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Free and forced polar motion and modern observations of the Chandler wobble

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ABSTRACT

In the absence of excitation, the Chandler wobble is closely a prograde motion along a circular arc. For a step excitation, the centre of the arc shifts, giving a secular motion but an equal and nearly opposite contribution to the Chandler wobble occurs, giving only a second order discontinuity in the pole path. To detect excitation events, we fit circular arcs by least squares to the unequally spaced data, weighting by the inverse of the square of the accompanying standard errors. A break is determined if extrapolation along the circular arc leads to a forecast pole position for which the next measured position lies outside a circle of acceptance.

We find that often for quite long periods of time, there seems to be relatively little continuous excitation, leading to the conclusion that much of the excitation comes from sudden events. In particular, we are encouraged that a break in the pole path was found 11 days before the December 26, 2004 Sumatra-Andaman Islands earthquake ($M = 9.1$).

1. Introduction

Classically, the Earth's wobble, or the motion of the rotation axis within the Earth, has been measured by observations of the associated latitude variation. Euler had suggested, in 1765, a rigid Earth would have a free wobble with a period, in sidereal days, near the reciprocal of the dynamical ellipticity, or close to 10 months. The existence of the associated latitude variation was confirmed by simultaneous observations at Berlin and Waikiki in 1891. Since the two stations are close to 180° apart in longitude, the variations, as expected, were found to be opposite in phase. In the same year, S.C. Chandler, an amateur astronomer, announced that his analysis of long series of latitude variations showed that there were two principal components, an annual term and 14-month variation, 40% longer than Euler's period for a rigid Earth. It was rapidly shown that the lengthened period was due to the fluidity of the oceans and elastic yielding of the solid Earth, and the motion is now called the Chandler wobble. These discoveries led to the establishment of the International Latitude Service (ILS) in 1895 with five stations all operating on latitude 39°08′ North, beginning regular observations in 1899. Details of the early work are given in the marvelous, pioneering book '*The Rotation of the Earth*' by Munk and MacDonald (1960) which opened up the subject to the general geophysical audience.

The original ILS stations used Visual Zenith Telescopes (VZTs). The inclusion of additional stations and instruments followed the formation of the International Polar Motion Service (IPMS) as the successor to the ILS in 1962. Meanwhile, the Bureau International de l'Heure had established an independent set of observatories to monitor polar motion to correct the effect of polar motion on Universal Time (UT). These incorporated instruments such as the Photographic Zenith Tube (PZT) with improved stability and measurement accuracy. These developments are described in detail in the masterful successor to Munk and MacDonald's book, *The Earth's Variable Rotation* by Lambeck (1980).

The co-ordination of polar motion observations using modern space measurement techniques, such as Very Long Baseline Interferometry (VLBI), Lunar Laser Ranging (LLR) and Satellite Laser Ranging (SLR), began with the formation of the International Earth Rotation Service (IERS) in 1987 as the successor to both the BIH and the IPMS. In contrast to the classical astronometric techniques used by the IPMS and BIH, which often yielded pole positions differing by as much as ten centiseconds of arc (one centisecond of arc is close to one foot of pole displacement), modern space measurement techniques yield pole positions with error levels reduced by three orders of magnitude.

The source of excitation of the free Chandler component of the polar motion has long been a subject of interest. The idea that earthquakes might have an effect on polar motion goes back at least to Milne (1906). They were dismissed as having a negligible effect on the assumption that the displacement fields extended only to a distance of a few fault lengths. However, Press (1965), using the elasticity theory of dislocations developed by Steketee

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(1958), showed that the displacement fields of earthquakes are very extensive, and found permanent offsets on strain meters in Hawaii following the 1964 Alaska event. This led Mansinha and Smylie (1967) to estimate that the very extensive displacement fields predicted by the elasticity theory of dislocations could have an important contribution to the excitation of the Chandler wobble and they later found a correlation between breaks in the pole path and large earthquakes (Smylie and Mansinha, 1968). Since the displacement fields were so extensive, studies in realistic Earth models (Smylie and Mansinha, 1971; Dahlen, 1971; Israel et al., 1973) quickly followed. Dahlen (1971) used a normal mode expansion to represent the effect on polar motion but it was shown that this was incomplete because in the static deformation, the variable y_1 in the Alterman, Jarosch and Pekeris notation, is discontinuous. Dahlen (1973) later corrected his calculation but the same erroneous assumption has been recently made by Gross and Chao (2006). The many theoretical questions involved in realistic Earth estimates of seismic excitation of the Chandler wobble, including the tensor nature of the focal force systems were finally resolved in a paper by Smylie et al. (1979).

The dramatic reduction in the error levels of polar motion measurements, particularly those made with the VLBI technique, prompts us to develop methods of analysis suited to these unequally spaced observations, allowing a new examination of breaks in the pole path that may be associated with seismic activity. Once again, we will find the breaks in the pole path are second order discontinuities, there being little instantaneous effect as the secular polar shift is balanced by a nearly equal and opposite contribution to the Chandler wobble.

2. Free and forced polar motion

Earth's rotation vector is conveniently expressed as

$$(m_1, m_2, 1+m_3)\Omega, \tag{1}$$

with $\Omega = 7.2921151467 \times 10^{-5}$ rad/s being the adopted mean rate of rotation. m_1, m_2, m_3 are then dimensionless quantities expressing Earth's instantaneous rotation. m_1, m_2 are the direction cosines of the axis of rotation with respect to the reference x_3 axis, and m_3 is the relative change in the axial spin rate. As mentioned before, one centisecond of arc subtended at the geocentre closely corresponds to a displacement of the pole of one foot. Since the pole never departs from the reference pole by more than 10 or 15 feet, the dimensionless quantities m_1, m_2, at most, are of order 10^{-6}.

Excluding large scale polar wander, the Liouville equations for polar motion are then accurately linearizable to (Munk and MacDonald, 1960, Ch. 6; Lambeck, 1980, Ch. 3)

$$\frac{d\boldsymbol{m}}{dt} - i\sigma_0 \boldsymbol{m} = \boldsymbol{f}(t), \tag{2}$$

with the complex phasor $\boldsymbol{m} = m_1 + im_2$ representing the pole position and $\boldsymbol{f}(t)$ a complex valued excitation function. The complex Chandler angular frequency is

$$\sigma_0 = \left(\sigma_c + \frac{i}{\tau}\right), \tag{3}$$

where τ is the damping time. The damping time is given by $\tau = 2Q/\sigma_c$ with σ_c the real Chandler angular frequency. In the absence of excitation, the free wobble is

$$\boldsymbol{m}(t) = \boldsymbol{C}e^{i\sigma_0 t}, \tag{4}$$

with $\boldsymbol{C}$ a complex constant. The free polar motion is then prograde along a circular spiral at the Chandler angular frequency σ_c, slowly decaying with damping time $\tau \approx 10$ years. With excitation,

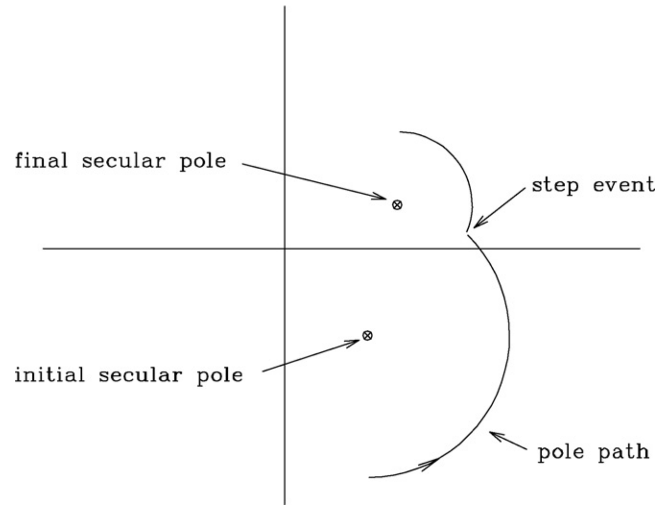

Fig. 1. Pole path for step excitation.

the polar motion is found by integration to be

$$\boldsymbol{m}(t) = e^{i\sigma_0 t}\int_{-\infty}^{t} e^{-i\sigma_0\lambda}\boldsymbol{f}(\lambda)d\lambda. \tag{5}$$

A sudden redistribution of mass within the Earth, such as associated with a major earthquake, produces off-diagonal components of the inertia tensor represented by the complex number $\Delta c = \Delta c_{13} + i\Delta c_{23}$. To terms of order $\sigma_c/\Omega \approx 1/436$, it produces a Chandler wobble $-(\Omega\,\Delta c/\sigma_0 A)e^{i\sigma_0 t}$ and a secular polar shift $(\Omega\,\Delta c/\sigma_0 A)$ (Smylie and Mansinha, 1968). To this order of approximation, there is no instantaneous change in pole position as the secular polar shift and Chandler wobble at $t = 0$ cancel. The pole path for this kind of step excitation is illustrated in Fig. 1.

The annual motion can be removed by a least squares fit to the overall polar motion. Once the annual motion has been removed, in the absence of excitation and damping, the pole path should be a prograde circular motion around the secular pole, with co-ordinates (m_{1_0}, m_{2_0}), at a uniform angular rate $\sigma_c = 2\pi/T_c$, where T_c is the period of the Chandler wobble. The jth pole position will have associated with it a unique time t_j. The angle turned through to the jth pole position is then

$$\alpha_j = \sigma_c \sum_{k=1}^{j-1}(t_{k+1} - t_k) \quad \text{for } j \geq 2, \ \ \alpha_1 = 0. \tag{6}$$

If standard errors $(\epsilon_{1_j}, \epsilon_{2_j})$ are available for the pole co-ordinates (m_{1_j}, m_{2_j}) to fit a circular arc by least squares to n successive pole positions, we minimize the sum

$$S = \sum_{j=1}^{n}\left(\frac{[m_{1_j} - m_{1_0} - a\cos(\phi + \alpha_j)]^2}{\epsilon_j^2} + \frac{[m_{2_j} - m_{2_0} - a\sin(\phi + \alpha_j)]^2}{\epsilon_j^2}\right), \tag{7}$$

with $\epsilon_j^2 = \epsilon_{1_j}^2 + \epsilon_{2_j}^2$, and where a is the radius of the fitted arc and ϕ is the angle that the radius to the first pole position makes with the x-axis. The geometry of the arc fitted to five successive pole positions is illustrated in Fig. 2. The sum (7) has four adjustable free parameters, the co-ordinates of the secular pole (m_{1_0}, m_{2_0}), the radius a of the circular arc and the angle ϕ the radius arm to the first pole position makes with the x-axis. The normal equations for the least squares fit of the circular arc are found by setting its

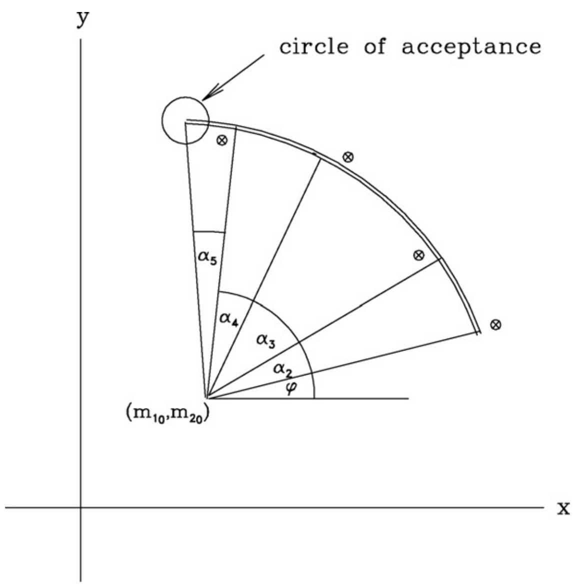

Fig. 2. Least squares arc fitting.

partial derivatives with respect to these parameters to zero. Setting the partial derivative of S with respect to m_{1_0} to zero gives the first normal equation as

$$s_1 - m_{1_0}\Sigma - ua\cos\phi + va\sin\phi = 0, \quad (8)$$

where

$$s_1 = \sum_{j=1}^{n} \frac{m_{1_j}}{\epsilon_j^2}, \qquad u = \sum_{j=1}^{n} \frac{\cos\alpha_j}{\epsilon_j^2}, \qquad v = \sum_{j=1}^{n} \frac{\sin\alpha_j}{\epsilon_j^2},$$

$$\Sigma = \sum_{j=1}^{n} \frac{1}{\epsilon_j^2}. \quad (9)$$

A second normal equation is obtained by setting the partial derivative of S with respect to m_{2_0} to zero, giving

$$s_2 - m_{2_0}\Sigma - ua\sin\phi - va\cos\phi = 0, \quad (10)$$

where

$$s_2 = \sum_{j=1}^{n} \frac{m_{2_j}}{\epsilon_j^2}. \quad (11)$$

Defining

$$w = \sum_{j=1}^{n} \left[\frac{m_{1_j}\cos\alpha_j + m_{2_j}\sin\alpha_j}{\epsilon_j^2} \right], \quad (12)$$

$$z = \sum_{j=1}^{n} \left[\frac{m_{1_j}\sin\alpha_j - m_{2_j}\cos\alpha_j}{\epsilon_j^2} \right], \quad (13)$$

on setting the partial derivative of S with respect to ϕ to zero, we find the third normal equation

$$z\cos\phi + w\sin\phi - m_{1_0}(u\sin\phi + v\cos\phi) + m_{2_0}(u\cos\phi - v\sin\phi) = 0. \quad (14)$$

Finally, the fourth normal equation is found by setting the partial derivative of S with respect to a to zero, giving

$$w\cos\phi - z\sin\phi - m_{1_0}(u\cos\phi - v\sin\phi) - m_{2_0}(u\sin\phi + v\cos\phi) - a\Sigma = 0. \quad (15)$$

Eqs. (8), (10), (14), and (15) are the normal equations for the least squares fit of the circular arc to the unequally spaced pole position observations, taking account of the standard errors of the measurements. They may be regarded as four equations in the four unknowns m_{1_0}, m_{2_0}, $a\cos\phi$, and $a\sin\phi$. If we multiply (14) through by $\sin\phi$ and add it to (15) multiplied through by $\cos\phi$, we find

$$um_{1_0} + vm_{2_0} + a\Sigma\cos\phi = w. \quad (16)$$

Similarly, multiplying (15) through by $\sin\phi$ and subtracting it from (14) multiplied through by $\cos\phi$, yields

$$vm_{1_0} - um_{2_0} - a\Sigma\sin\phi = z. \quad (17)$$

Solving Eq. (8) for m_{1_0} gives

$$m_{1_0} = \frac{s_1 - ua\cos\phi + va\sin\phi}{\Sigma}, \quad (18)$$

while solving Eq. (10) for m_{2_0} gives

$$m_{2_0} = \frac{s_2 - ua\sin\phi - va\cos\phi}{\Sigma}. \quad (19)$$

Substitution of these expressions in Eq. (16) yields

$$a\cos\phi = \frac{us_1 + vs_2 - w\Sigma}{u^2 + v^2 - \Sigma^2}, \quad (20)$$

while substitution in Eq. (17) yields

$$a\sin\phi = \frac{us_2 - vs_1 + z\Sigma}{u^2 + v^2 - \Sigma^2}. \quad (21)$$

In turn, substitution of these expressions in (18) produces

$$m_{1_0} = \frac{uw + vz - s_1\Sigma}{u^2 + v^2 - \Sigma^2}, \quad (22)$$

while substitution in (19) produces

$$m_{2_0} = \frac{vw - uz - s_2\Sigma}{u^2 + v^2 - \Sigma^2}. \quad (23)$$

The circular arc fitted to n successive pole positions can then be used to predict the $(n+1)$ th pole position. The predicted pole co-ordinates of the $(n+1)$ position are

$$m_{1_0} + a\cos(\phi + \alpha_{n+1}) = m_{1_0} + (a\cos\phi)\cos\alpha_{n+1} - (a\sin\phi)\sin\alpha_{n+1}, \quad (24)$$

and

$$m_{2_0} + a\sin(\phi + \alpha_{n+1}) = m_{2_0} + (a\sin\phi)\cos\alpha_{n+1} + (a\cos\phi)\sin\alpha_{n+1}. \quad (25)$$

An objective criterion can be constructed to determine breaks in the pole path. The squared radius from the predicted pole position to the actual position is multiplied by the ratio of the squared error to the average squared error for the whole record. If the root of this quantity is less than the radius of the specified circle of acceptance, no break is detected and the least squares fit to the circular arc is continued, if it is equal or greater than the radius of the specified circle of acceptan ce, a break is found and a new circular arc is begun.

Of interest, is whether the largest earthquake of the period 1984.0–2007.0, the magnitude 9.1 Sumatra-Andaman Islands event on December 26, 2004, shows up in the pole path. In Fig. 3, we show the arcs fitted for a circle of acceptance of radius 0.01 arcsec. The first arc runs from June 8, 2004 to December 14, 2004. The second arc runs from December 15, 2004 to June 30, 2005. No break is detected for more than a half year before the quake and for more than a half year after the quake. The detected break occurs 12 days before the earthquake.

In order to show the fit of the arcs to the actual pole path, we over plot the VLBI pole positions, showing their standard errors

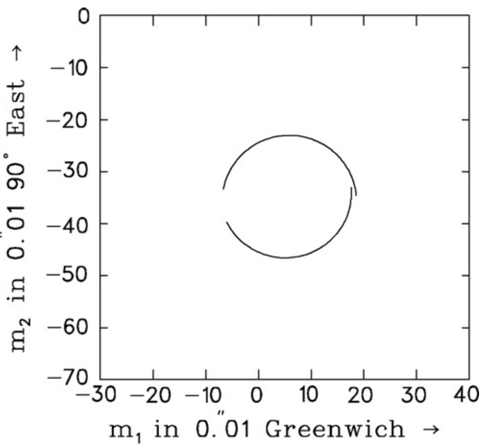

Fig. 3. 2004–2005 fitted arcs.

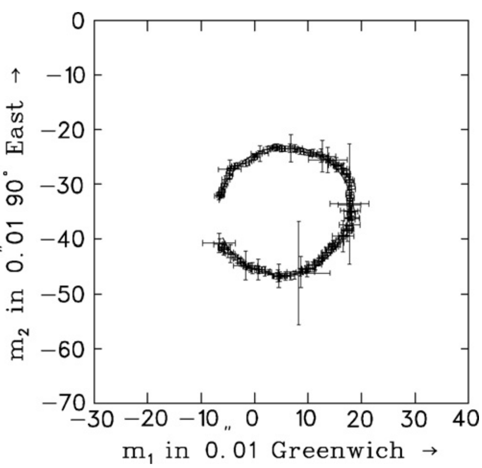

Fig. 4. Over plot of the VLBI pole positions, with their standard errors inflated by a factor of one hundred for clarity, on the fitted circular arcs.

inflated by a factor of one hundred for clarity, on the fitted arcs in Fig. 4.

3. Conclusions

Of great interest is the evidence found that the largest earthquake of the period, the magnitude 9.1 Sumatra-Andaman Islands event of December 26, 2004, is found to be reflected in a break in the pole path. The break occurs 12 days before the quake. Many of the breaks found by Smylie and Mansinha (1968) in their analysis of the classical BIH pole path also occurred before large quakes. This suggests the the far field displacement develops before the final release of stress by slip on the fault plane.

While this study is preliminary in nature, it opens up the analysis of modern observations made with new space techniques such as VLBI. Much geophysical information is likely contained in these observations.

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