Module 1: September 2011

Something to think about: Earth rotation and the Moon's retreat: Since the Apollo astronauts placed corner retroreflector panels on the Moon in 1969, we have been measuring the distance to the Moon continuously. We find that the Moon retreats from the centre of the Earth by about ***3.85 cm/yr***. This is a consequence of angular momentum from the Earth's rotation being taken up by the Moon. Angular momentum in the Earth-Moon system must be conserved.

As angular momentum is being lost by the Earth to the Moon, the Earth's rotation slows. Given that the Earth's moment of inertia about its rotation axis is $0.3308\, m_E a_E^2$ where m_E is Earth's mass and a_E Earth's average radius (***6371 km***), by how much of an increase in Earth's rotation period does this transfer of angular moment account for in ***1 year***?

[Note, those of you who were in my Earth Physics class last year did this problem on the midterm exam. Most of you found an answer of approximately ***22 µsec/year***. I shall bring the method of calculation to class next day.]

The Gravity noteset that I introduced in class showed that the measured increase in Earth's rotation period over the past 40 years has amounted to something less: about ***17.1 µsec/year***. There is a "***spin-down deficit***"! We partially account for this by a continuing secular change in the angular momentum of the atmosphere and ocean circulation but most of the effect is known to be due to the melting of the icecaps and the resulting plastic adjustment of the Earth which must produce a still-continuing decrease in the Earth's axial moment of inertia.

Let us look into this effect: If we were to remove the mass equivalent to an icesheet ***1.5km*** thick over an area of $10 \times 10^9\ km^2$ centred at a latitude of ***65ºN*** and distribute this mass evenly into the oceans, what increase or decrease in the Earth's axial moment of inertia would arise?

Most of this melt adjustment would have taken place quickly between 12000 and 8000 years ago. The Earth, though, is still adjusting to that unloading and it is the effect of this continuing adjustment that we see in this record of these past 40 years. Consider the movement of an amount of mass to a latitude line at ***75ºN*** to be taken from the equatorial latitude belt. How much mass displacement per year could account for the current "spin-down deficit" that we observe? You might take a belt of mass from the equator to be placed as a belt at ***75ºN.***

Some hints that will simplify your efforts:

1. The moment of inertia of a uniform sphere rotating about a diameter is $\boldsymbol{0.4mr^2}$ where $\boldsymbol{m}$ is the total mass of the sphere and $\boldsymbol{r}$ its radius.
2. The moment of inertia of a spherical shell with all its mass on its surface and rotating about a diameter is $\boldsymbol{0.666...mr^2}$
3. The moment of inertia of a circular band rotating on an axis through its centre which is normal to the plane of the band is $\boldsymbol{mr^2}$
4. Angular momentum is conserved!