<u>Geodynamics and Geomagnetism (EPSC 510)</u>

Midterm Test (Take home)

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Assigned: 20h30, October 27, 2016
Due: 12h00, October 31, 2016

<u>Instructions</u>

Answer both questions. You may, and are in fact encouraged to, work with others in the class but we ask each of you to submit an independent and separate answer to each question for separate marking.
Several links within this on-line midterm take you to necessary and/or useful and interesting information.

1. A question on Earth's slowing rotation and the Moon's retreat:

We have measured the average rate of retreat of the Moon to be **3.825 cm/year (+/- 0.0004 cm/year)** since lunar laser ranging was initiated in 1969 with the Apollo 11 landing and measured continually ever since. The Earth's rotation rate, consequently, has slowed down due to a transfer of angular momentum from the spinning Earth to the Moon's orbital momentum. If only the angular momentum transfer to the Moon from the spinning Earth is considered, what is the average rate of lengthening of the Earth's rotation period (LOD) since the ranging measurements were initiated. Offer your answer in terms of units of microseconds/year. Your answer will not accord with the best estimate of the rotational slowing averaged over the past 2700 years, namely **17.0 ± 0.005 µs/year**. Explain why.

Some data:

The semi-major axis of the Moon's orbit: $\mathbf{a_m = 384399\ km}$

The Moon's orbital period: $\mathbf{P_m = 27.321661\ days}$ (86 400 seconds/day)

The Moon's mass: $\mathbf{M_m = 7.342 \times 10^{22}\ kg}$

Earth's siderial rotation period: $\mathbf{P_e = 0.99726968\ days}$

Earth's radius: $\mathbf{r_e = 6371\ km}$

Earth's mass: $\mathbf{M_e = 5.97237 \times 10^{24}\ kg}$

Earth's moment of inertia: $\mathbf{I_e = 0.3308\ M_e\ r_e^2}$

For simplicity, you may well ignore the angular momentum change due to the balancing increase (from the Earth-Moon COM) of Earth's small orbit and assume a circular lunar orbit. These introduce only small 2^{nd} order differences in your answer.

… but if you are fussy with detail:

The Moon's orbital eccentricity: $\mathbf{e = 0.0549}$

The semi-major axis of the Earth's orbit about the Earth-Moon COM: $\mathbf{a_e = 4700\ km}$

Angular momentum of the Moon in its eccentric orbit: $\mathbf{\underline{L} = M_m\ a_m^2\ (1 - e^2)^{1/4}\ \underline{\omega}}$

On Keplerian orbits: http://sappho.eps.mcgill.ca/~olivia/GG/2016-Autumn/On_elliptic_orbits.pdf (Planetary and satellite orbits are approximately 2-body Keplerian)

On the history of the Moon's orbit: http://onlinelibrary.wiley.com/doi/10.1029/RG004i004p00411/pdf (Note that this paper precedes lunar ranging measurements.)

2. Isostasy, Rheology and Sea Level

a) Thickness of Continental Crust

Estimate the average thickness of the continental crust, using the concept of isostasy and all of the following assumptions:

The oceanic crust is in the upper 10% of the oceanic lithosphere.

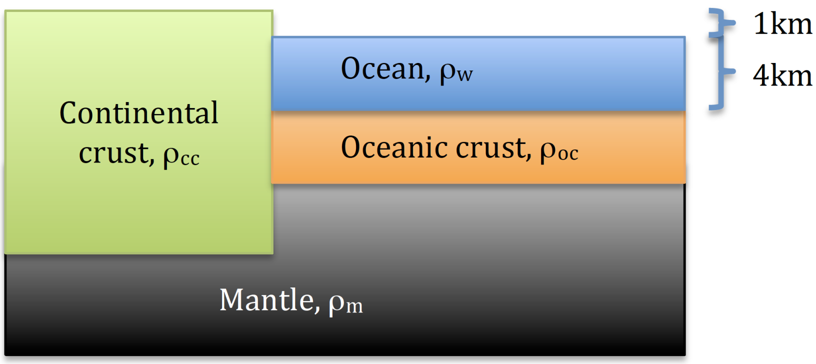
The average age of oceanic lithosphere is 40 Myr.

The average ocean depth is 4km.

The average elevation of land is 1km.

The density of the mantle under the oceans is the same as the density of the mantle under the continents.

The following cartoon might help you to think about the problem.



b) The Earth as a Maxwell Viscoelastic Material

Determine the stress, *F(t)*, as a function of time needed to maintain constant strain $e(t)=e_0$ in a 1-D Maxwell viscoelastic body. What are the elastic and viscous strains as a function of time (sketch them out on the same plot)? At what time are these two equal? What is the physical meaning of this time? Estimate this value for the Earth, approximating the Young's modulus as 10^{11} N/m^2 and the viscosity as 10^{21} Pa*s.

c) Ice Loss and Sea Level Rise

Let's imagine that marine-based sectors of the West Antarctic Ice Sheet catastrophically collapse over the next 100 years, contributing 2×10^6 km^3 of ice above floatation to the oceans. ("ice above floatation" means that we are considering only the ice volume left over after filling in the "hole" left behind in marine sectors freed of ice with water.) Estimate the global average sea level rise associated with this melt event. Now, using a figure from the course notes, estimate the sea level rise that would occur in the middle of the Atlantic Ocean immediately following the melt event. Assuming the Earth has a viscoelastic rheology, describe (in words, but sketches are welcome in addition to the words), how the gravitational equipotential sea surface elevation (*G*), solid surface elevation (*R*) and sea level (*SL*) would change through time at that location following the melt event, 100 years, 1,000 years and 1,000,000 years into the future.