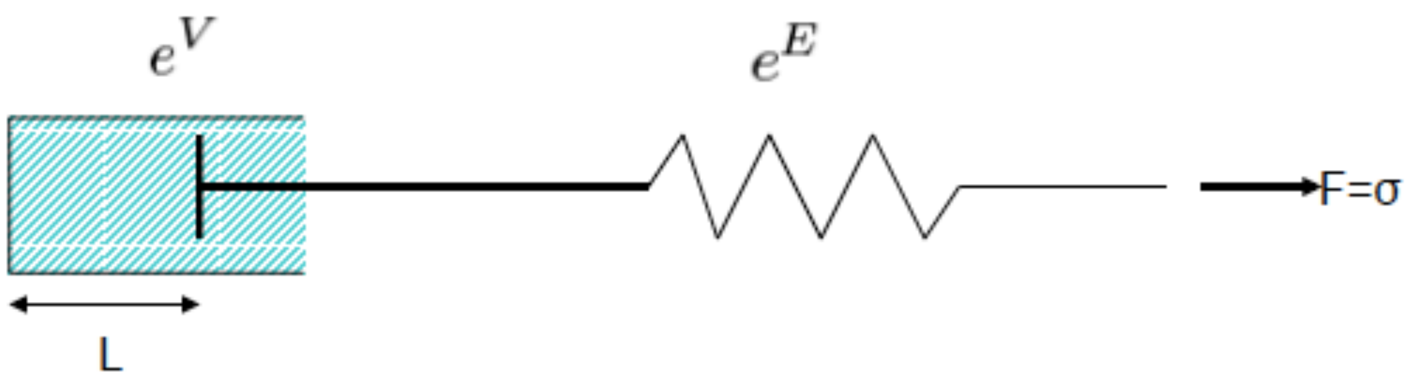
$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta}F$$

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

- What happens if we apply a constant stress of σ?



At t=0, when stress is first applied, the response is elastic

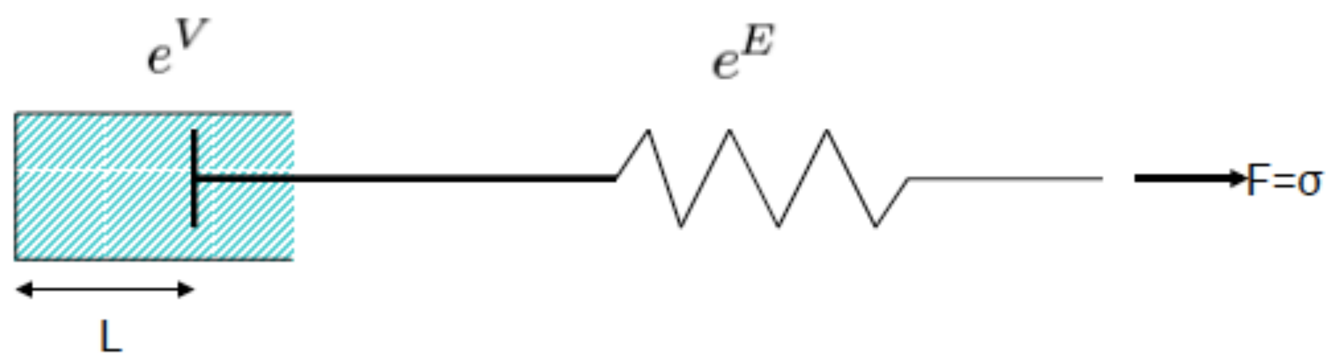
$$F = k \cdot e(t=0) \Longrightarrow e(t=0) = \frac{\sigma}{k}$$

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

• What happens if we apply a constant stress of σ?



Constitutive equation

$$\dot{e} = \frac{\dot{F}}{k} + \frac{F}{\eta} \qquad F = \sigma = \text{constant} \Longrightarrow \dot{F} = 0$$

$$\frac{de}{dt} = \frac{\sigma}{\eta} \Longrightarrow e(t) = \frac{\sigma}{\eta}t + c$$

but $e(t=0) = \frac{\sigma}{k}$

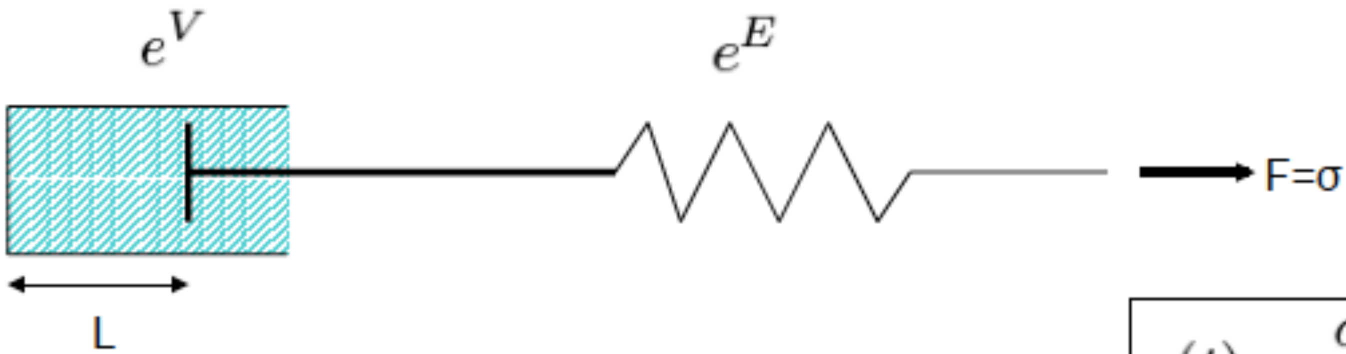
$$\boxed{e(t) = \frac{\sigma}{\eta}t + \frac{\sigma}{k}}$$

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

• What happens if we apply a constant stress of σ?



$$e(t) = \frac{\sigma}{\eta}t + \frac{\sigma}{k}$$

Strain Partitioning

$$e^E = \frac{\sigma}{k} \qquad e^V = \frac{\sigma}{\eta}t$$

When does viscous strain equal elastic strain?

$$\frac{\sigma}{k} = \frac{\sigma}{\eta}t \rightarrow \boxed{t = \frac{\eta}{k}}$$

Maxwell time!

Earth's mantle?

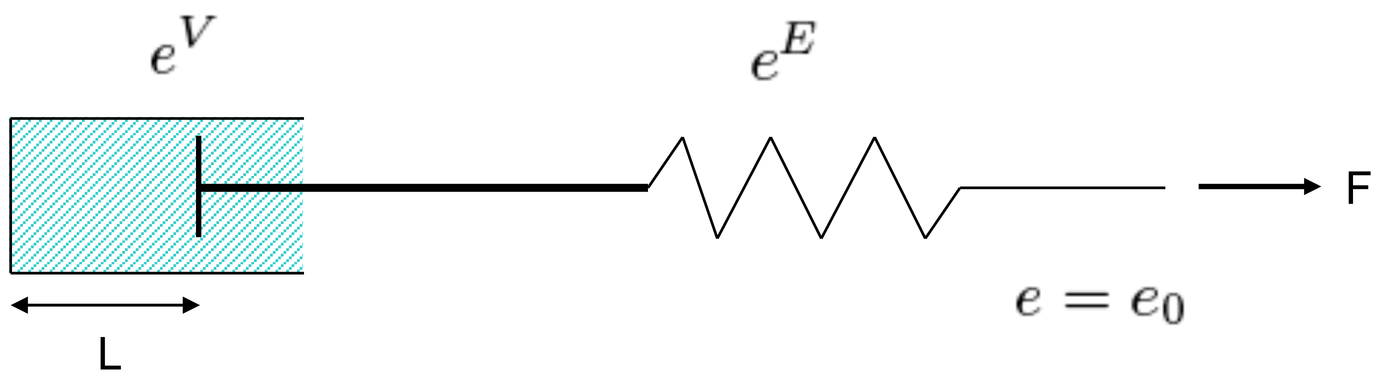
$$t = \frac{10^{21}\ \text{Pas}}{10^{11}\ \text{Pa}} = 300 \text{ years!}$$

Earth Rheology

1-D MODELS OF RHEOLOGY

3) Viscoelastic Body

- What happens if we apply a constant strain of $e = e_0$ at $t = 0$



$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta}F$$

Earth Rheology

RHEOLOGICAL CLASSIFICATIONS

- constitutive equation relates stress & strain
- in our examples: 1-D elastic, viscous, or Maxwell (linear) viscoelastic
- in general, require 3-D formulation for Earth's response --> tensor algebra (e.g. Hooke's Law in 3-D)
- viscoelastic rheology need not be linear, i.e., $F \propto (\dot{e})^m \quad m \neq 1$
- **Complications (and the exciting part!!)**: a single type of material can behave as a brittle, elastic, viscoelastic or viscous body depending on:

Physical Environment
- Pressure
- Temperature

Nature of Applied Forcing
- Force Magnitude
- Force Duration

Material rheology depends on P, T, F_M,F_D

e.g., silly putty, tar, glass

Earth Rheology

FORCE DEPENDENCIES

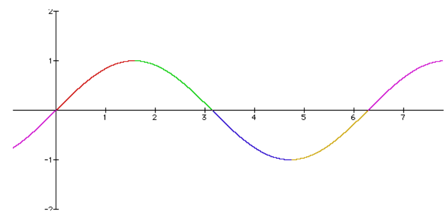
Force Duration:

- Dimensional analysis: Consider a Maxwell visco-elastic body

$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta}F$$

- Assume a forcing which varies harmonically:

$$F = F_0 e^{i\omega t} \Rightarrow$$
$$\dot{F} = i\omega F_0 e^{i\omega t}$$



- The material must respond with the same frequency but perhaps out of phase:

$$e = e_1 e^{i(\omega t + \phi)} = e_0 e^{i\omega t}$$

- Plugging e, F and F into the constitutive equation we obtain:

$$i\omega e_0 e^{i\omega t} = \frac{i\omega}{k} F_0 e^{i\omega t} + \frac{1}{\eta} F_0 e^{i\omega t} \Rightarrow \boxed{e_0 = F_0\left(\frac{1}{k} + \frac{1}{\eta i\omega}\right)}$$

Earth Rheology

FORCE DEPENDENCIES

Force Duration:

$$e_0 = F_0\left(\frac{1}{k} + \frac{1}{\eta i\omega}\right)$$

(A) Short-time scale forces: $\omega \to \infty \Rightarrow e_0 = \frac{F_0}{k}$ Elastic response

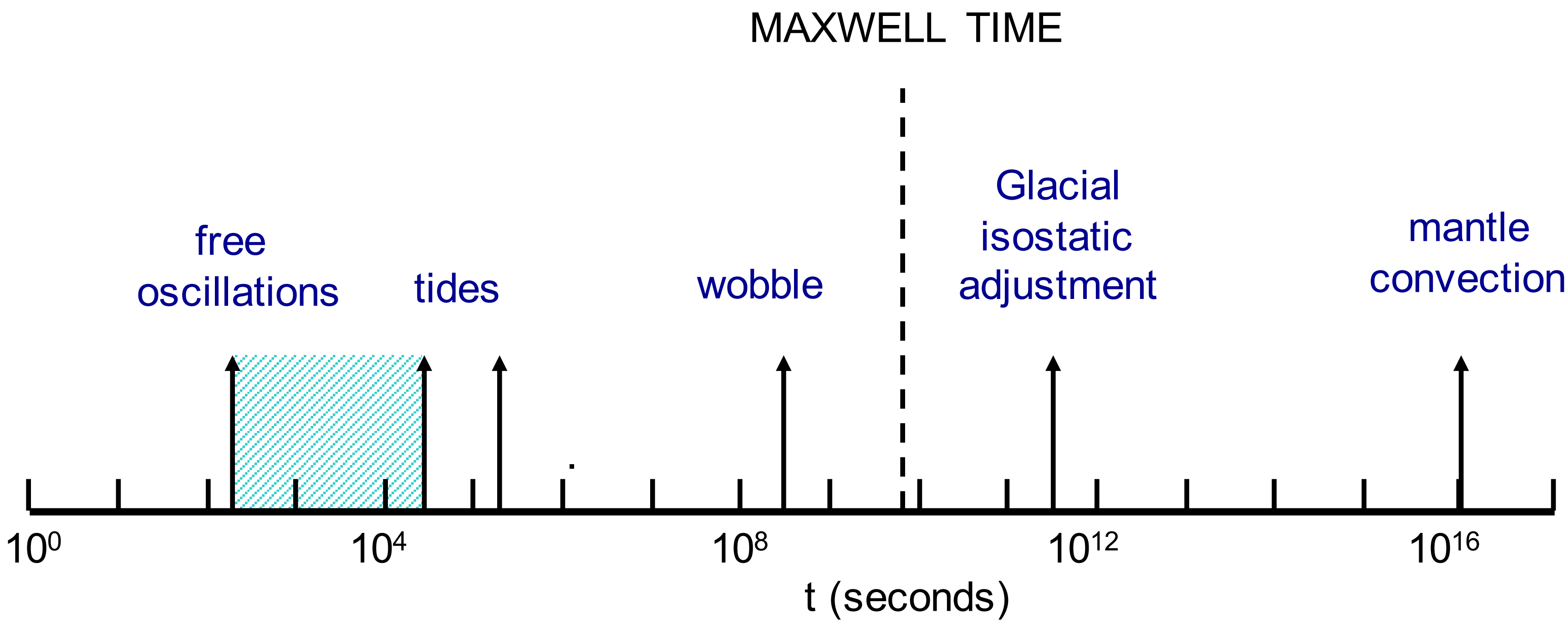
(B) Long-time scale forces: $\omega \to 0 \Rightarrow F_0 = \eta i\omega e_0 \Rightarrow F = \eta\dot{e}$ Viscous response

(C) Intermediate time scale forces: visco-elastic response - body can behave as a 'fluid' or a 'solid' depending on the time scale of the applied forcing

- The Earth has different rheological responses to different forcings
  - Seismic, tides: Elastic
  - Wobble, post-glacial rebound, polar wander: Visco-elastic
  - Convection: viscous
- By studying these forcings and the Earth's response to them, we can determine information on the Earth's interior

Earth Rheology

THE EARTH AS A VISCOUS SOLID!



- The Earth has different rheological responses to different forcings
- By studying these forcings and the Earth's response to them, we can determine information on the Earth's interior

Earth Rheology

ENVIRONMENT DEPENDENCIES

$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta}F$$

- Both k & η are functions of pressure P & temperature T
- ν is a much stronger function of P,T (in general)
- to a first approximation: $\eta = \eta(P,T) = \eta(T)$
- experiments have shown: $\eta(T) = \eta_0 \exp(-a[T - T_0])$

  $\eta(T_0) = \eta_0$

- "a" is a complex function of thermodynamic properties
- thus, small changes in temperature produce exponentially large changes in viscosity!!!

$$\dot{e} = \frac{1}{k}\dot{F} + \frac{1}{\eta_0 \exp(-a[T - T_0])}F$$

Cases:

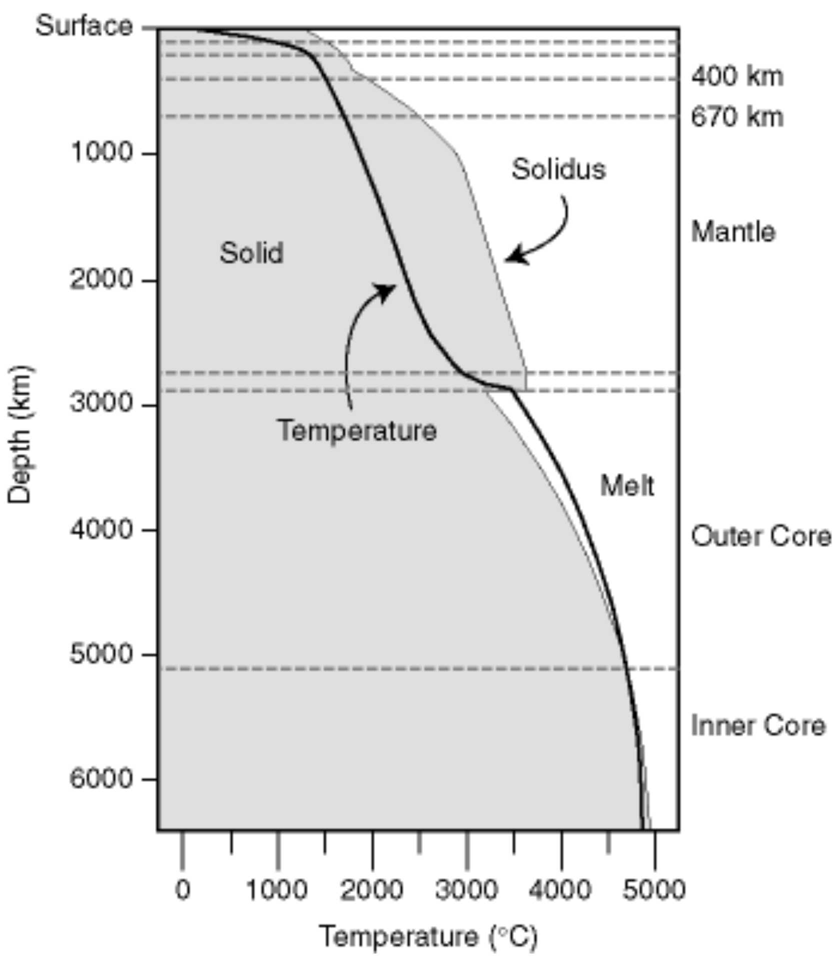
(1) Cold … means η large so: $\dot{e} \cong \frac{1}{k}\dot{F}$ Elastic response

(2) Hot … means η small so: $\dot{e} \cong \frac{1}{\eta}F$ Viscous response

(3) Warm … intermediate T so: viscoelastic response … similar to tar

Earth Rheology

RHEOLOGY & THERMAL REGIME OF THE EARTH



- Earth surface: T ~ 0C
- Define lithosphere as: the thin outer shell of the Earth whose lower boundary is constrained to coincide with the T=1400C isotherm
- according to figure, this occurs around depth=100km
- 1400C isotherm is somewhat arbitrary but is chosen on rheological grounds
- For T<1400C, mantle materials will behave elastically (or as brittle materials if forcing magnitude is large enough) no matter what the time scale of the geophysical process (in general)
- this portion of the mantle involves the "plates". Plates exist because of the temperature dependence of viscosity!
- Individual plates can move around (plate motion) & elastically deform (plate bending, for example), but they are said to be RIGID because they do not viscously deform

Why we care

- To understand questions like:
  What is a plate?
  Why does it exist?
  What drives the plates?
  we require knowledge of the Earth's rheological behaviour

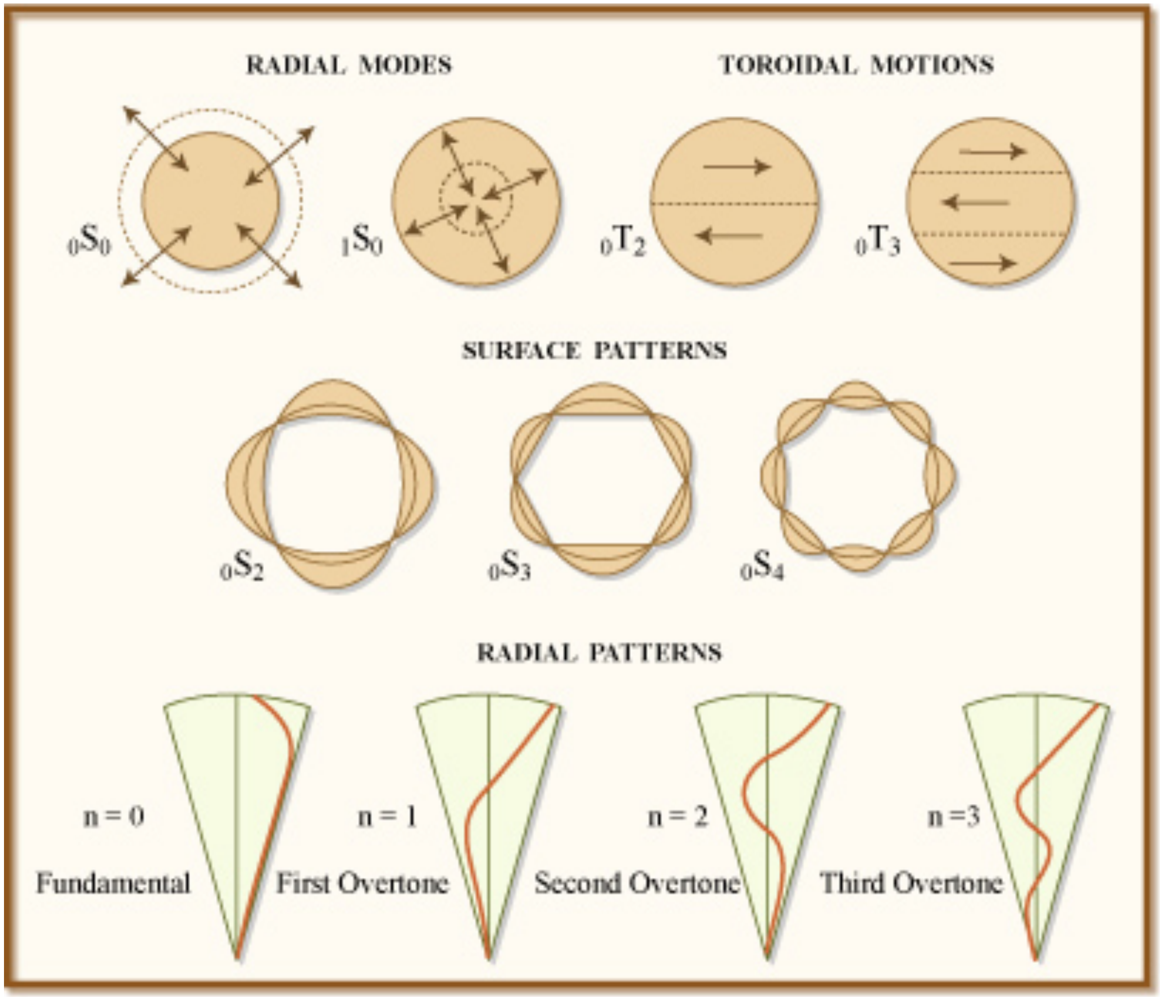
- Rheology of Earth materials tells us how Earth responds to forcings associated with:

Forcing:		Response:
•Large earthquakes	--->	seismic waves
•Gravitational pull of the sun and moon	--->	tides
•Melting of glaciers	--->	post-glacial rebound
•Temperature contrast between CMB & surface + internal heating	--->	mantle convection

- These forcings have a wide range of characteristic time scales
- We can observe the responses and use this information to constrain elastic and viscous properties of the Earth's interior
  - examples: seismology tells us about the Earth's elastic structure
    post-glacial rebound tells us about the Earth's viscosity profile

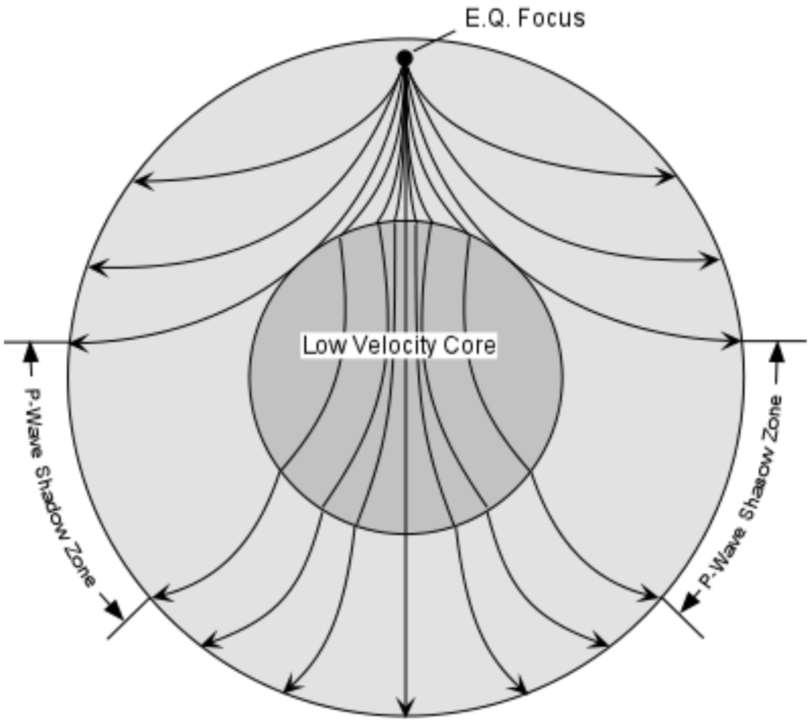
Seismology

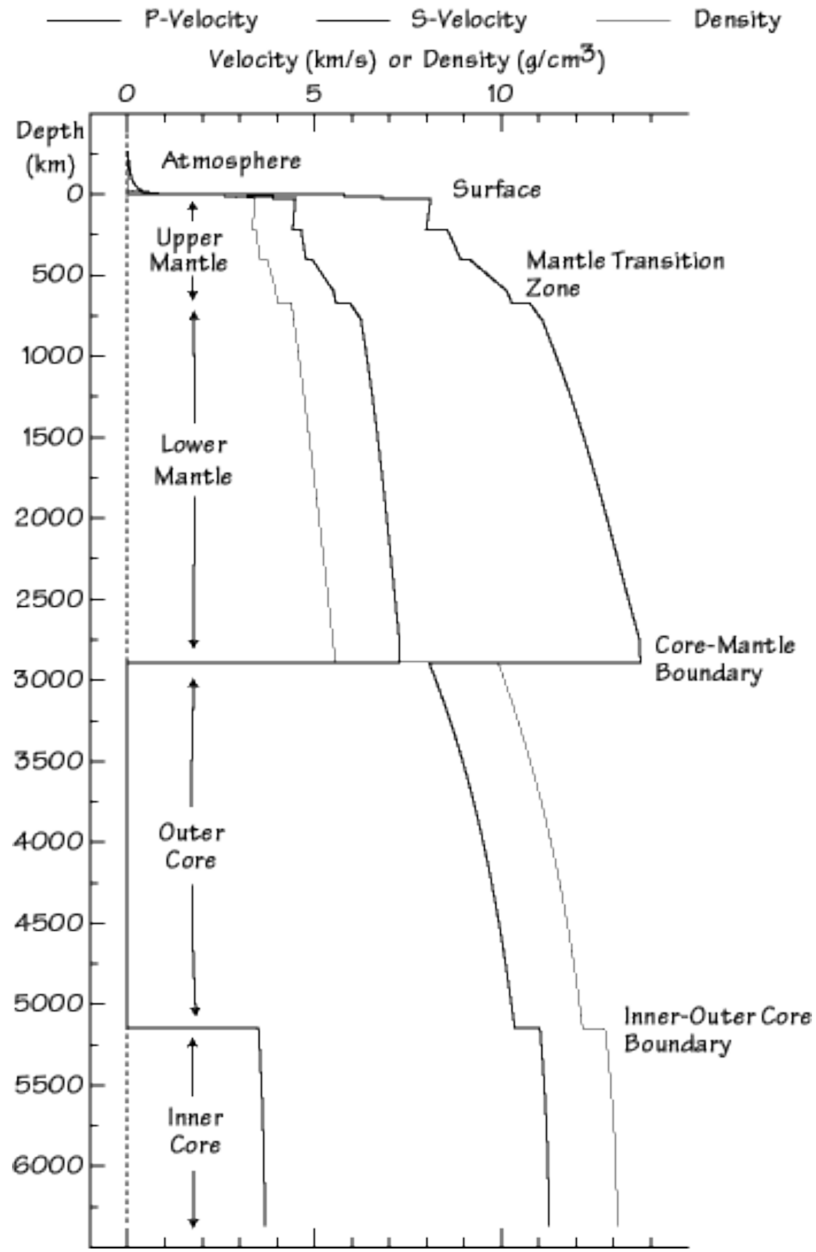
•Free oscillations (earth ringing due to forcing like an earthquake)



Seismology

•Body wave data (track paths of seismic rays through Earth)





Preliminary Reference Earth Model "PREM" gives density and seismic wave speeds as a function of radius
-wave speeds related to Young's modulus (elastic property)

Glacial Isostatic Adjustment (post glacial rebound)

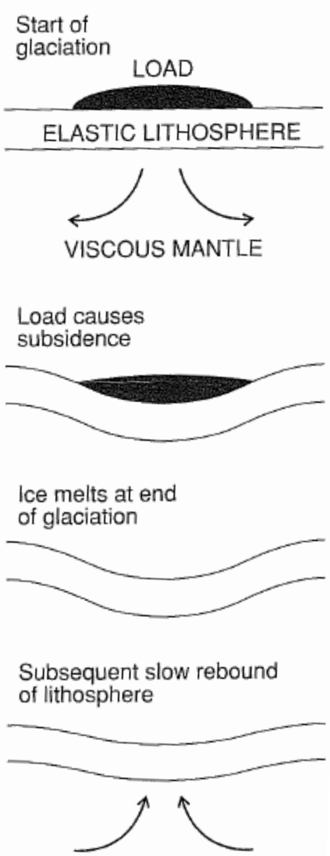


Figure 5.18. The deformation and uplift which occur as a result of loading and unloading of an elastic lithospheric plate overlying a viscous mantle.

- A load like a glacier causes the lithosphere to subside due to isostasy
- Upon removal of the load (melting the glacier), the system will deform again to try and reach isostasy
- The rate of defomation is dependent on both the rigidity of the lithosphere and the viscosity of the mantle
- We can investigate the rate of uplift for different sized loads to determine the viscosity of different regions of the mantle