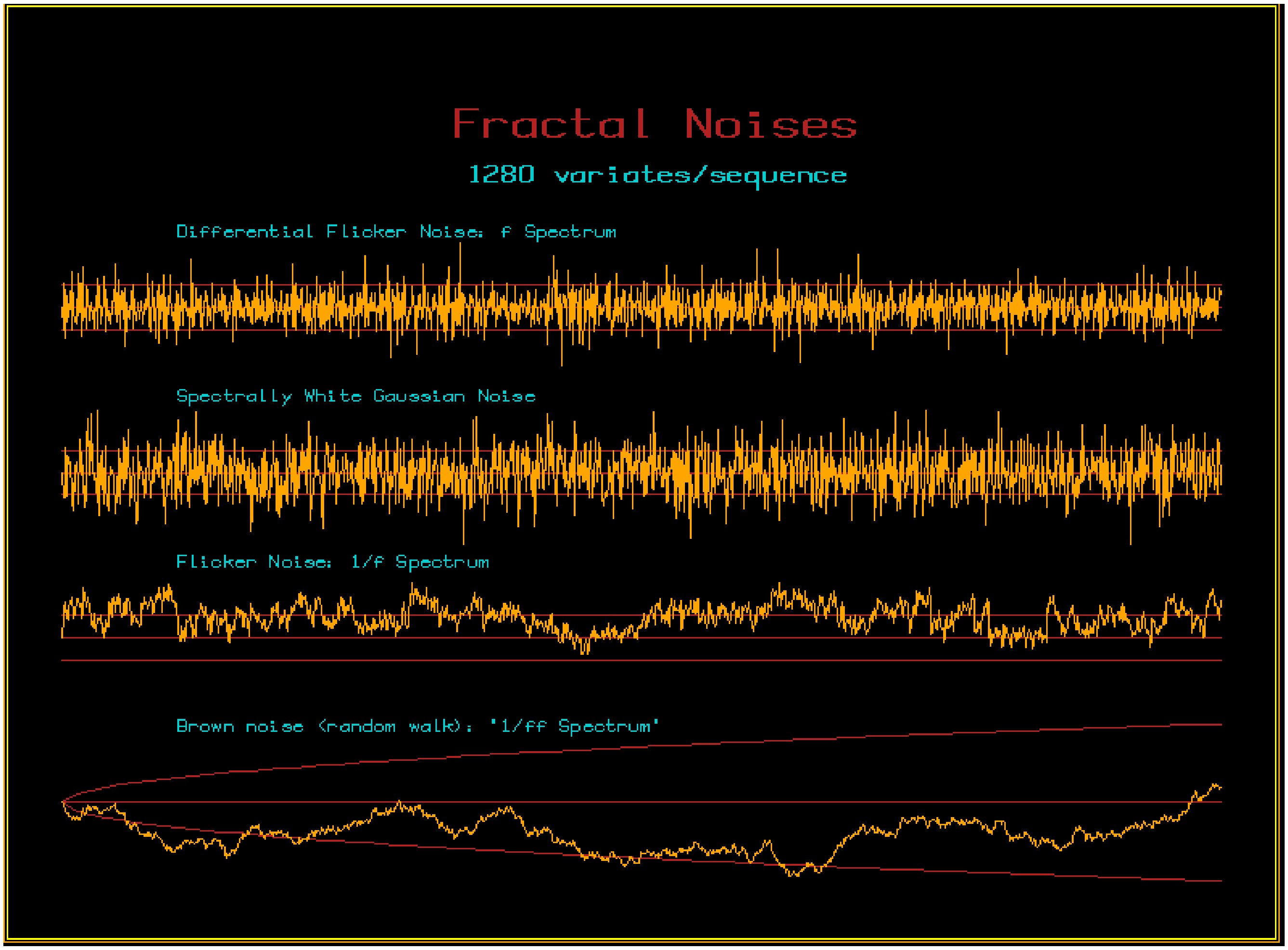
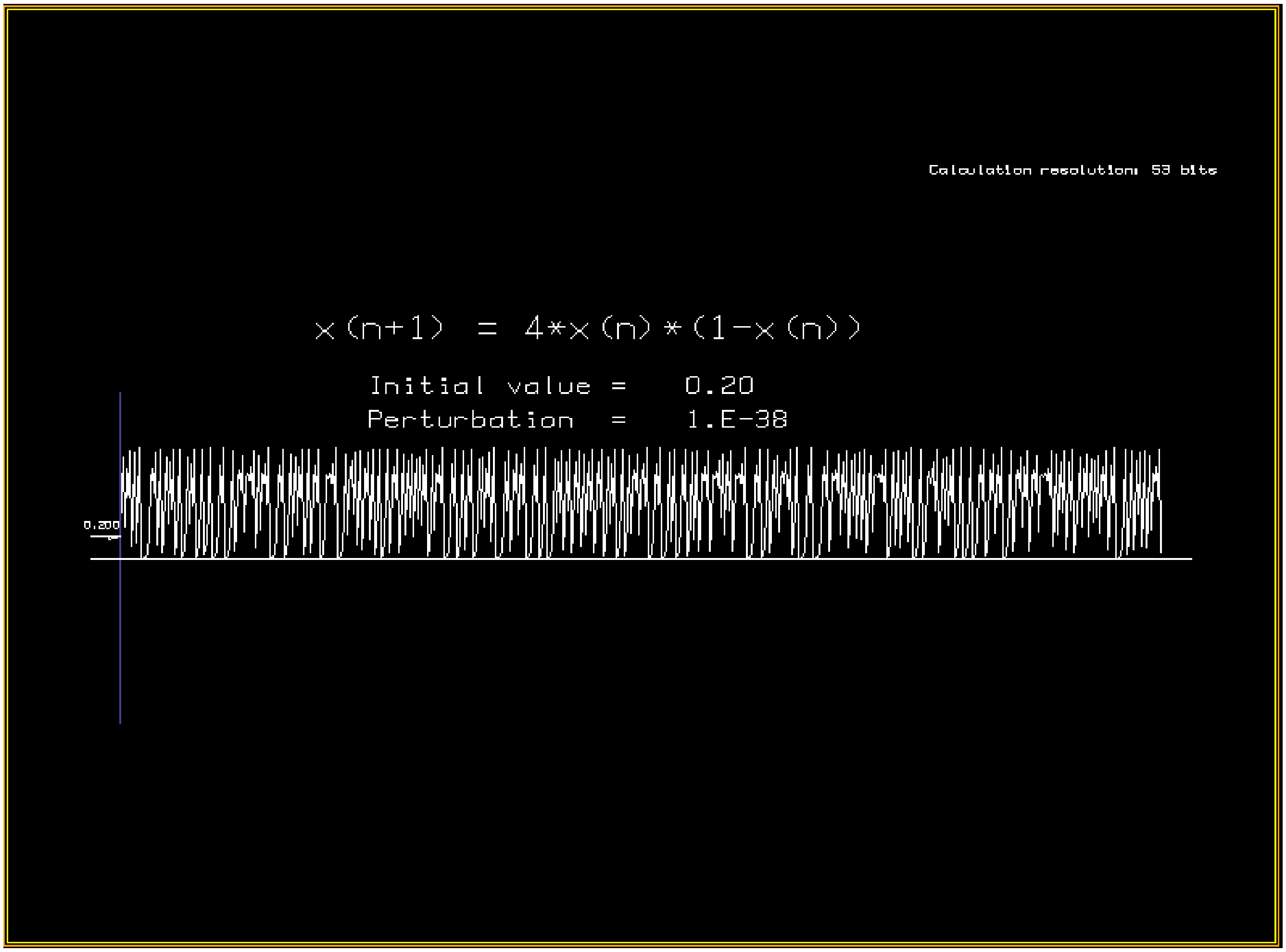
Noises and chaos



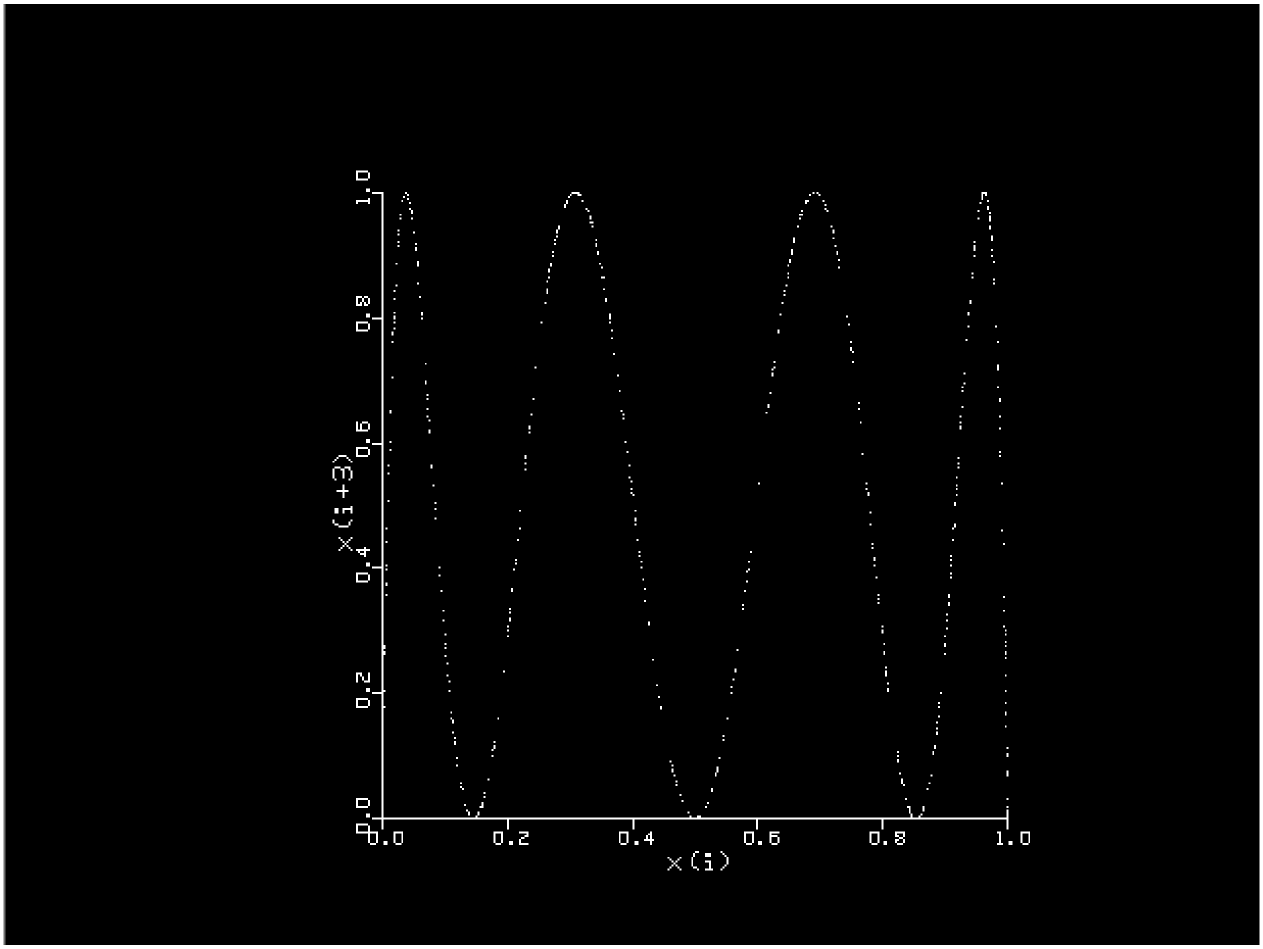
These are the “fractal noises”. Fractal means that they are self-similar on all scales.



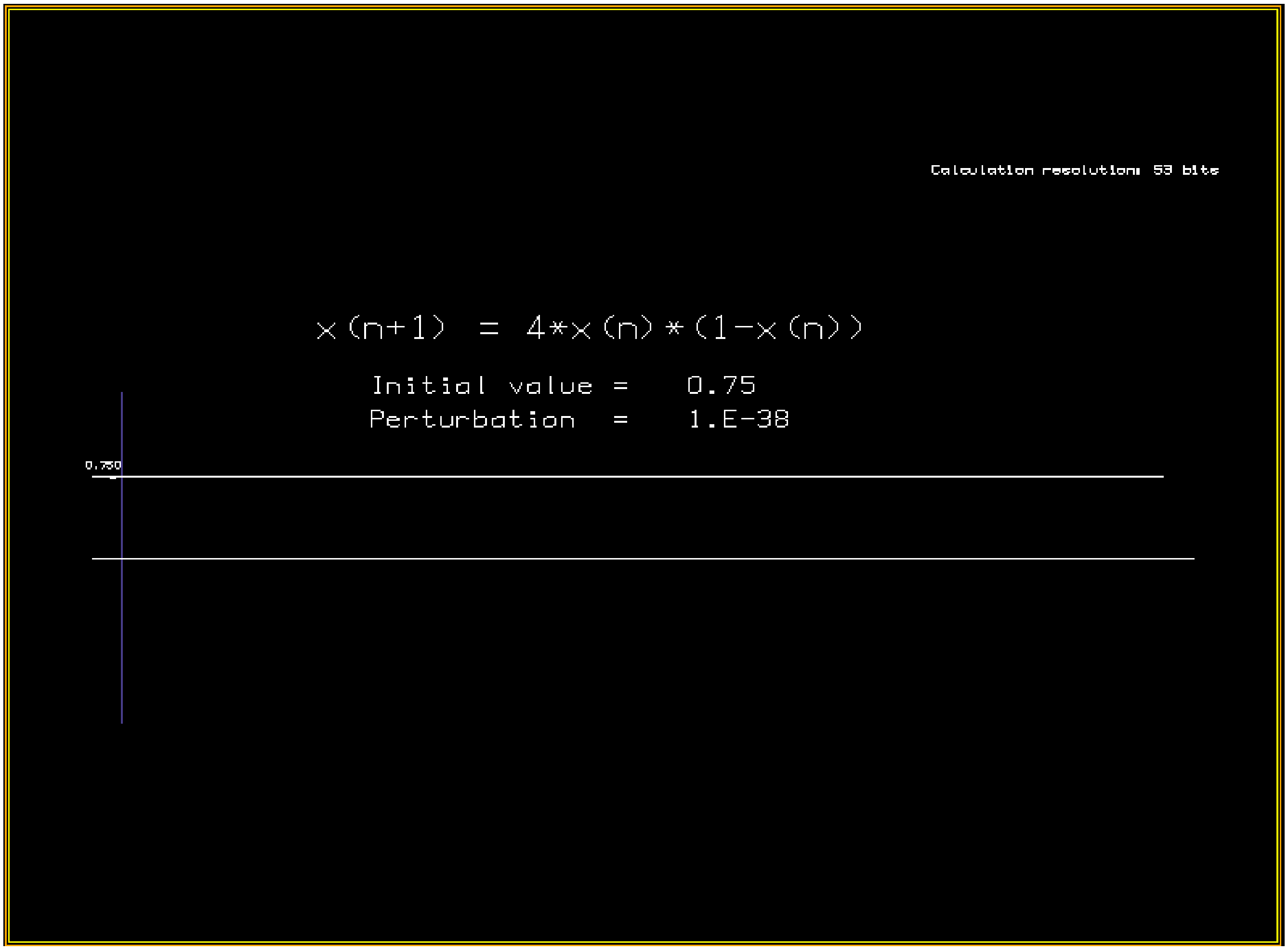
This is the simplest Verhultz' chaotic system. It is sometimes called the "quadratic chaotic model". The future is essentially unpredictable in spite of its purely deterministic recursion. This is a simulation, of course, and computers approximate the calculation step by step, here with 53bit resolution. The perturbation was entered as exactly 0.

These calculations are done using the 64-bit IEEE-754 floating point format. In this number format, any number described with a mantissa that is less than 2^{-53} is indistinguishable from zero. That is the resolution of the mantissa is equivalent to about 16 decimal places. That is a number described as 0.00000000000000001 is "translated" as exactly 0. Note that if this same value were described as an exponential in IEEE format as 0.1e-17 the number would be properly recognized as such. On the other hand if one were (as is done in this code) add the number 0.1e-17 to 0.20, the sum would be seen by the computer as exactly 0.20, or rational fraction 1/5.

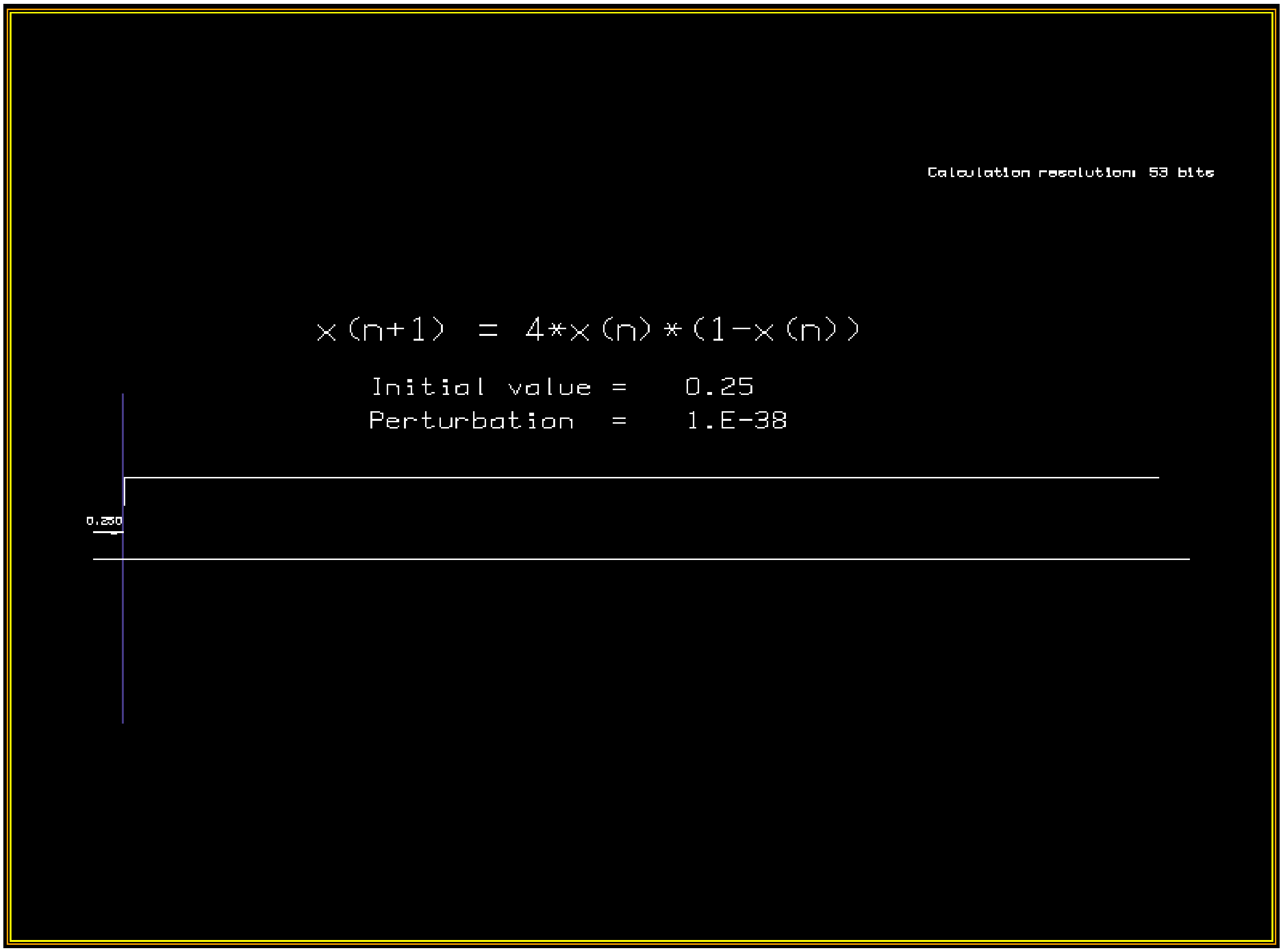
As the software that plots the diagram above is written for 32-bit IEEE-754 format, which cannot distinguish any number less than 1.18×10^{-38} from zero, a zero perturbation is written on the plotted diagram as 1.0e-38. That is 0 and 1.0e-38 are not distinguishable by the software.



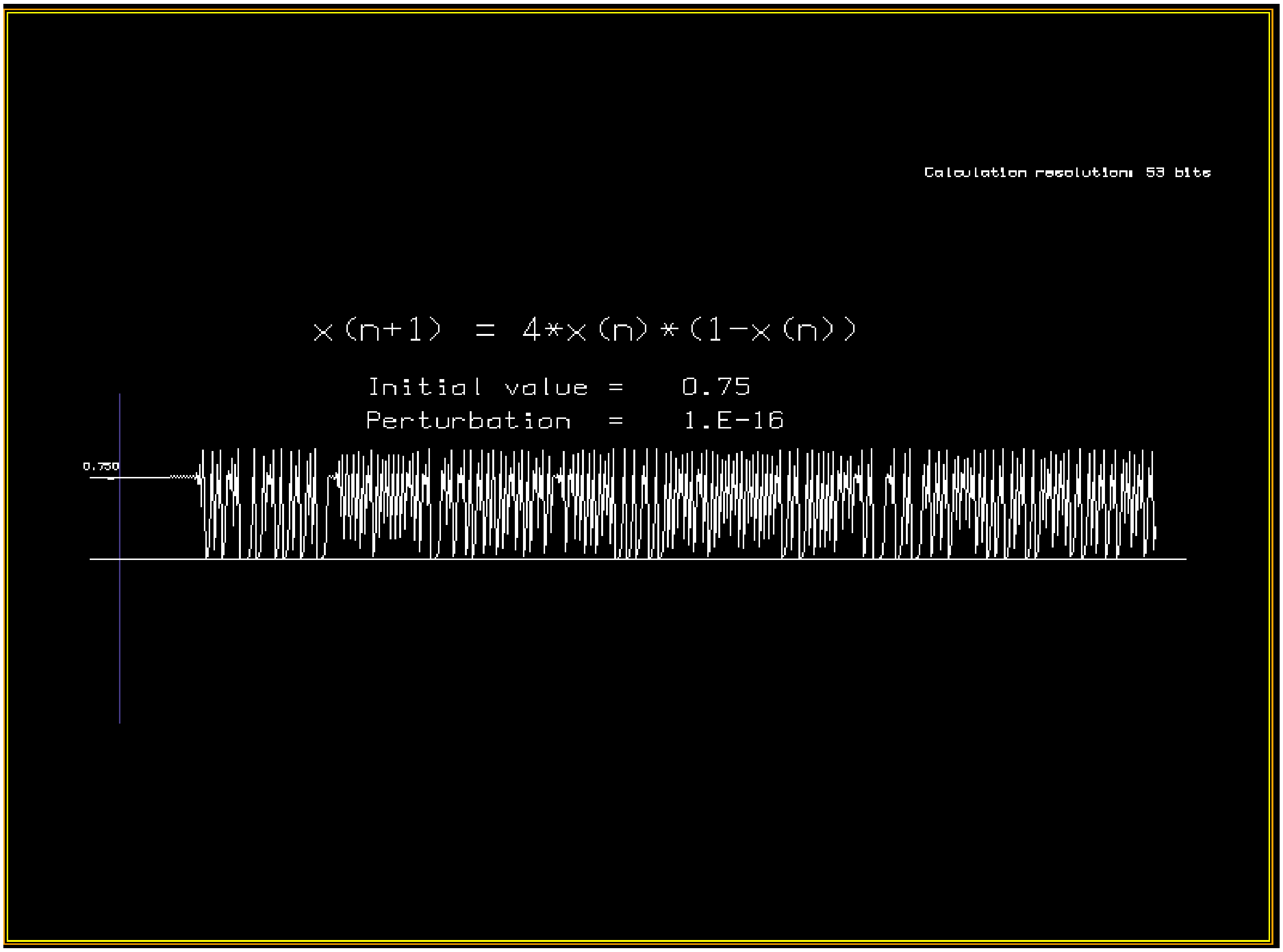
This diagram shows how the current value relates to that 3 steps earlier. Essentially, the full range of values in the interval (0, 1) are covered.



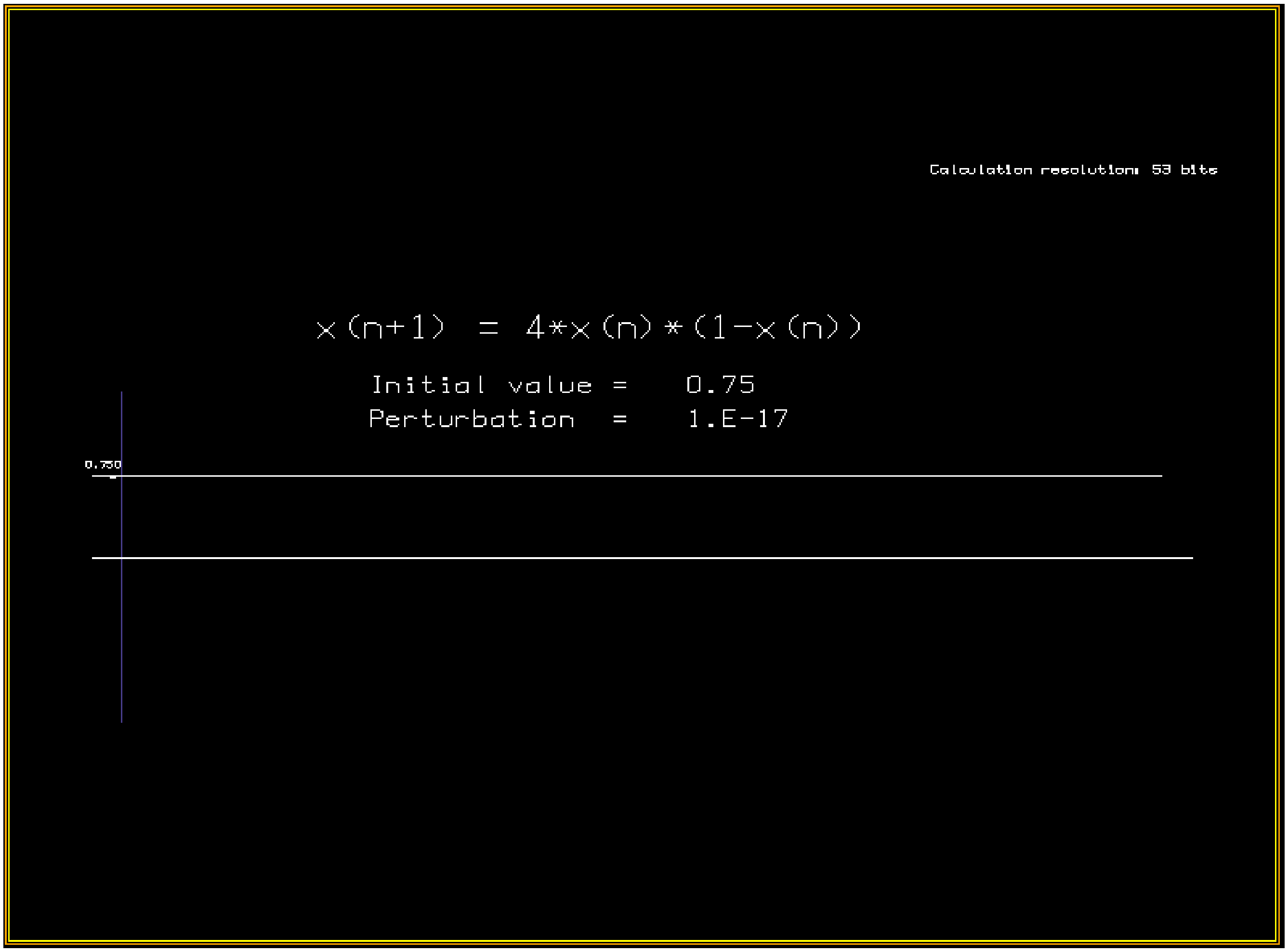
The Verhultz system does, though have a repeatable value of 0.75. Starting with that rational number, ¾, it continues to hold that value in recursion. The value 0.75 is called a “strange attractor” solution of the recursion. There is another...



If we start the Verhultz system at ¼, it immediately jumps to the ¾ continuing recursion... another strange attractor!



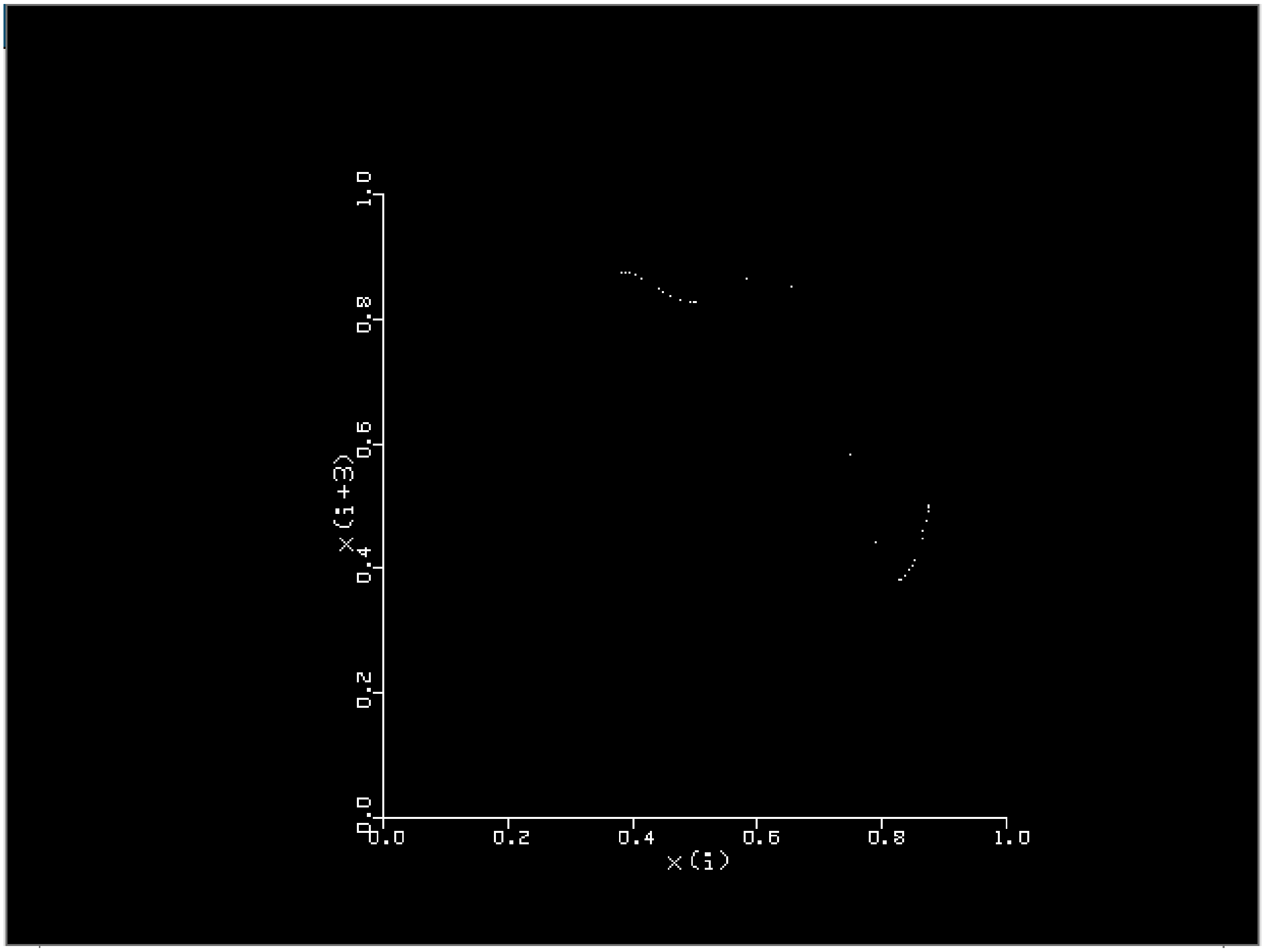
If we start at ¾ plus a very small perturbation from ¾, the system evolves away from the ¾ value with time. Here we have started at 0.7500000000000001 and the system falls into its chaotic behaviour within about 100 recursions.



This diagram is exactly as that shown immediately above but with a perturbation entered as 0.00000000000000001. You might note that the calculated recursion is identical to that for a perturbation of exactly 0. When one adds 1 x 10^{-17} to 0.75, even in double precision format, the computer does not see the sum as different from exactly 0.75 or (rational fraction) ¾.

You might wonder how the recursion evolves if the scale factor for the recursion is less than exactly 4. With a scale factor of exactly 4, the recursion is properly chaotic and while it does evolve with much oscillation for values that are somewhat less than 4, these recursions are not properly chaotic.

With a scale factor of 3.5, the recursion continues with a rather regular oscillation.



You might note that certain values in this oscillation are entirely avoided.

In this non-chaotic regime, you might not that there are several phase jumps occurring at seemingly regular intervals. If this were to be argued as a model for the geodynamo, pole reversals are happening every two or three steps with the “phase” of the reversals changing every 100 or so cycles.