

A wide, flat beach at low tide with a path leading to the water under a cloudy sky.

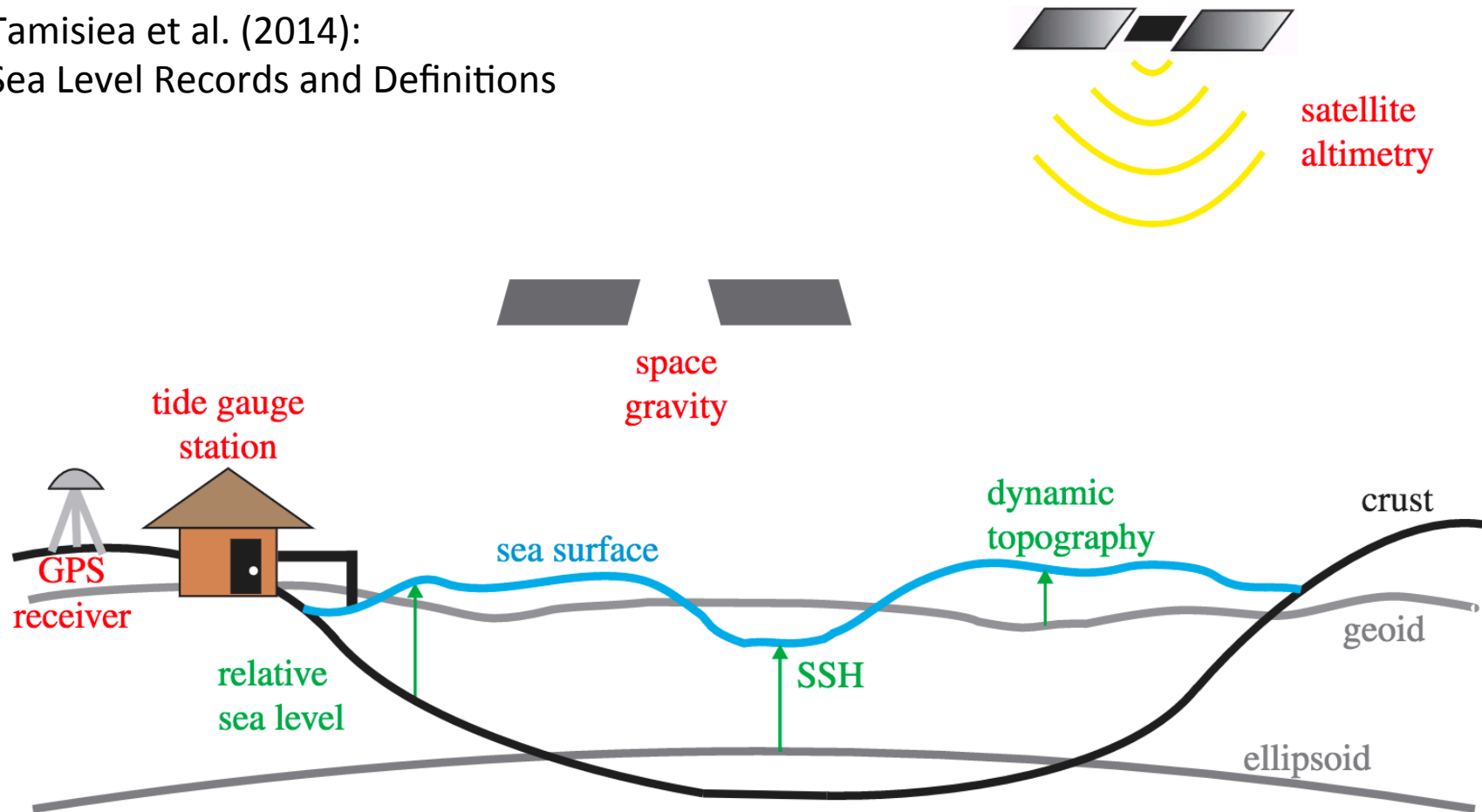
# EPSC510 Module 2

## Lecture 3: Sea Level Change Continued

James Bay, Ontario, photo credit: Natalya Gomez 2008

# Sea Level Records and Definitions

Tamisiea et al. (2014):  
Sea Level Records and Definitions

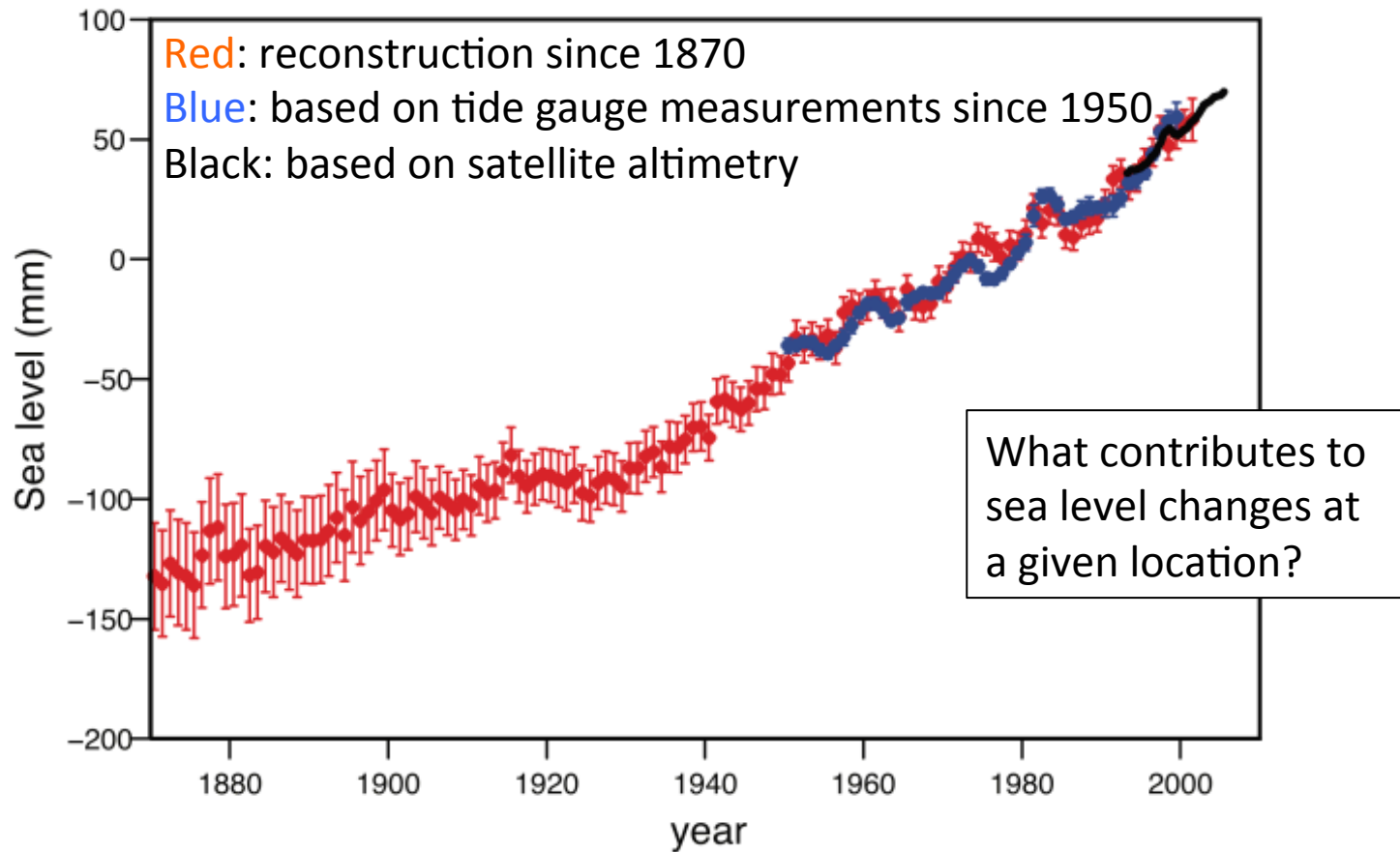


**Figure 1.** Simple schematic illustrating the relationship between sea surface height (SSH), the geoid, and dynamic topography. Included on the figure are representations of different components of the observing system and their respective measurement: GPS (or GNSS) for crustal deformation, satellite gravity for the geoid, altimetry for SSH and tide gauges for relative sea level.



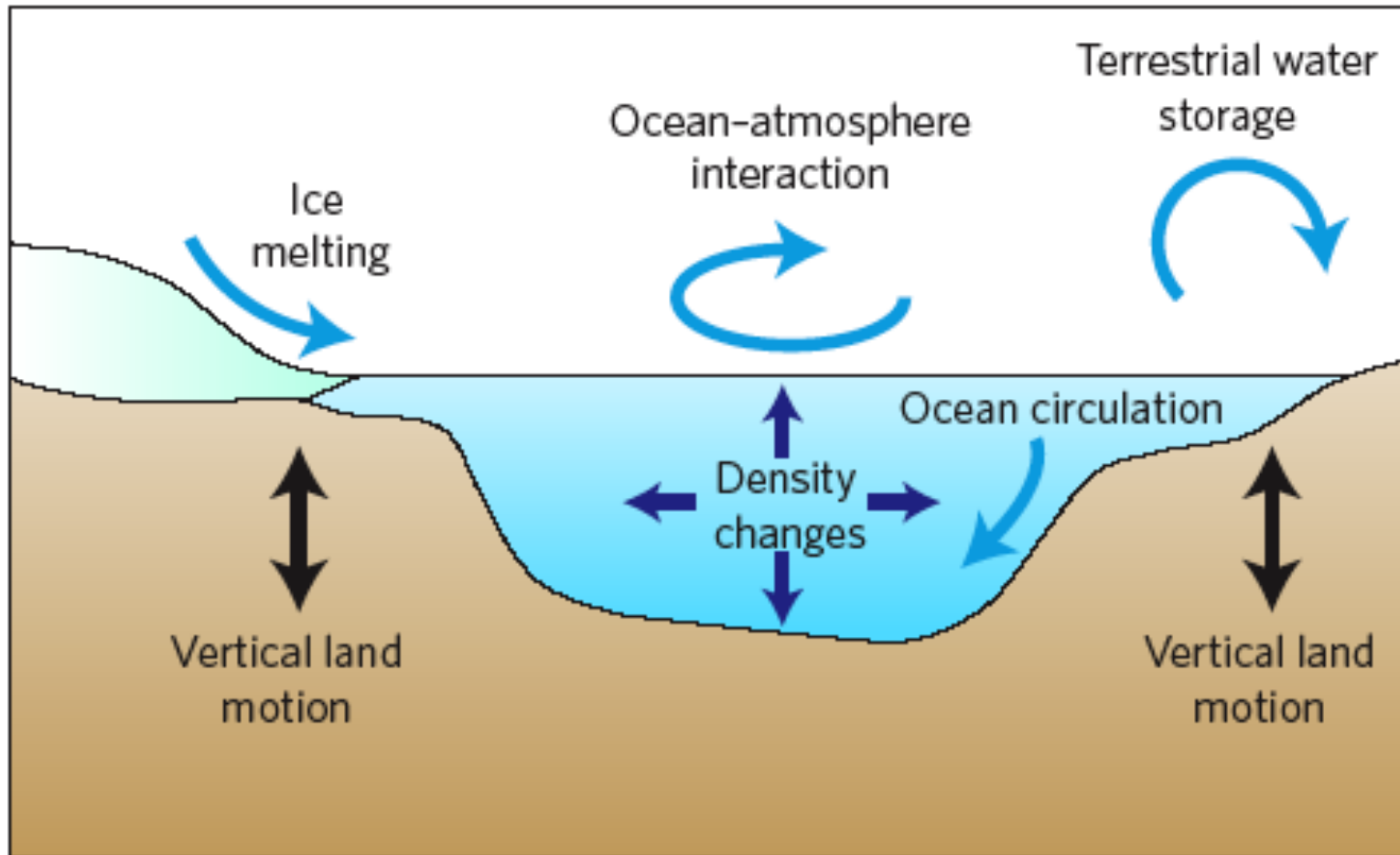
# Global Average Sea Level Change

## Annual averages of global sea level



Bindoff et al., (2007). Observations: Climate Change and Sea Level. In: *Climate Change 2007: The Physical Science Basis. Contribution of WG1 to the 4<sup>th</sup> Assessment Report of the IPCC*. USA

# Sea Level Change: Physical Processes



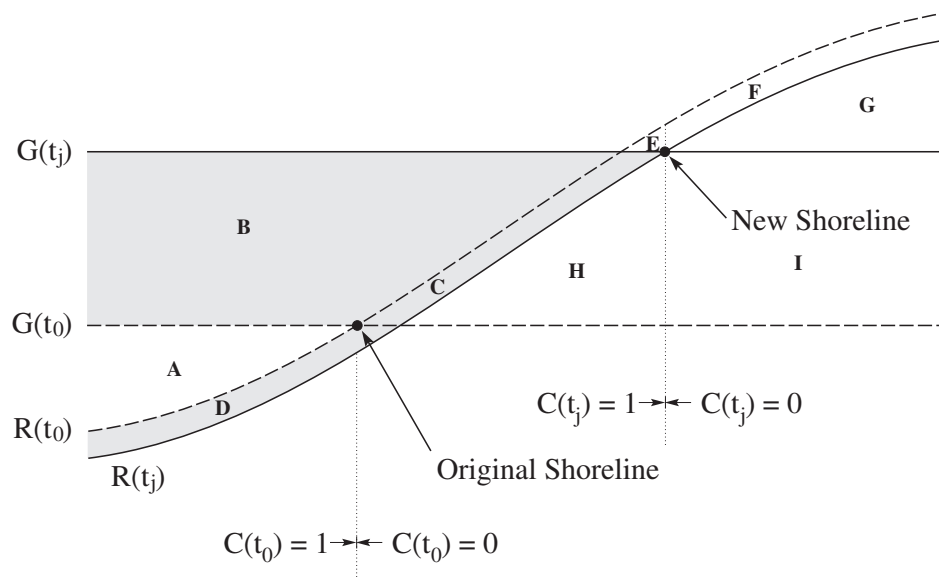
Contributions to Modern Sea Level Change

# The Sea Level Equation

## Generalized Sea-Level Equation

$$\Delta S(\theta, \psi, t_j) = \left[ \Delta \mathcal{S}\mathcal{L}(\theta, \psi, t_j) + \frac{\Delta \Phi(t_j)}{g} \right] C(\theta, \psi, t_j) - T(\theta, \psi, t_0) [C(\theta, \psi, t_j) - C(\theta, \psi, t_0)]$$

Projection of sea-level change at  $t_j$  onto the ocean function at time  $t_j$



A topographic correction term to account for shoreline migration

# The Sea Level Equation

## SEA LEVEL MODEL INGREDIENTS

$$\Delta \mathcal{S}\mathcal{L}(\theta, \psi, t) = \Delta \mathcal{S}\mathcal{L}(\Delta I, \Delta S, \Delta \omega)$$

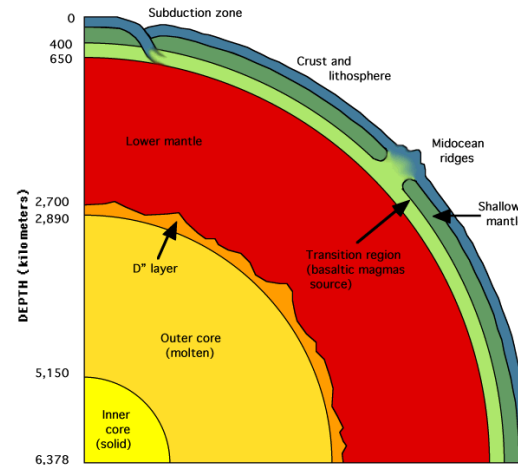
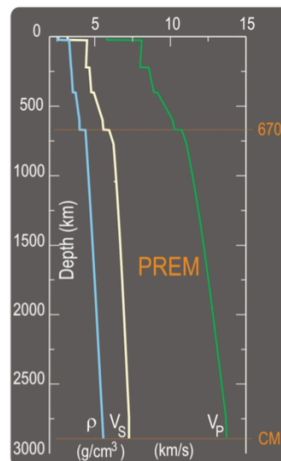
EARTH FORCING

Ice location and thickness  
(inferred from modeling  
GIA data, e.g. sea level)

EARTH MODEL

Earth geometry and elastic,  
density and viscosity\*\*  
structure (PREM, GIA data)

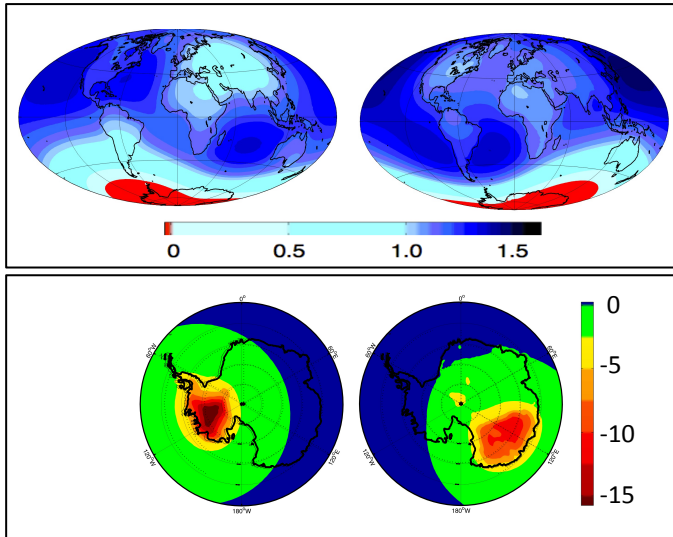
Physics...



\*\*strong  
temperature  
dependence,  
not well  
constrained

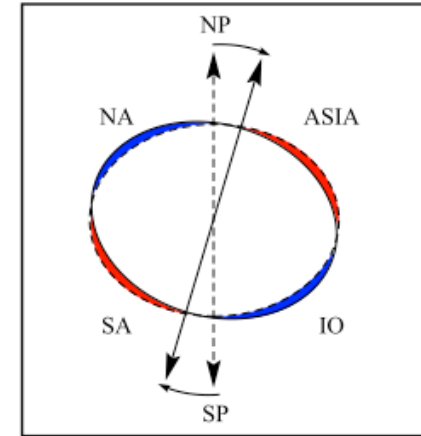
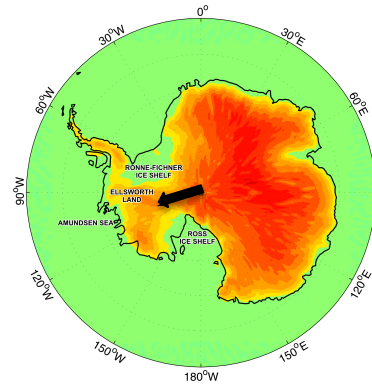


# Sea Level Physics: short timescales

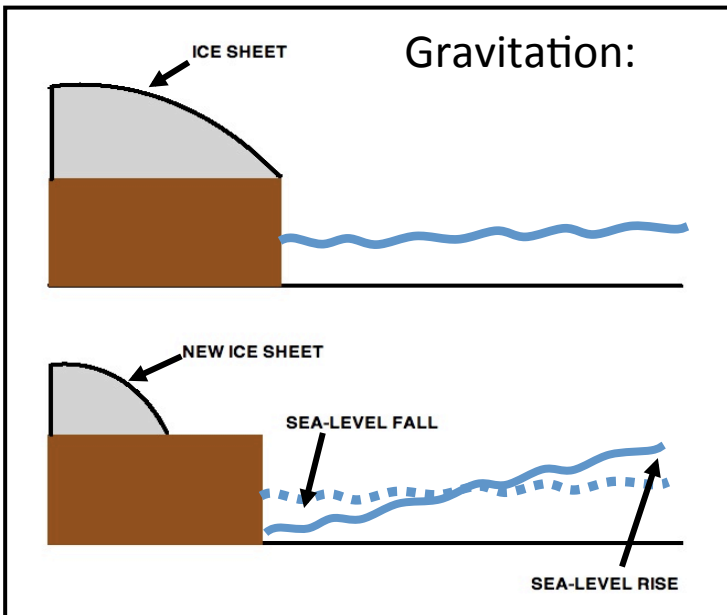


Gomez et al., GJI, 2010

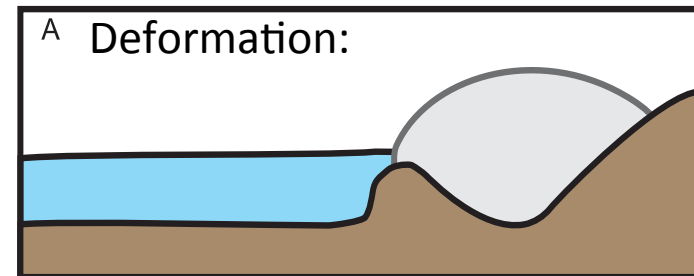
Rotation:



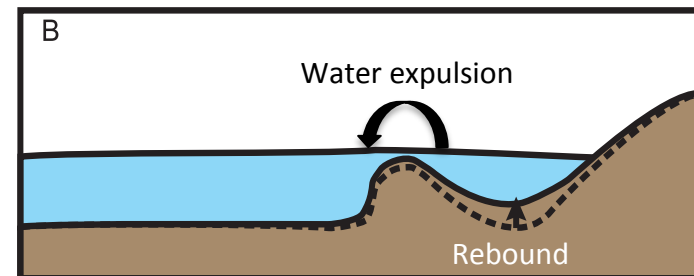
Gravitation:



A Deformation:



B



# A bit of history

Woodward  
(1888)

## On Postglacial Sea Level

W. E. Farrell\*

Cooperative Institute for Research in Environmental Sciences, University of Colorado/NOAA,  
Boulder, Colorado 80302, USA

J. A. Clark

Institute of Arctic and Alpine Research and Dept. Geological Sciences, University of Colorado,  
Boulder, Colorado 80302, USA

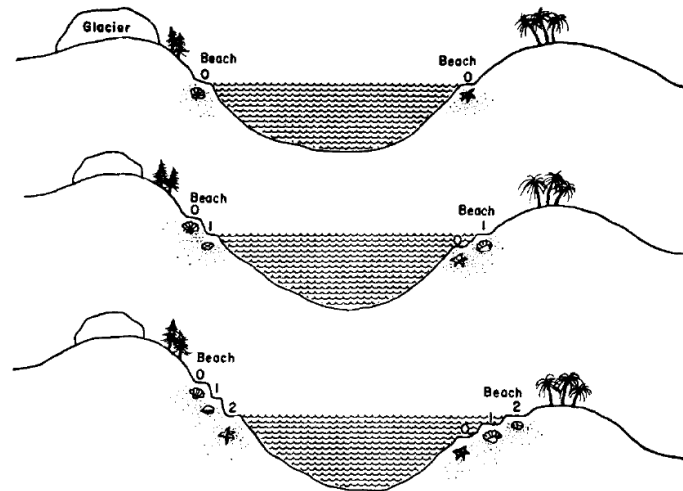



FIG. 6. Schematic illustration of the sea level changes occurring on a viscoelastic earth model. (a) The situation in the isostatic state before any melting of the ice.

... Next major  
developments by:  
Richard Peltier,  
Kurt Lambeck,  
Jerry Mitrovica,

*Geophys. J. R. astr. Soc.* (1976) **46**, 647–667.


From Farrell & Clark, 1976: spherically symmetric, rigid Earth density field. The gravitational potential on and outside the Earth's surface is


$$\phi(r) = \frac{\Gamma M_E}{r}, \quad r \geq a$$

where  $\Gamma$  is Newton's gravitational constant and  $M_E$  is the Earth's total mass. Suppose we extract a mass  $M_I$ , representing an ice sheet, from a thin spherical shell (the ocean) at  $r = a$  and locate  $M_I$  at a point on the Earth's surface. ( $M_E - M_I$  is still spherically symmetric.) If  $\theta$  measures the angular distance away from the point mass, the new gravitational potential field with reference to origin 0 is

$$\phi^*(r, \theta) = \frac{\Gamma(M_E - M_I)}{r} + \frac{\Gamma M_I}{\sqrt{(r^2 + a^2 - 2ar \cos \theta)}}$$

Along the Earth's surface the new potential field


$$\phi^*(a, \theta) = \frac{\Gamma(M_E - M_I)}{a} + \frac{\Gamma M_I}{2a \sin(\theta/2)}$$

is not constant and hence the spherical surface  $r = a$  is not a possible sea level. There is a nearby equipotential, however, at a radial distance  $\varepsilon(\theta)$  away from  $a$  on which

$$\phi^*(a + \varepsilon, \theta) = \phi(a).$$

Since  $M_I \ll M_E$ , we expect  $\varepsilon \ll a$  and, to first order in  $\varepsilon$ ,

$$\phi^*(a + \varepsilon, \theta) = \phi^*(a, \theta) + \varepsilon \frac{\partial \phi^*(a, \theta)}{\partial r}.$$

But to sufficient accuracy,  $\partial \phi^*/\partial r = \partial \phi/\partial r = -g$ , the acceleration of gravity at the Earth's surface, hence

Writing  $g/\Gamma = M_E/a^2$ , we have

$$\Delta S\mathcal{L} \longrightarrow \varepsilon(\theta) = \frac{M_I a}{M_E} \left( \frac{1}{2 \sin(\theta/2)} - 1 \right).$$


Along the surface  $r = a + \varepsilon$ ,  $\phi^*$  is constant, and thus the surface represents a possible sea level, but it is not the actual sea level because we have not allowed for the volume lost from the ocean and added to the ice. If  $a + \varepsilon$  is an equipotential, then for any constant  $c \ll a$ ,  $a + \varepsilon + c = a + \varepsilon^*$  is also an equipotential, and  $c$  can be found by conserving the total mass in the system. It is easy to show that the volume contained between the two surfaces  $r = a$  and  $r = a + \varepsilon$  is zero, so to conserve mass we must have

$$\int_0^\pi 2\pi \rho_\omega c a^2 \sin \theta d\theta + M_I = 0$$

where  $\rho_\omega$  is the density of sea water. This yields

$$\frac{\Phi}{g} \longrightarrow c = - \frac{M_I}{M_E} \frac{a \rho_E}{3 \rho_\omega}$$






$$\varepsilon^*(\theta) = \frac{M_I a}{M_E} \left( \frac{1}{2 \sin(\theta/2)} - 1 - \frac{\rho_E}{3\rho_\omega} \right). \quad (1)$$

Since  $\phi^*(a + \varepsilon^*, \theta)$  is constant, and the mass lost by the ocean (that contained between the surface  $r = a$  and  $r = a + \varepsilon^*$ ) equals the mass of the point ice cap  $\varepsilon^*$  is the correct change in sea level for this idealized example.  $\varepsilon^*$  is the change in sea level because it is the difference between the radial distance to the final sea surface and the radial distance to the initial sea surface.

**Table 1**

*Relative sea level change on a rigid Earth when water from the ocean is frozen at a point*

	$\theta$ (degrees)	$R$
	1	+30.54
	10	+1.65
~2200 km 	20	+0.05
	45	-0.83
	60	-1.00
	90	-1.16
	180	-1.28

# Outline

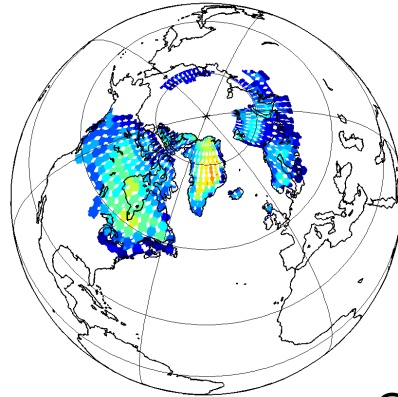
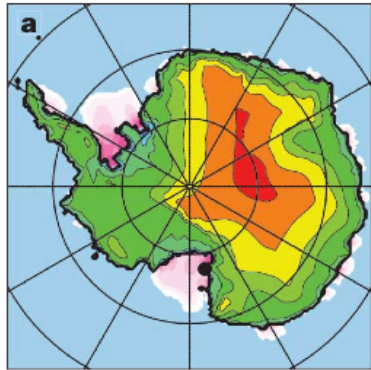
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This Class: Sea Level Change Continued...

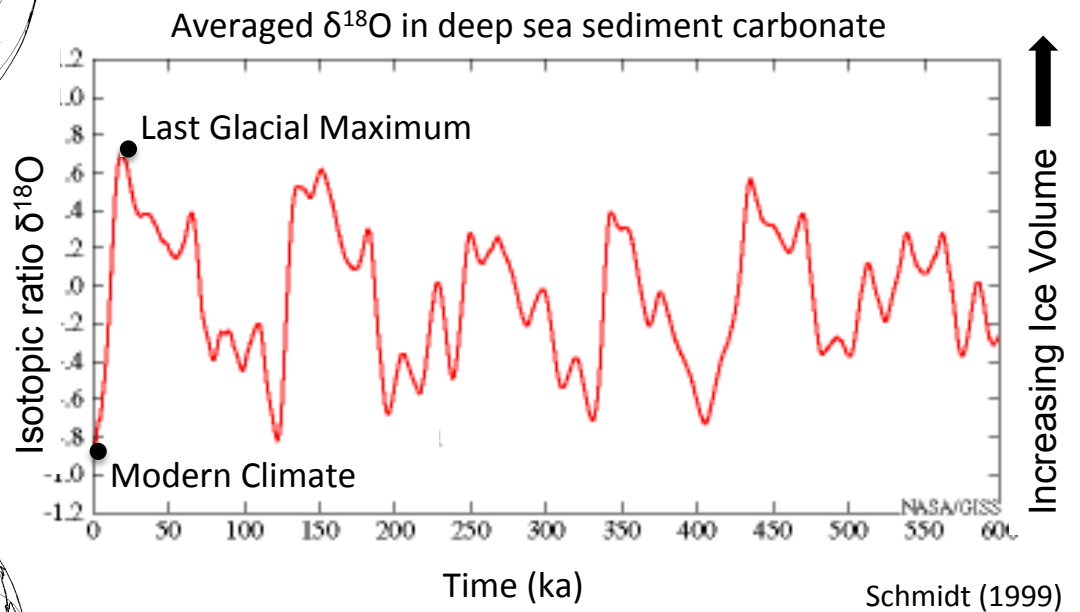
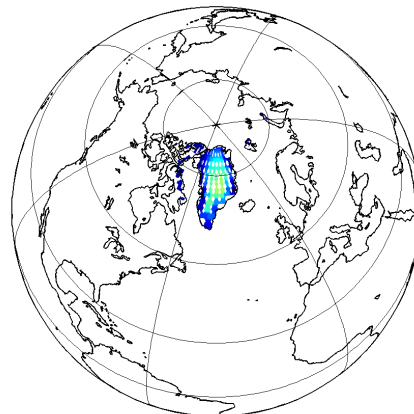
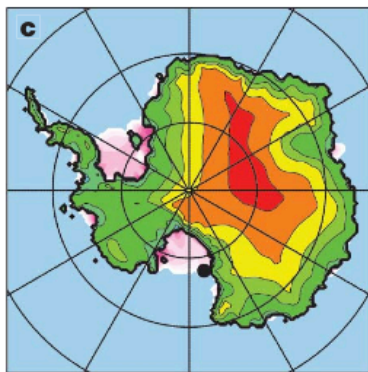
1. Sea level change and GIA on ice age timescales.
2. An Example Calculations
3. Applications
  1. Short timescale modern: 20<sup>th</sup> Century Tide Gauge Analysis
  2. Short timescale paleo: Meltwater Pulse 1A (~14ky ago)
  3. GIA: Archaeological evidence for recent acceleration in sea level rise (Holocene – last 2 ky)
  4. Ice age timescale: Sea Level during the Last Interglacial (~125 ky ago)

# Sea Level Physics: Ice-Age Timescales

## Last Glacial Maximum



## Modern Climate



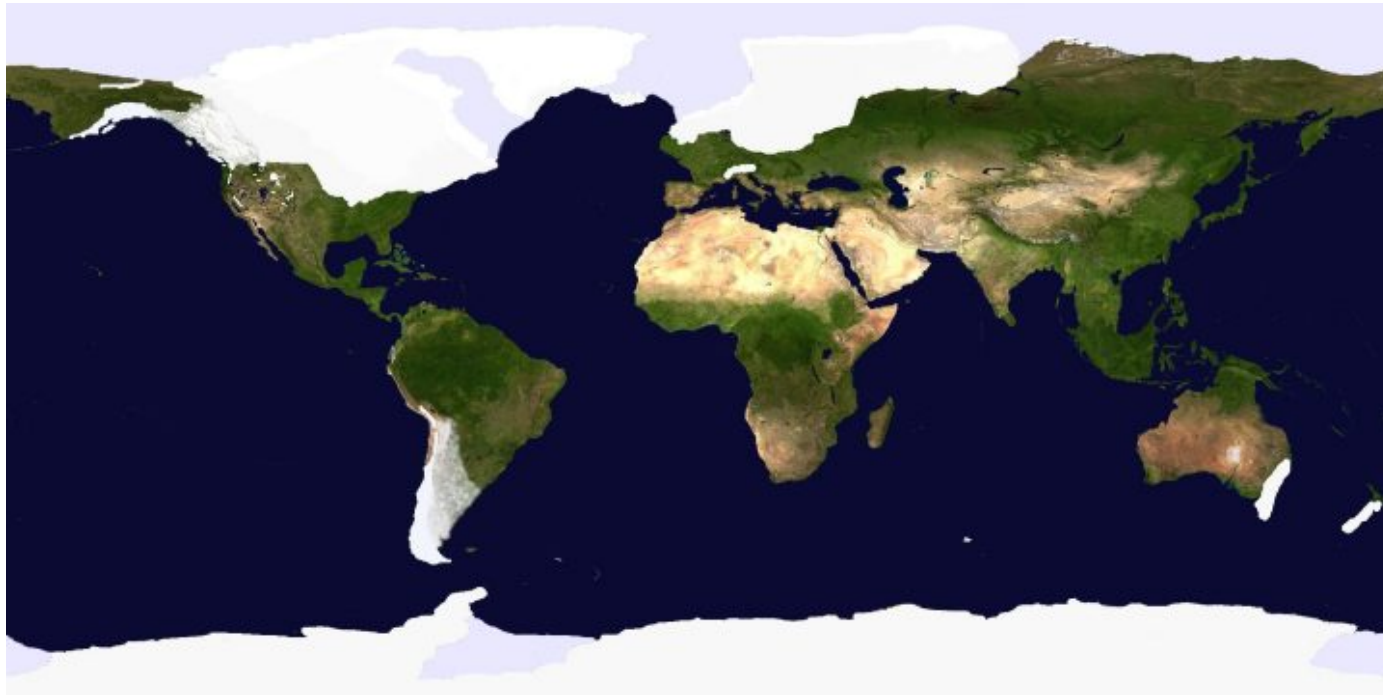
Pollard & DeConto (2009)

Peltier (2004) ICE5G

# Sea Level Physics: Ice-Age Timescales

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LGM

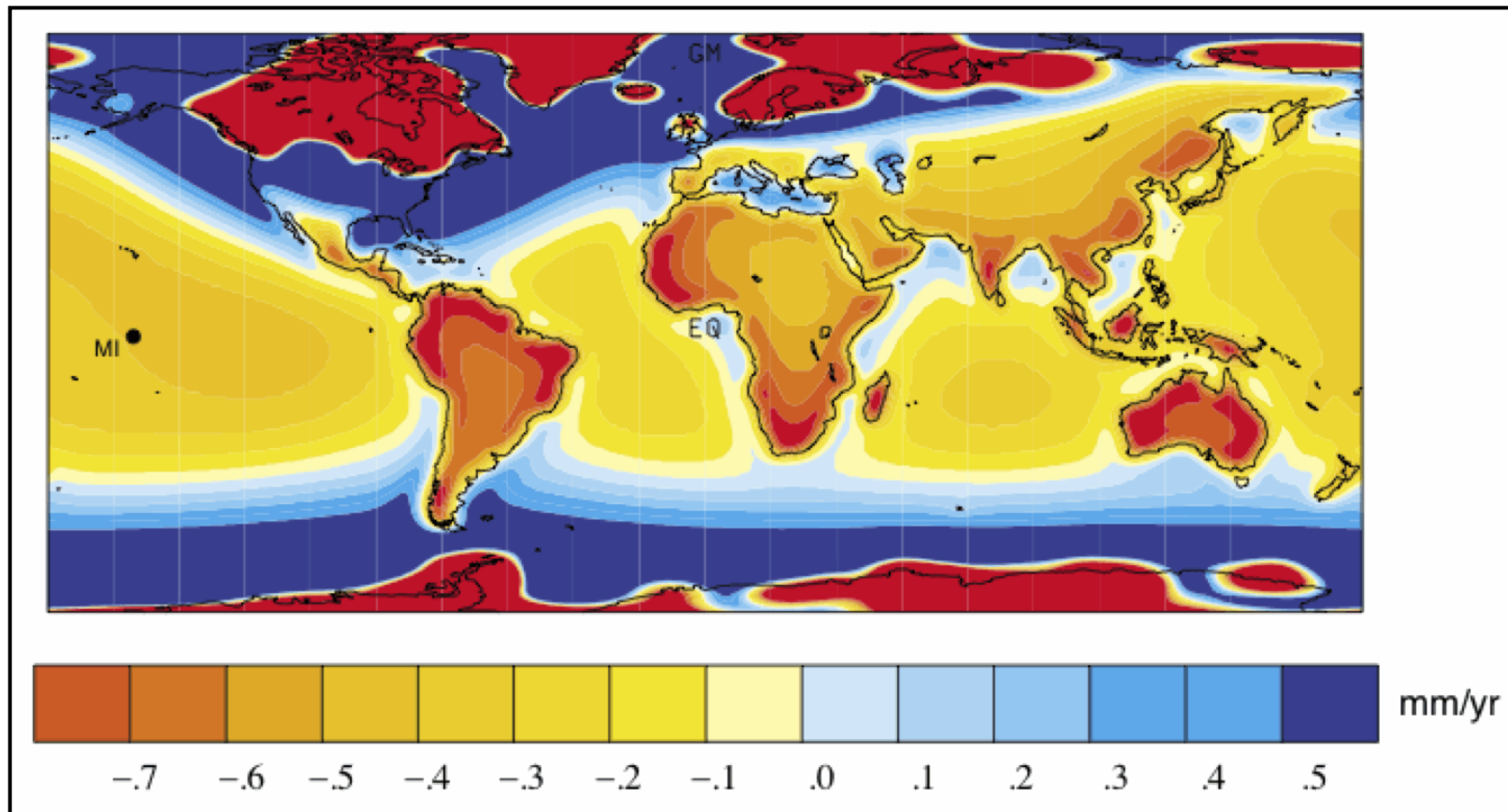


... difference in ice volume relative to the present-day is sufficient to raise globally averaged sea level by ~130 m (~ ½ of this is associated with the Laurentide ice sheet)



## 2b. Sea Level Physics: Ice-Age Timescales

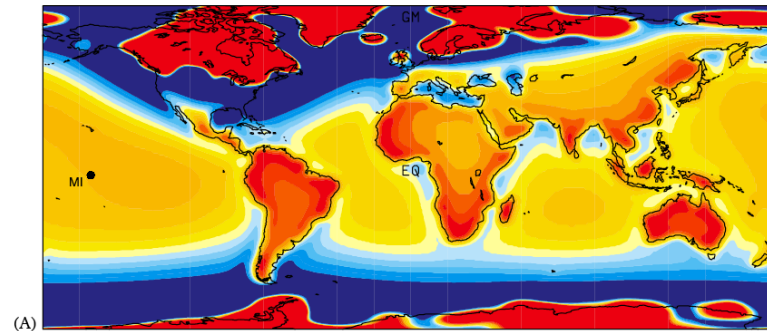
Numerical prediction of the present-day rate of change of global sea level due to ongoing GIA effects from the last ice age



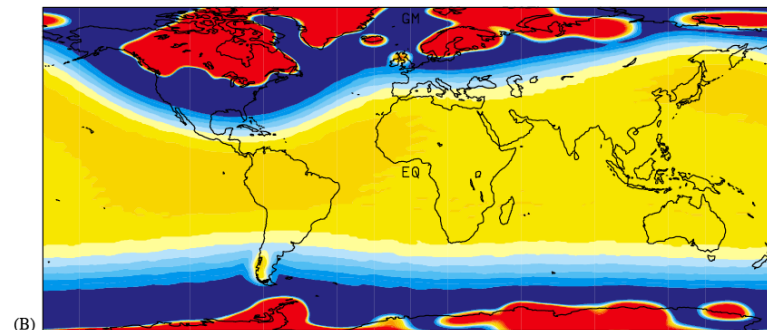
Mitrovica and Milne (2002)

## 2b. Sea Level Physics: Ice-Age Timescales

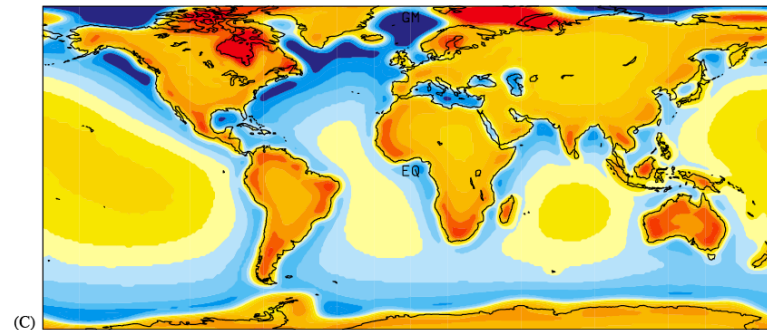
Numerical Prediction  
of Present-Day Rate  
of Global Sea-Level  
Change Due to  
Ongoing GIA



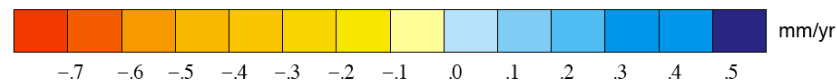
TOTAL



ICE



OCEAN

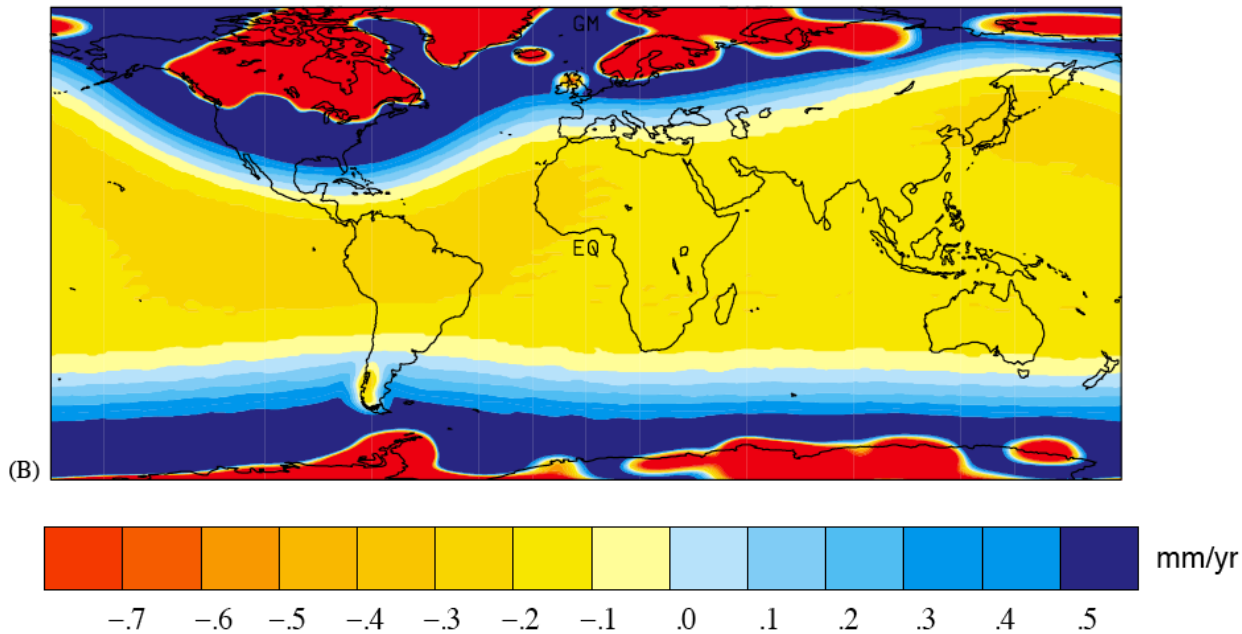


Mitrovica and Milne (2002)

## 2b. Sea Level Physics: Ice-Age Timescales

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Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA

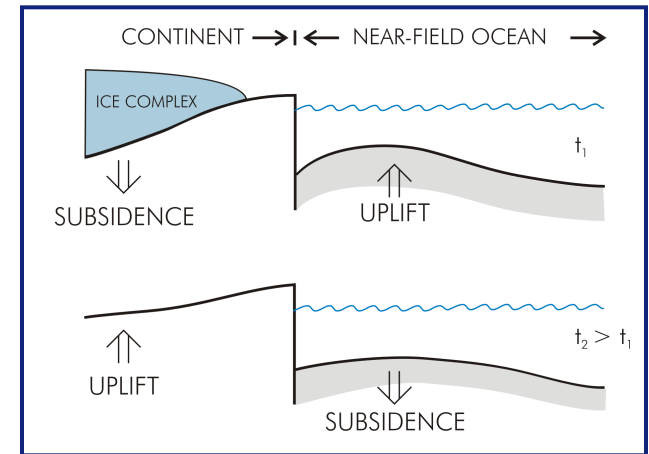
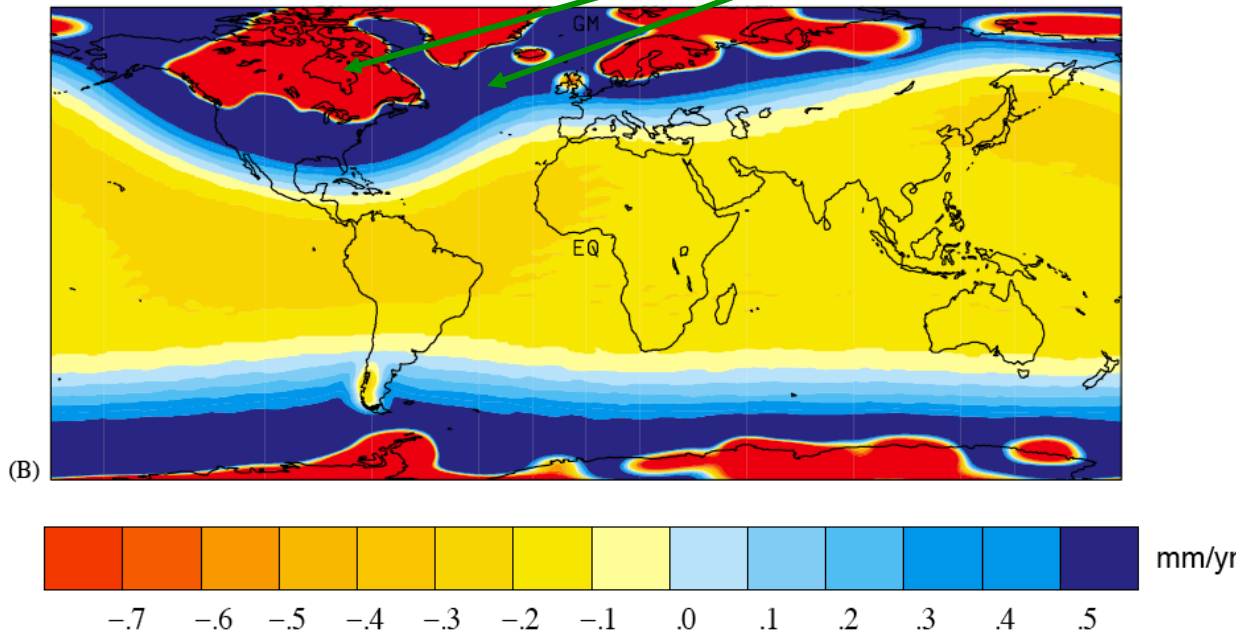


Ice Signal

## 2b. Sea Level Physics: Ice-Age Timescales

Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA

NEAR FIELD



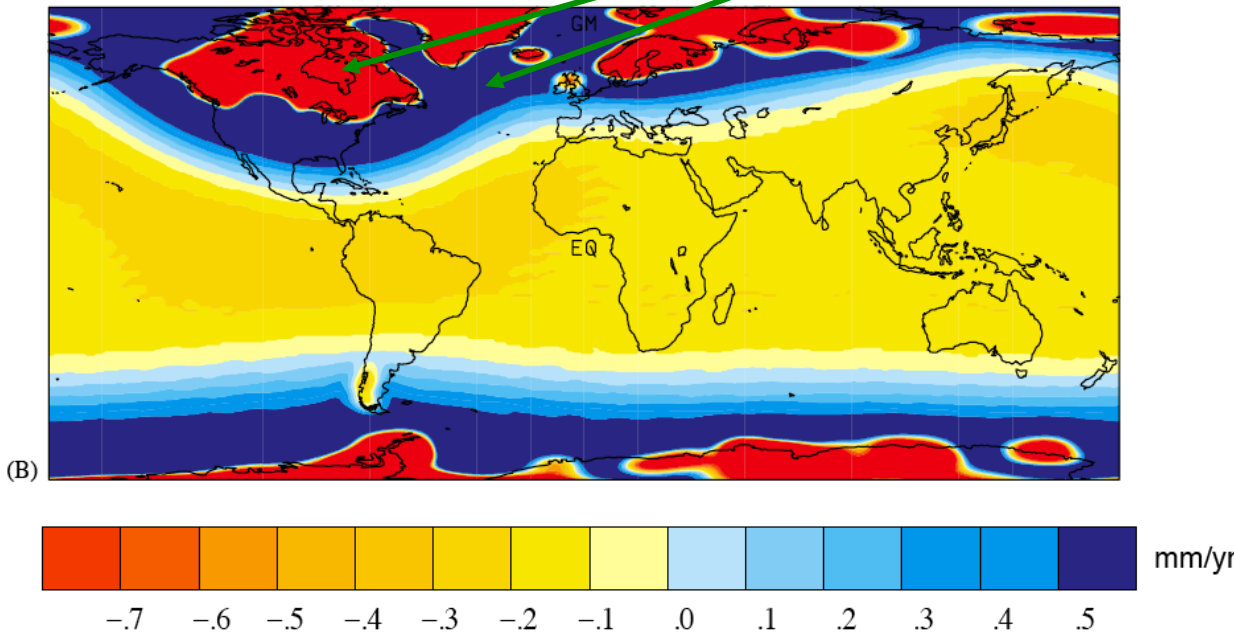
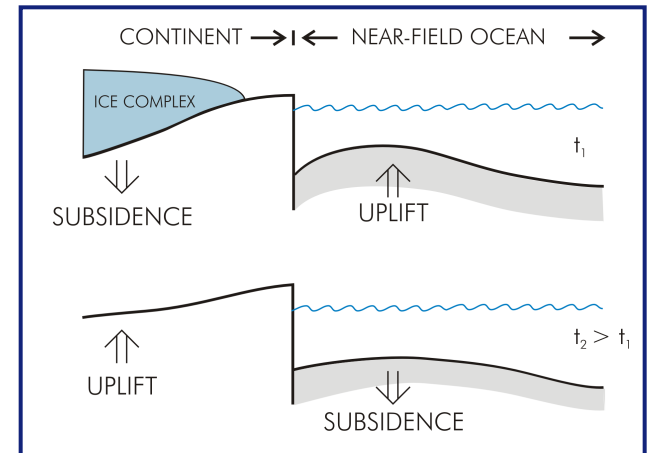
Ice Signal



## 2b. Sea Level Physics: Ice-Age Timescales

Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA

NEAR FIELD

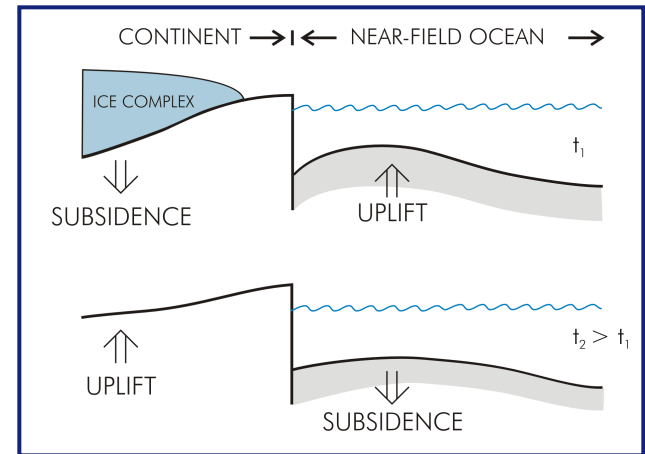
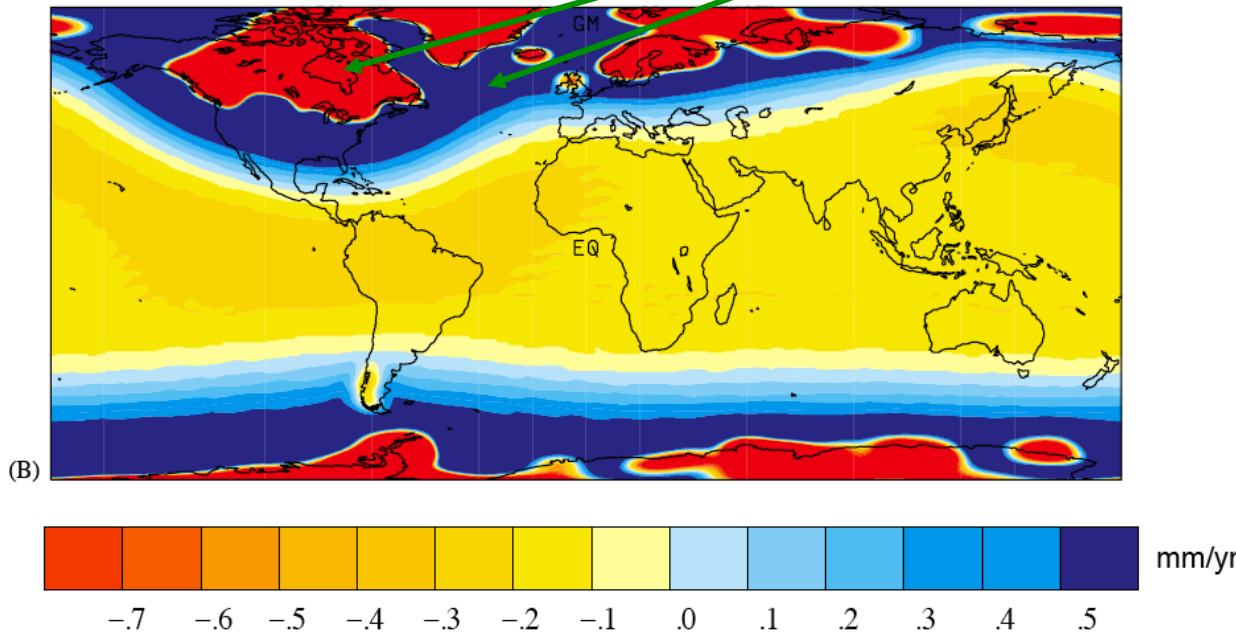


Ice Signal

## 2b. Sea Level Physics: Ice-Age Timescales

Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA

NEAR FIELD

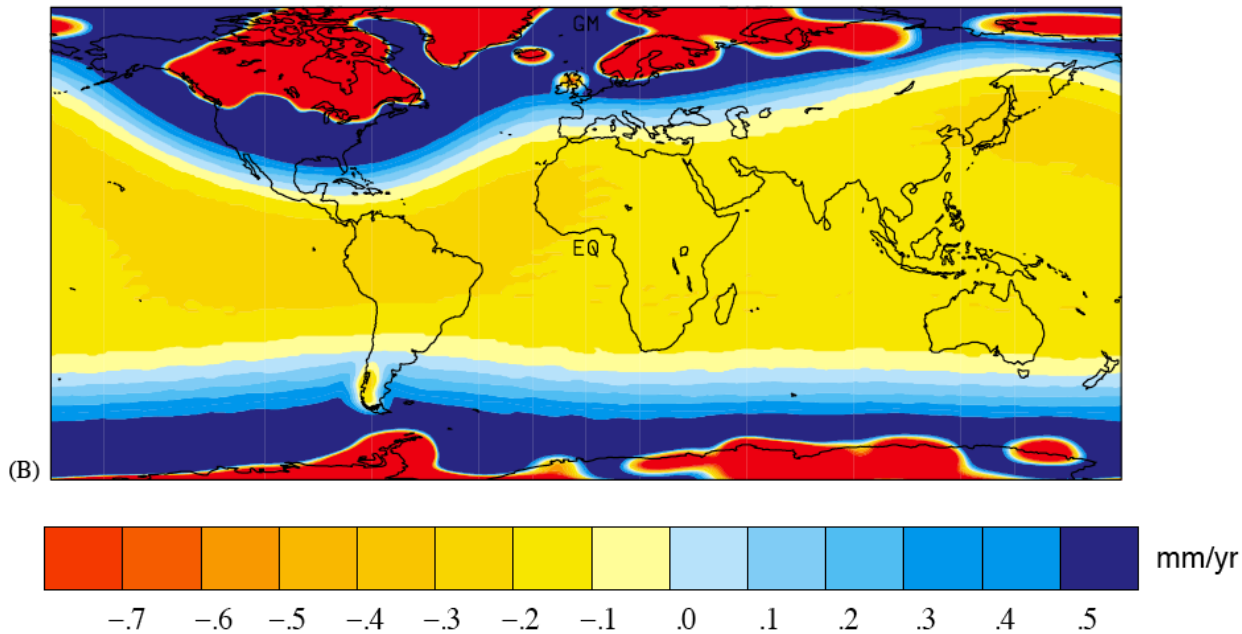


Ice Signal

## 2b. Sea Level Physics: Ice-Age Timescales

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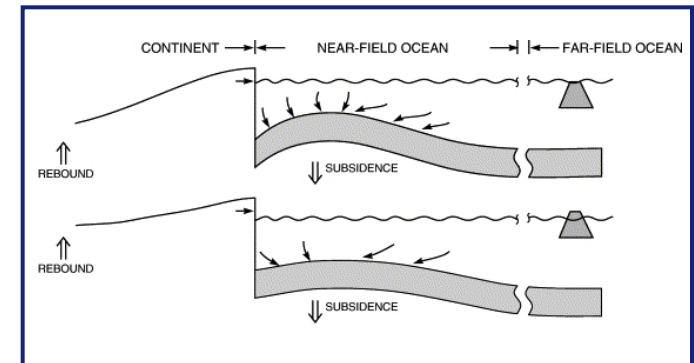
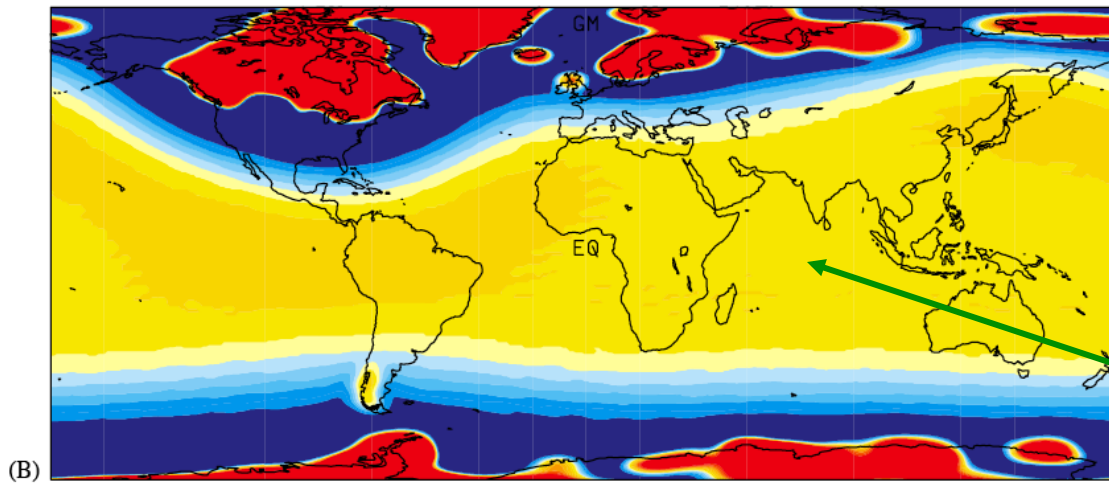
Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA



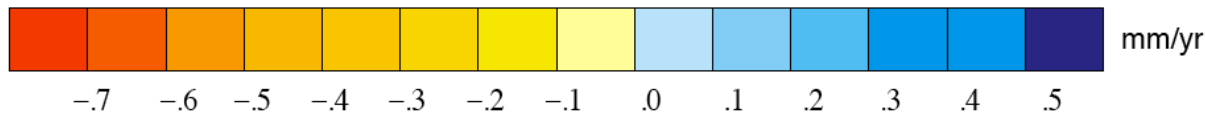
Ice Signal

## 2b. Sea Level Physics: Ice-Age Timescales

Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA



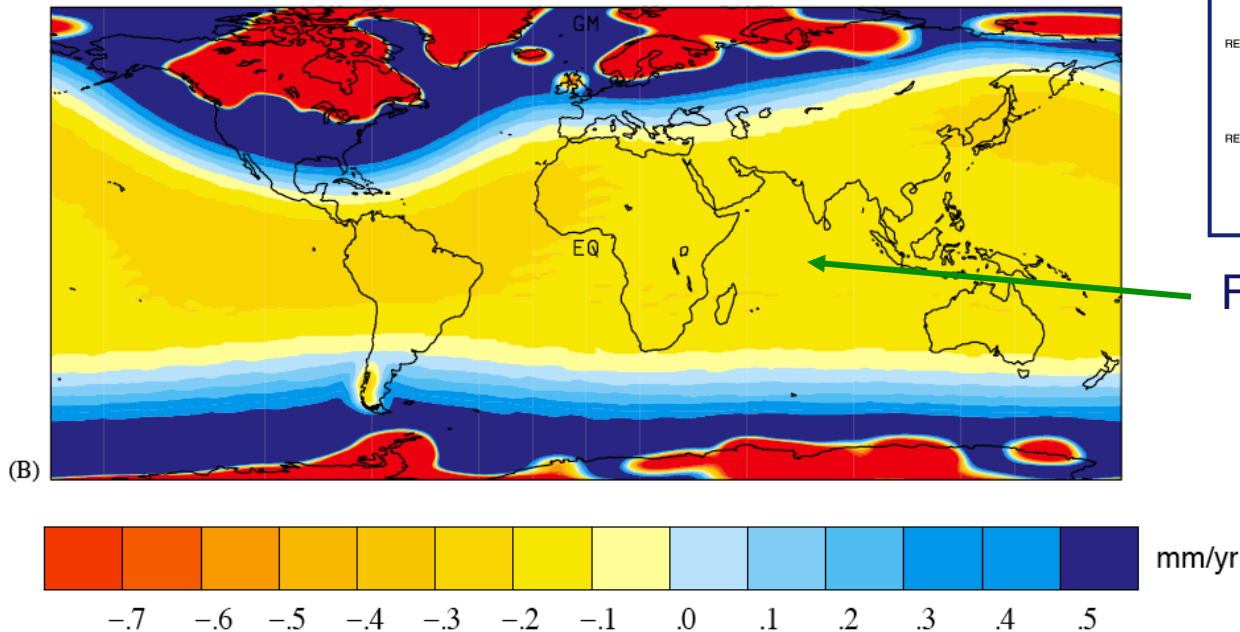
FAR FIELD



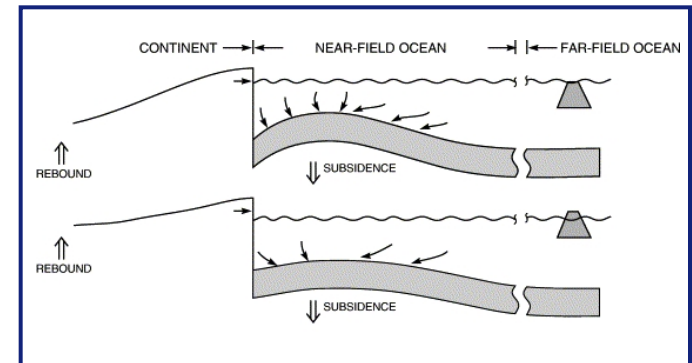
Ice Signal

## 2b. Sea Level Physics: Ice-Age Timescales

### Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA



Ice Signal



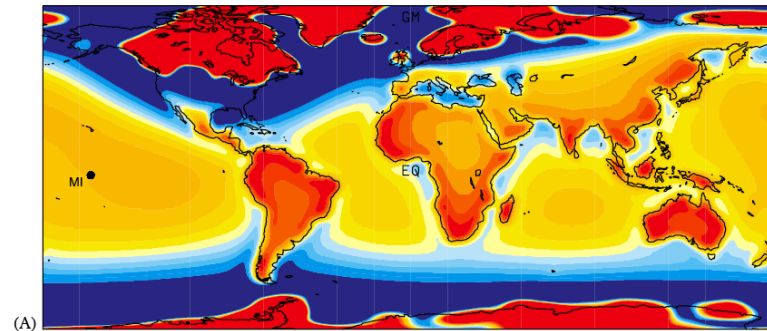
FAR FIELD



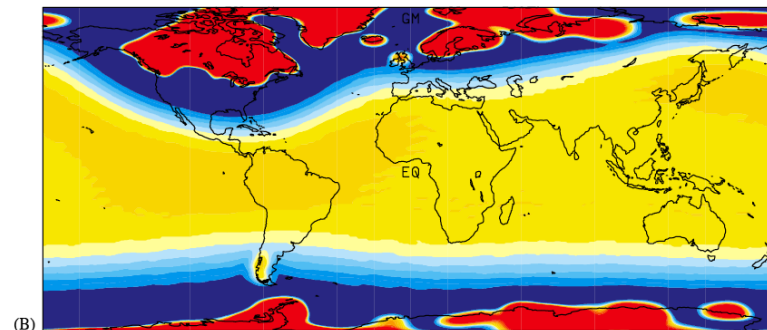
Equatorial Ocean Syphoning!

## 2b. Sea Level Physics: Ice-Age Timescales

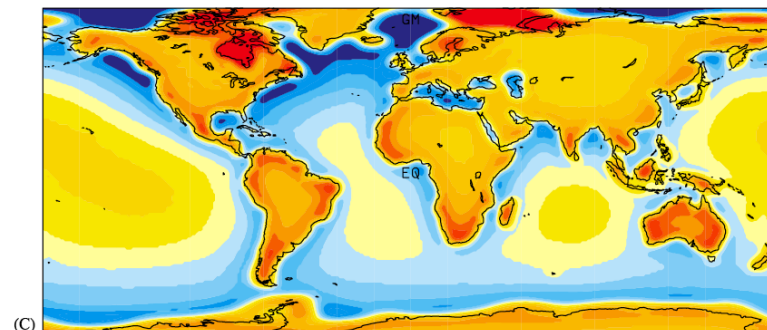
Numerical Prediction  
of Present-Day Rate  
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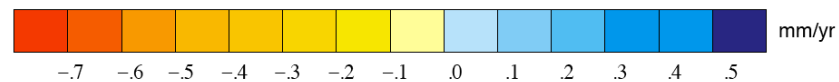
TOTAL



ICE



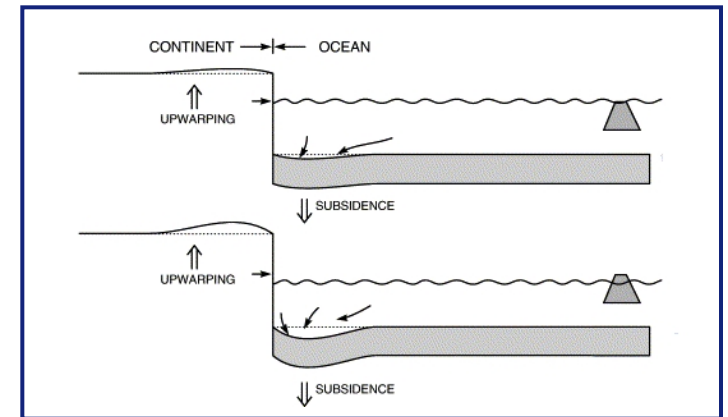
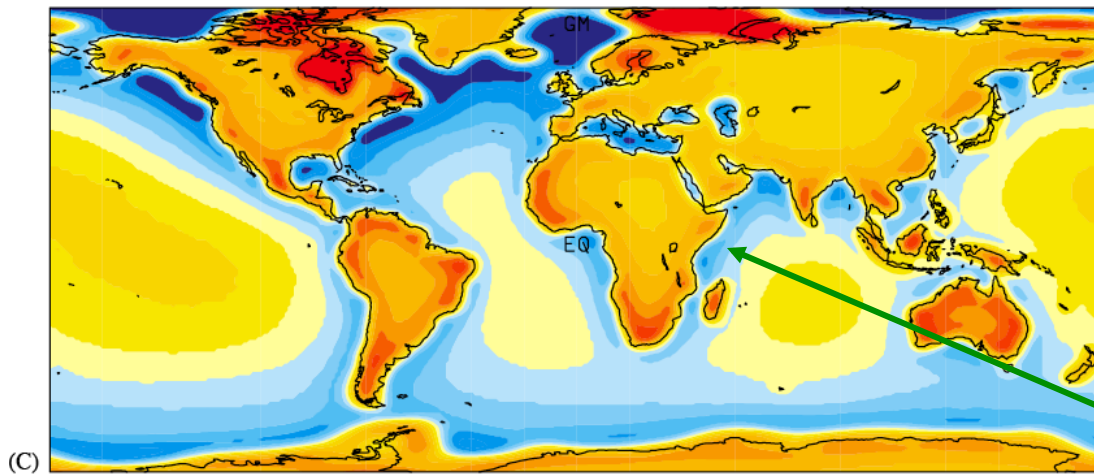
OCEAN





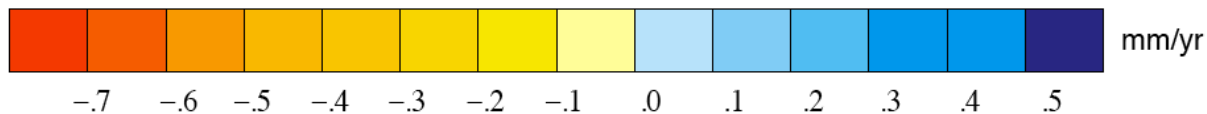
## 2b. Sea Level Physics: Ice-Age Timescales

### Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA



Continental Levering!

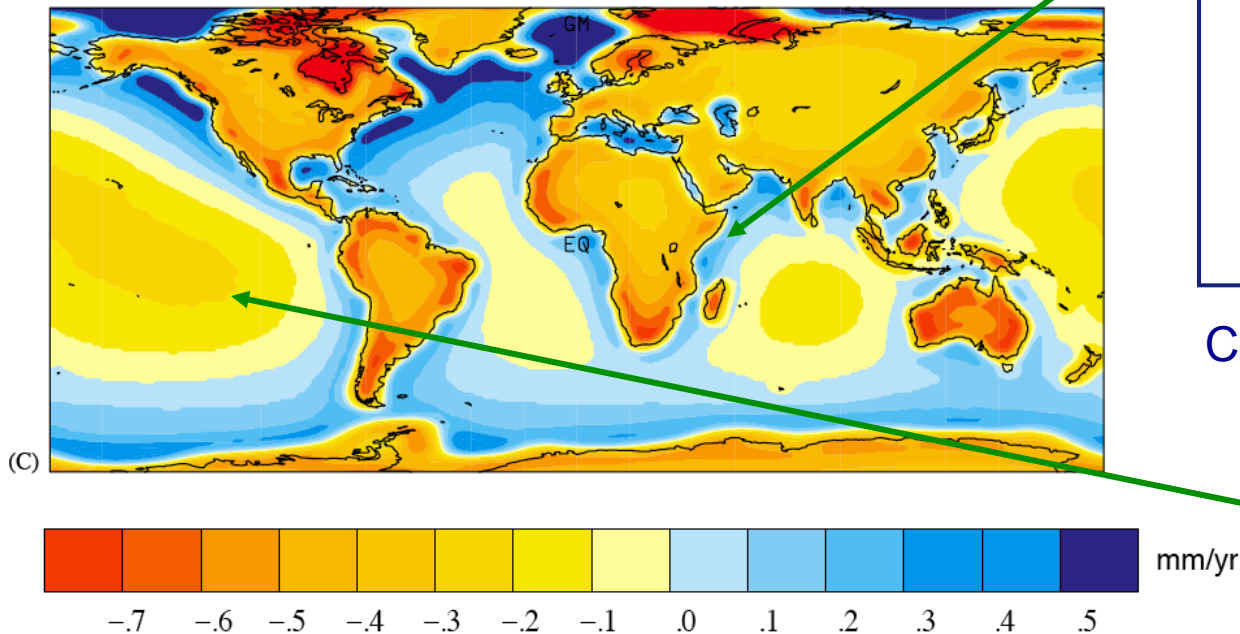
FAR FIELD



Ocean Signal

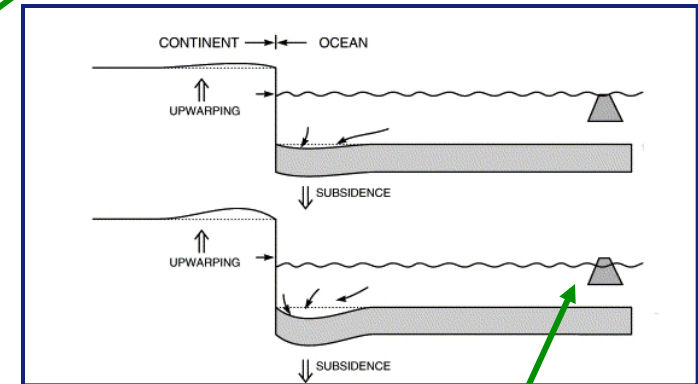
## 2b. Sea Level Physics: Ice-Age Timescales

Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA



Ocean Signal

FAR FIELD



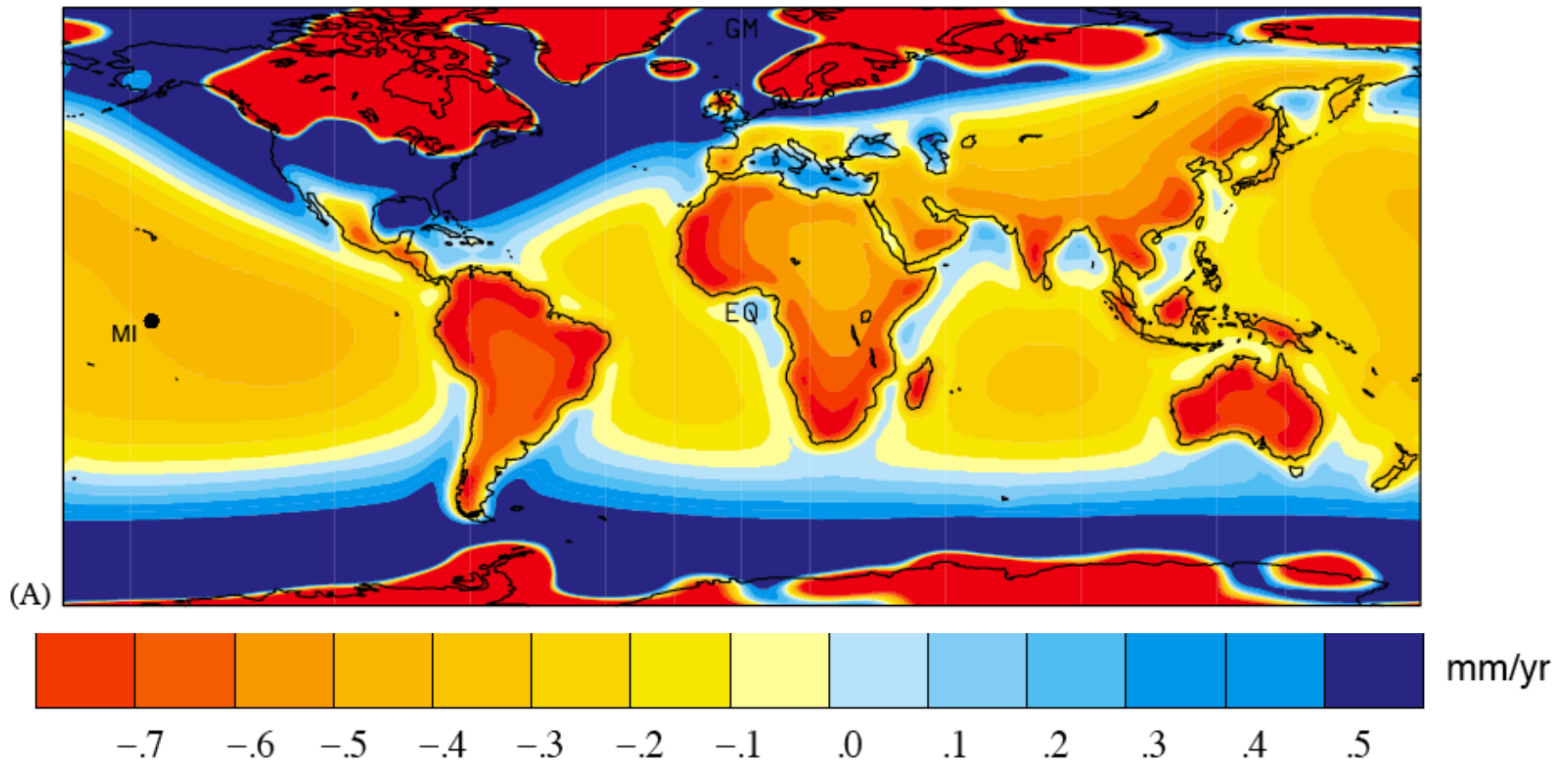
Continental Levering!

2nd contribution to equatorial ocean syphoning



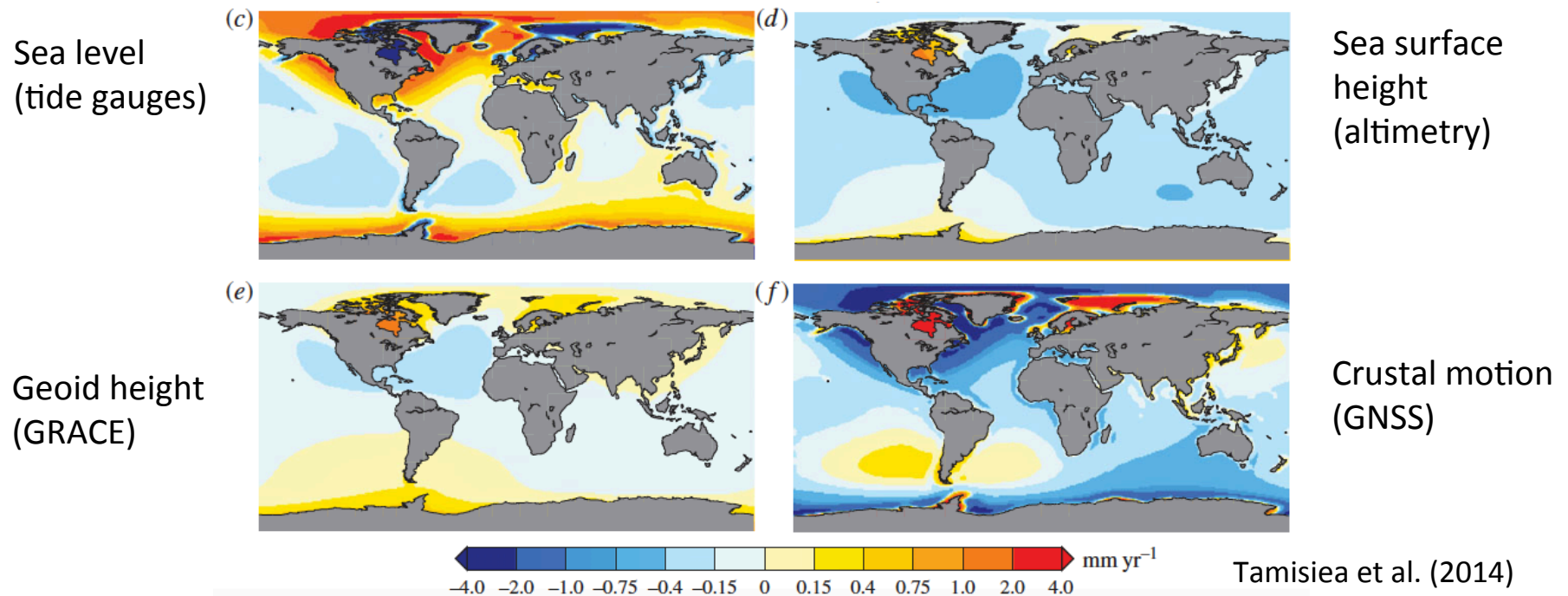
## 2b. Sea Level Physics: Ice-Age Timescales

Numerical Prediction of Present-Day Rate of Global Sea-Level Change Due to Ongoing GIA



## 2b. Sea Level Physics: Ice-Age Timescales

### Expressions of GIA in modern sea-level records

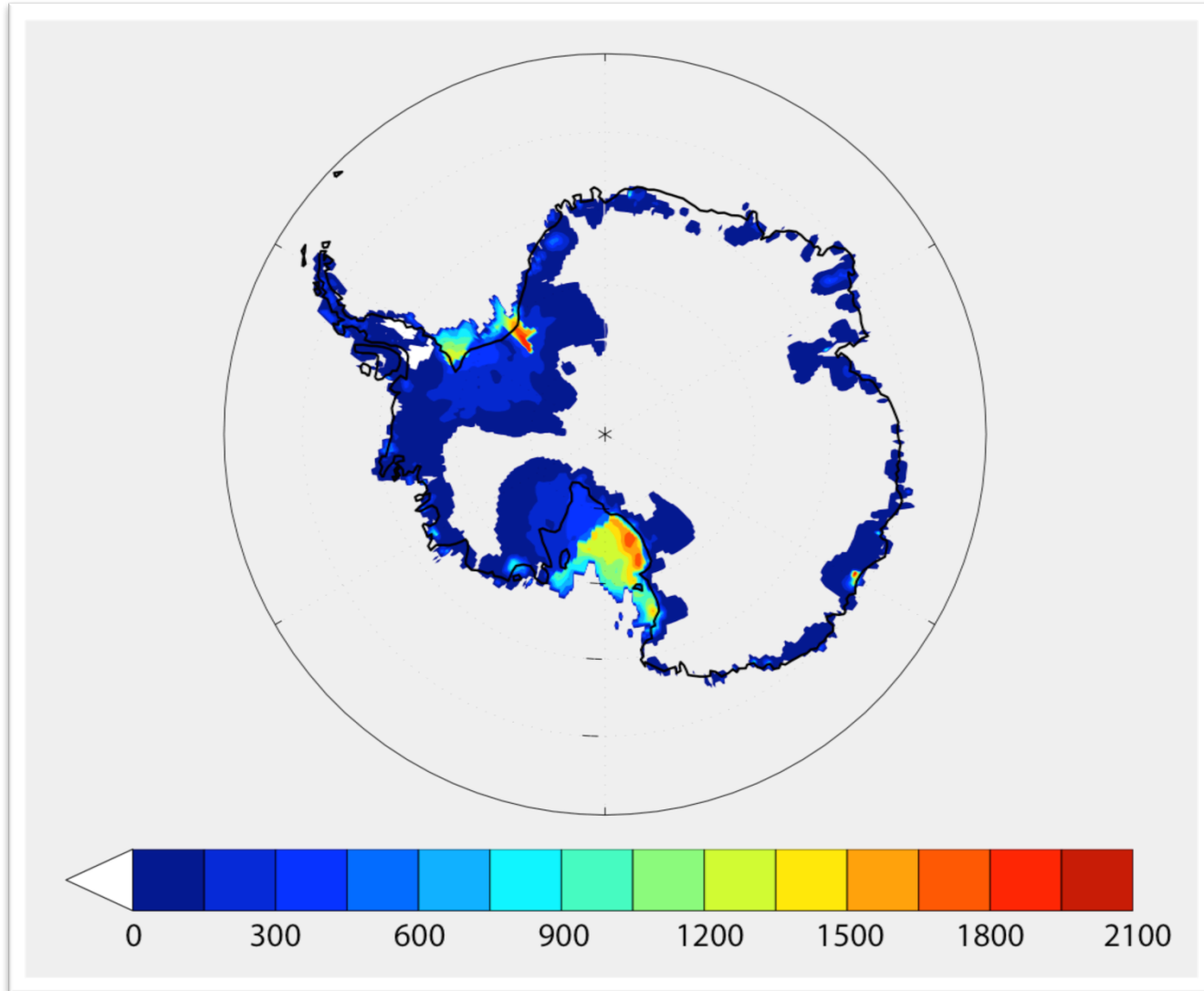


All modern observations of sea-level-related quantities are impacted by past ice and ocean loading changes!

# 3. Sea Level Change Example

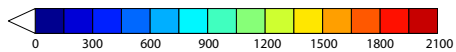
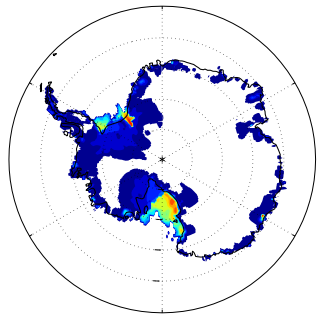
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Ice Loss Scenario:  
eustatic value = 1.8 m (after filling the holes)

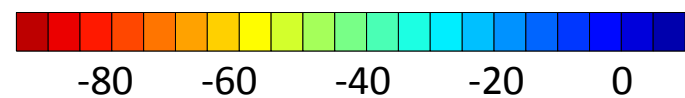
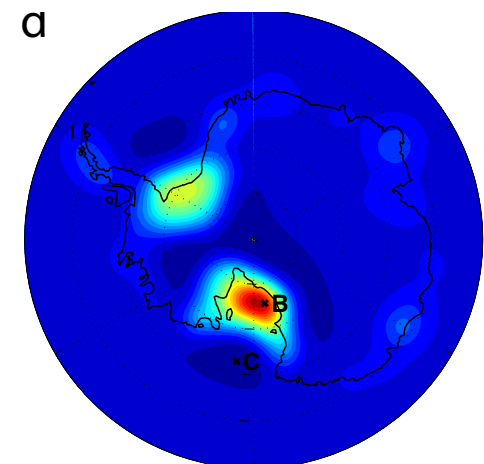
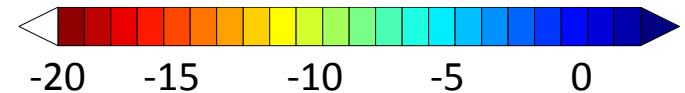
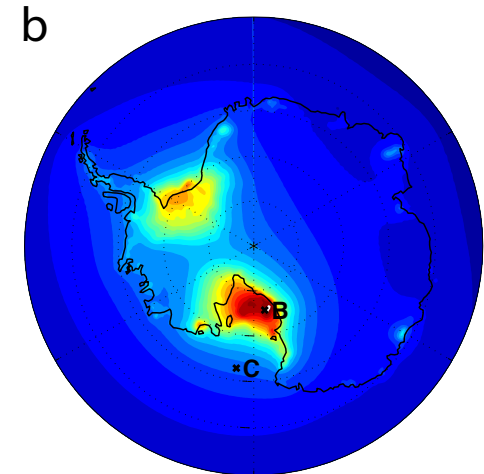
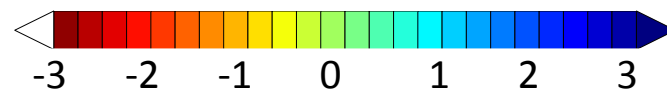
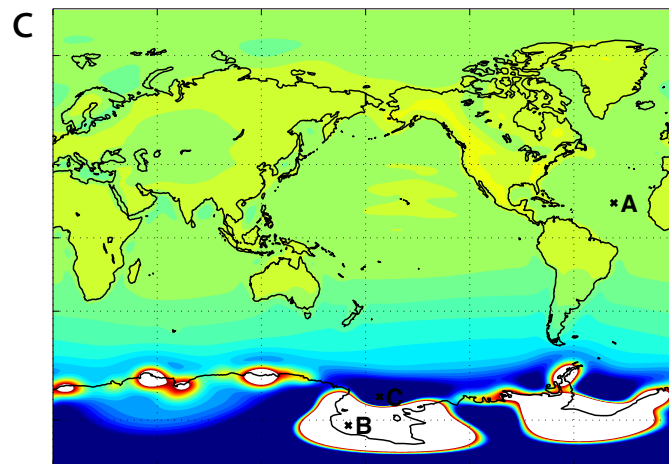
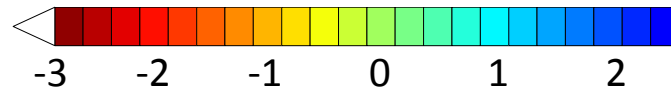
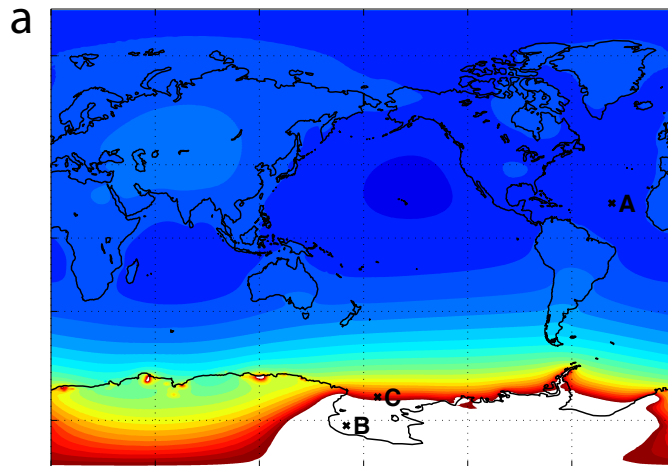


# 3. Sea Level Change Example

Elastic SL change immediately after ice sheet retreat

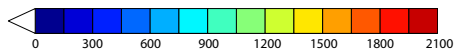
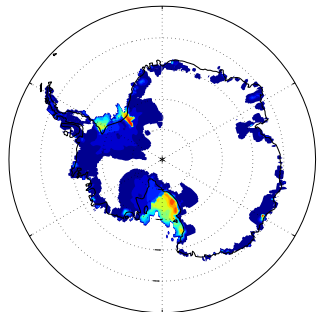
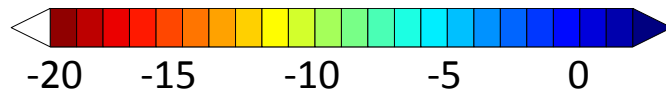
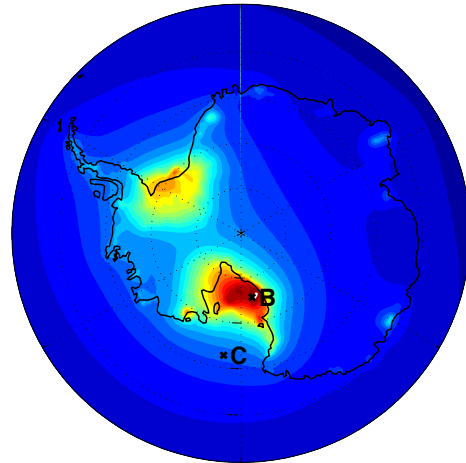


SL change over next 10 ky (ice remains constant)

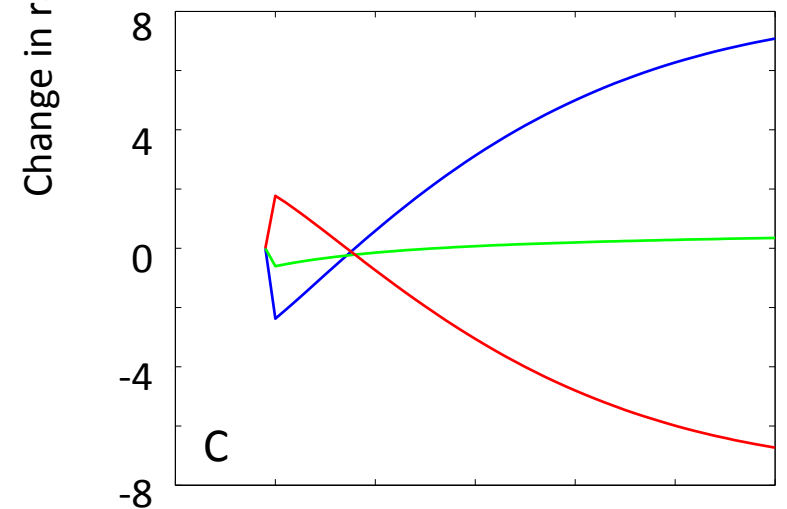
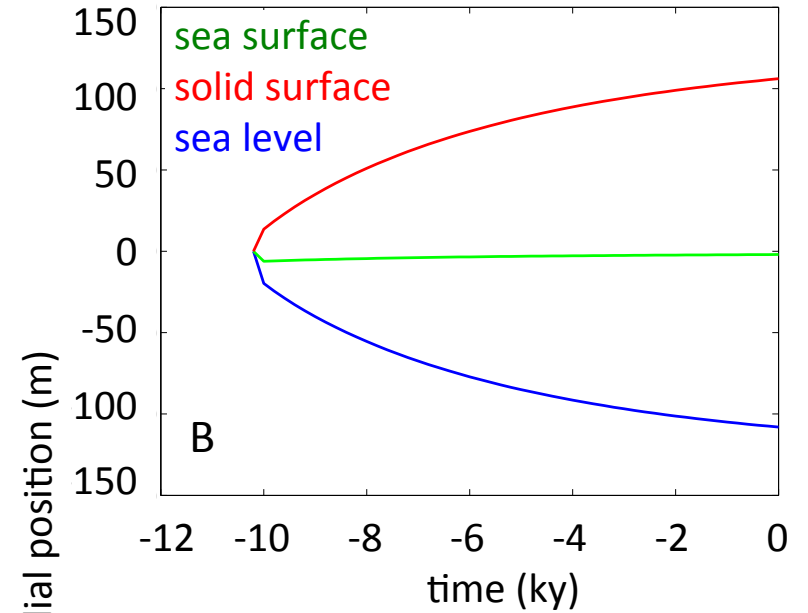
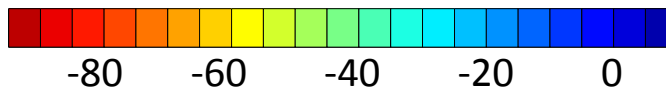
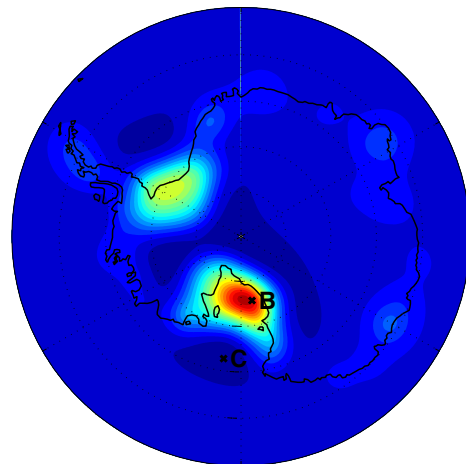


# 3. Sea Level Change Example

Elastic SL change immediately after ice sheet retreat

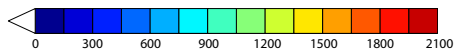
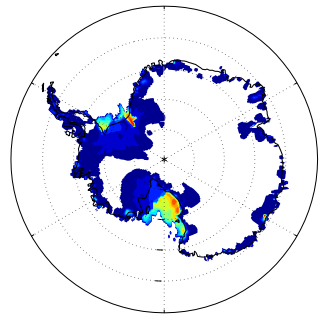
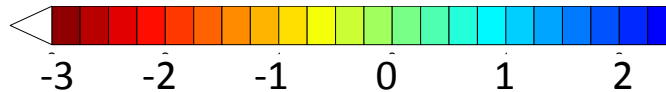
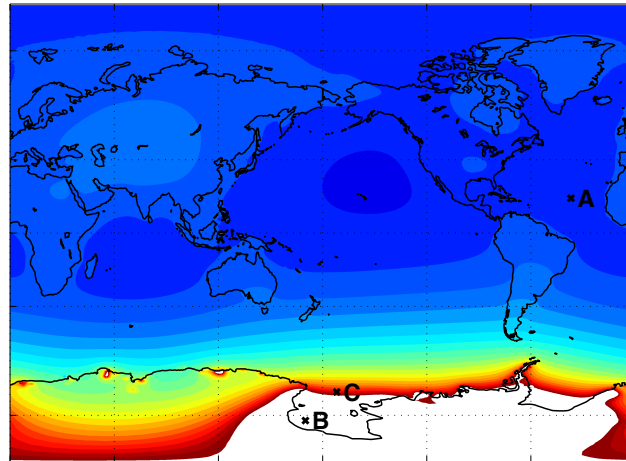


SL change over next 10 ky (ice remains constant)

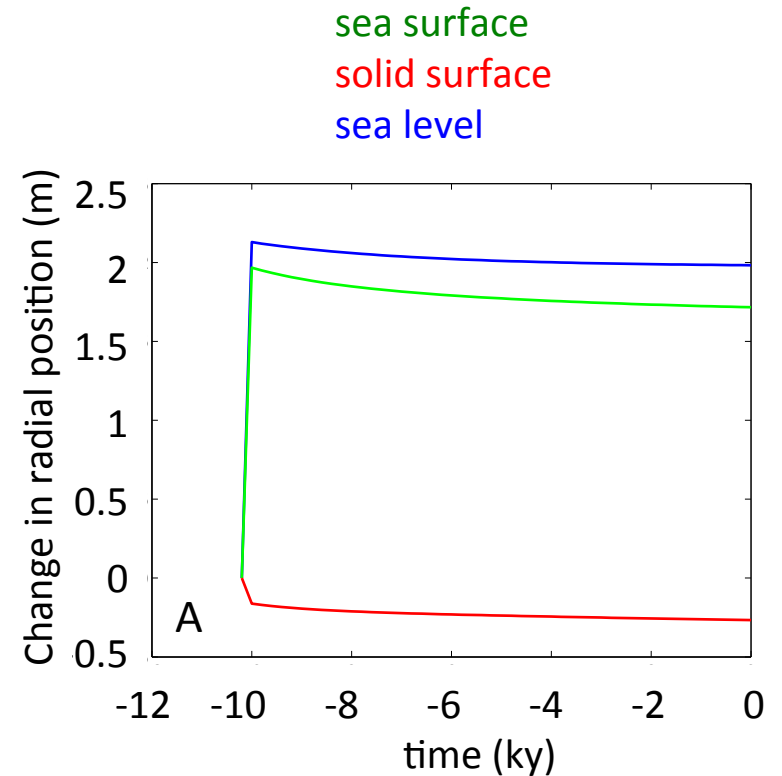
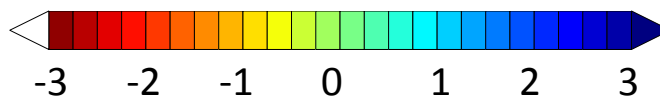
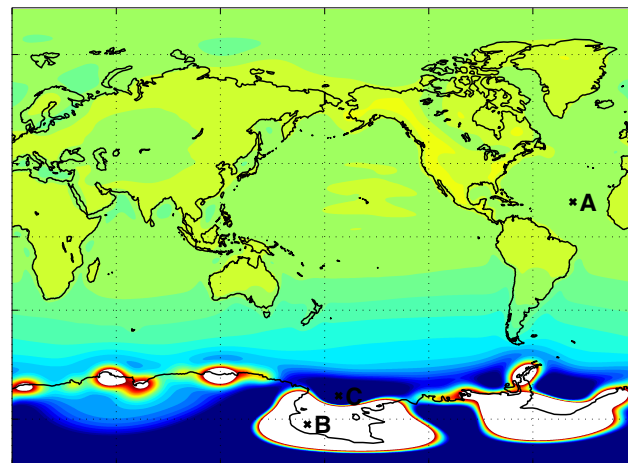


# 3. Sea Level Change Example

Elastic SL change immediately after ice sheet retreat



SL change over next 10 ky (ice remains constant)



# Quick Mental Break!

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# Outline

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This Class: Sea Level Change Continued...

1. Sea level change and GIA on ice age timescales.
2. An Example Calculations
3. Applications
  1. Short timescale modern: 20<sup>th</sup> Century Tide Gauge Analysis
  2. Short timescale paleo: Meltwater Pulse 1A (~14ky ago)
  3. GIA: Archaeological evidence for recent acceleration in sea level rise (Holocene – last 2 ky)
  4. Ice age timescale: Sea Level during the Last Interglacial (~125 ky ago)



# Outline

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This Class: Sea Level Change Continued...

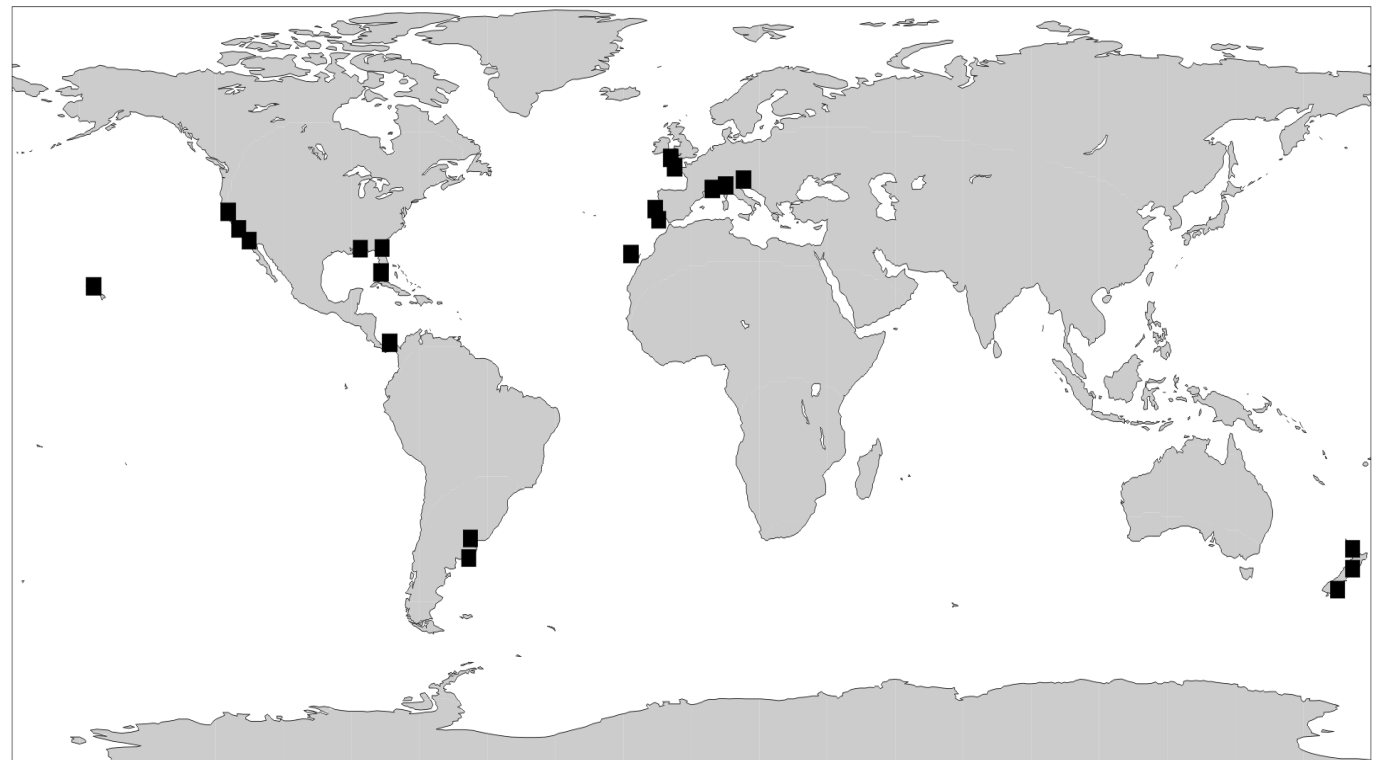
1. Sea level change and GIA on ice age timescales.
2. An Example Calculations
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  1. Short timescale modern: 20<sup>th</sup> Century Tide Gauge Analysis
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# Application to 20<sup>th</sup> century tide gauge analysis

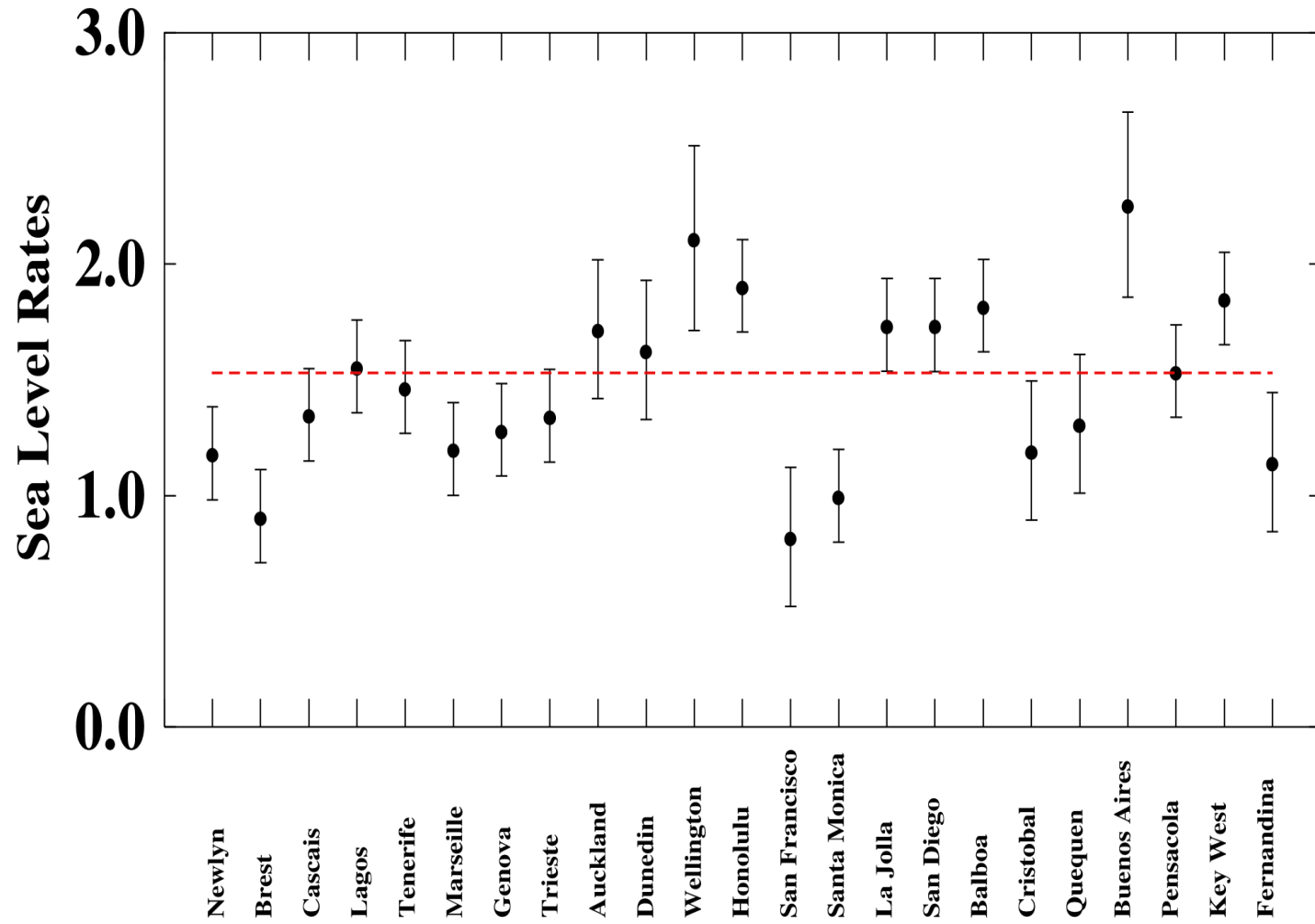
## Recent mass balance of polar ice sheets inferred from patterns of global sea-level change

Jerry X. Mitrovica\*, Mark E. Tamisiea\*, James L. Davis†  
& Glenn A. Milne‡

(2001)

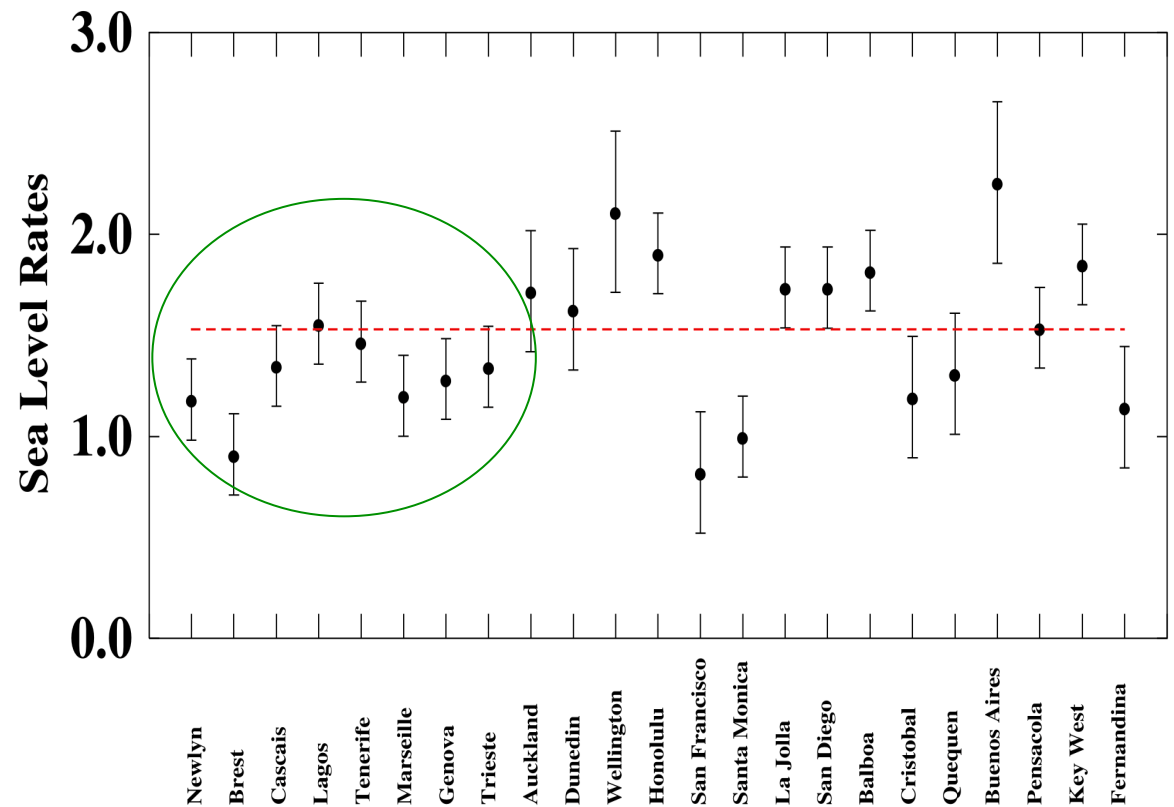
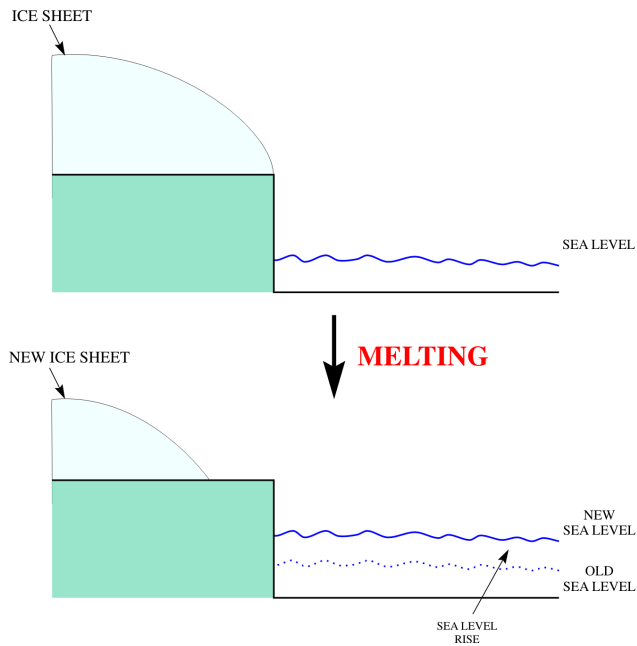


# Application to 20<sup>th</sup> century tide gauge analysis



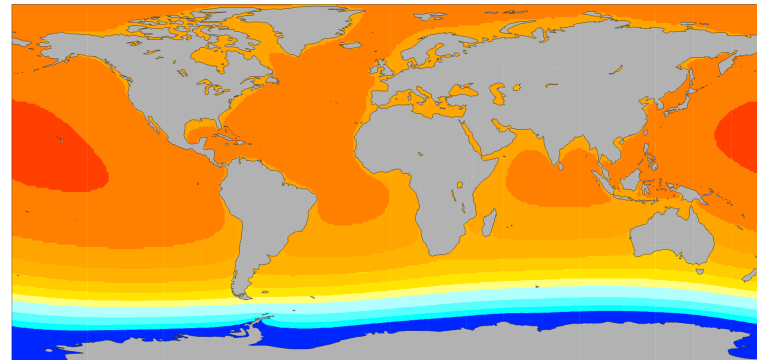
# Application to 20<sup>th</sup> century tide gauge analysis

## The Bathtub Model?

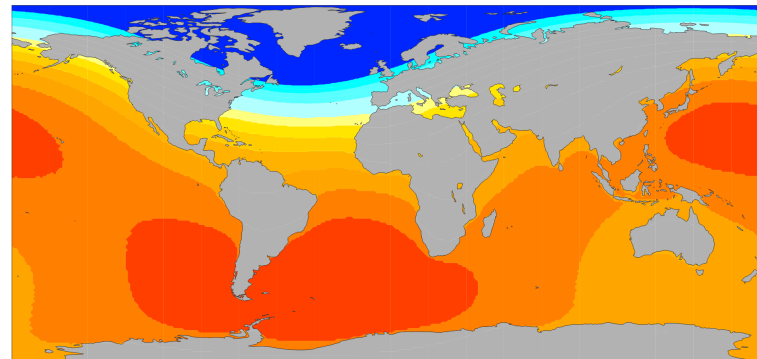


# Application to 20<sup>th</sup> century tide gauge analysis

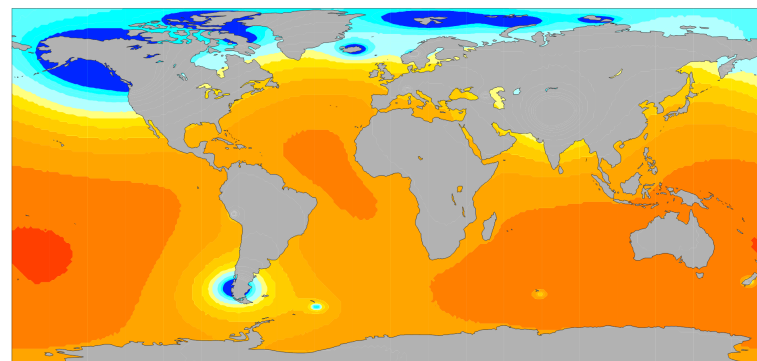
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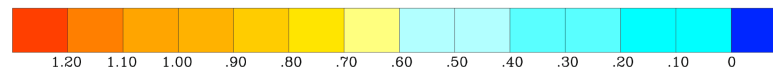
Antarctic melting



Greenland melting



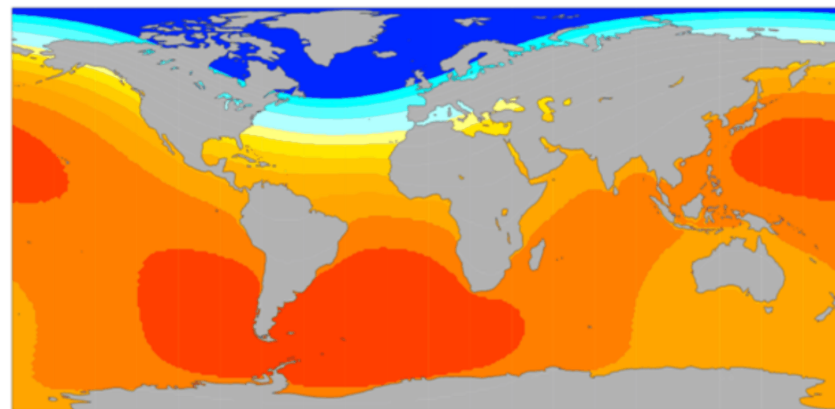
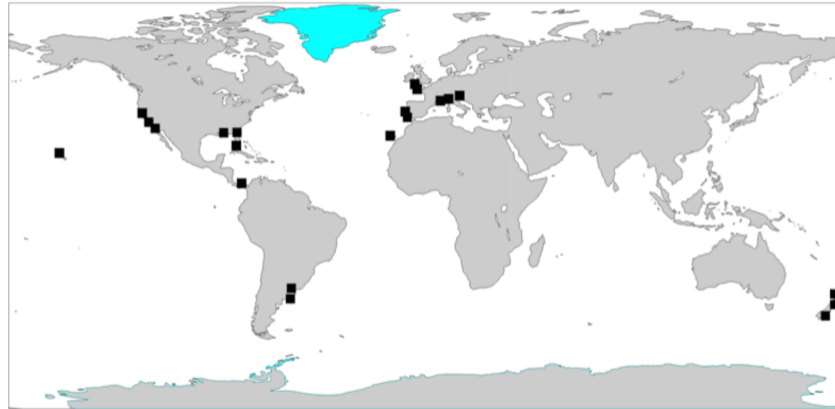
Glacier melting



Mitrovica et. al.  
(2001)

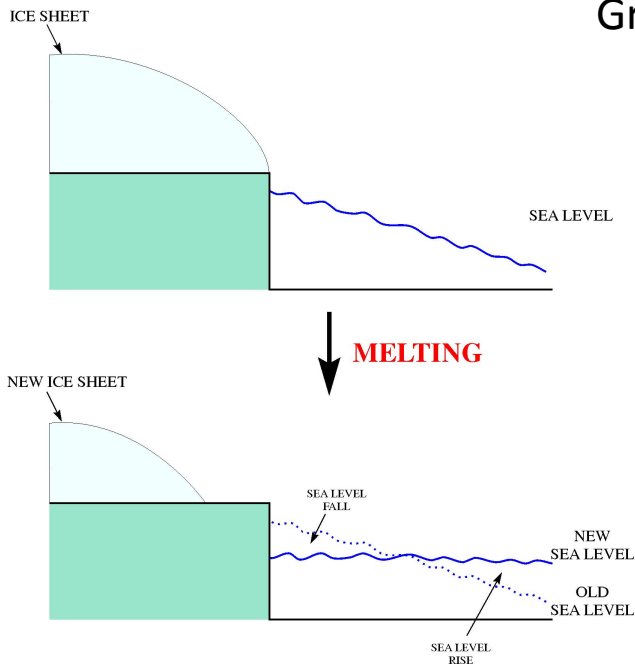
# Application to 20<sup>th</sup> century tide gauge analysis

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# Application to 20<sup>th</sup> century tide gauge analysis

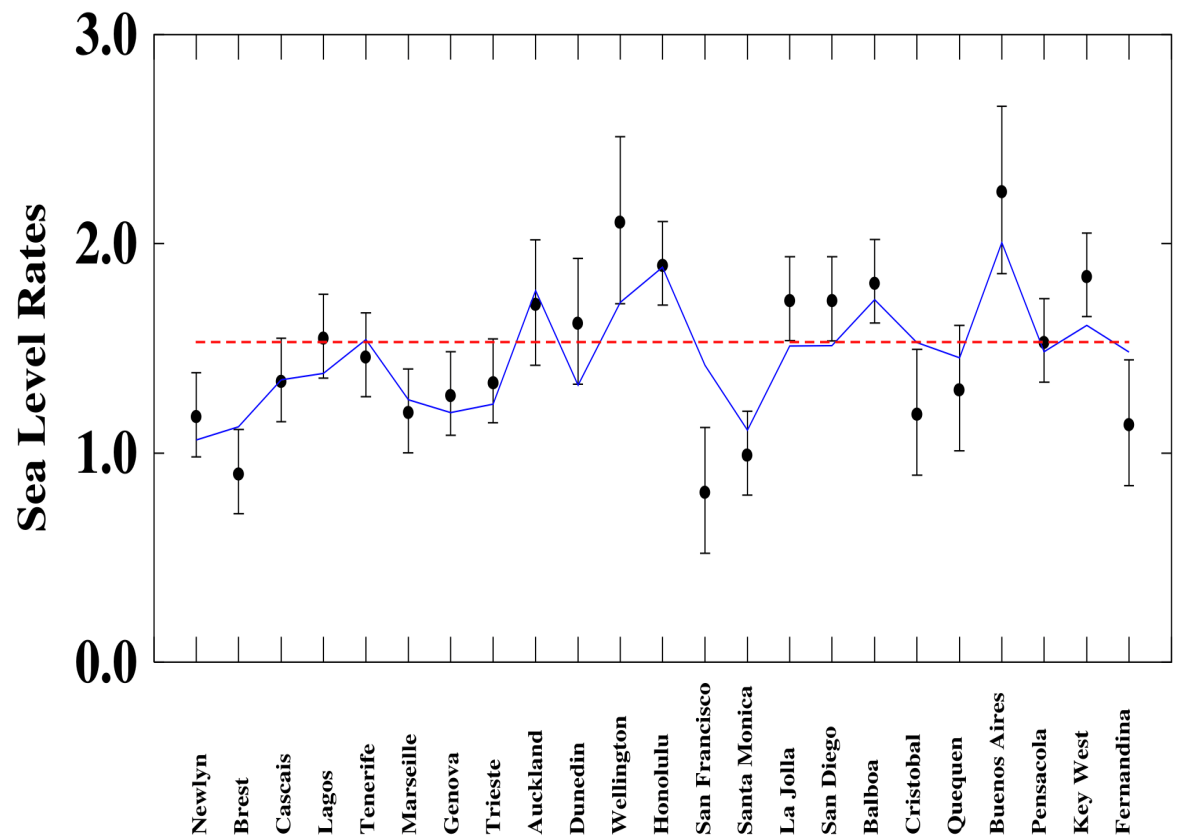
## The Correct Physics ...



This analysis:

Sea Level Change =

Greenland + Antarctica + mountain glaciers + the rest

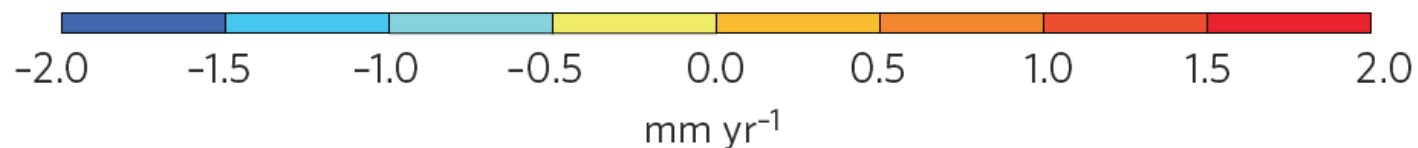
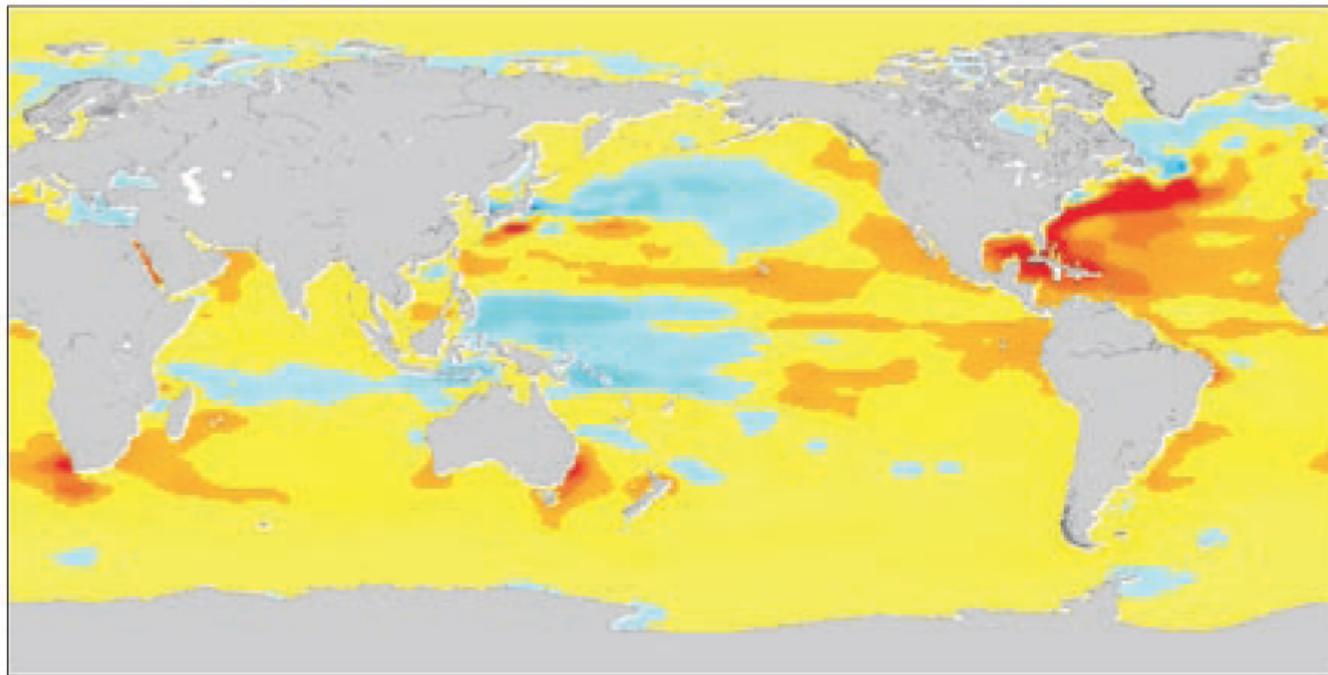




# Application to 20<sup>th</sup> century tide gauge analysis

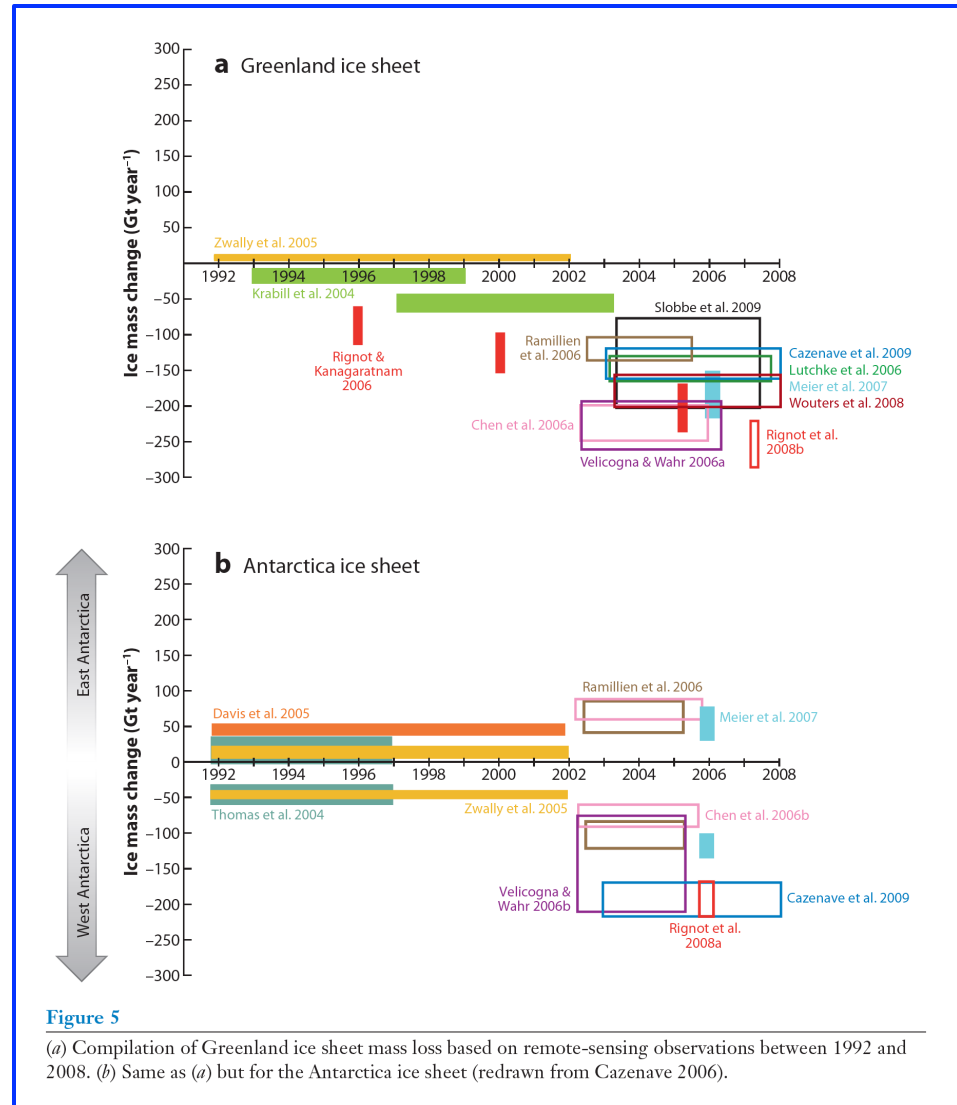
---

Issue: We need another fingerprint!



Steric effects (ocean temperature and salinity changes) – 1950-2003: Berge-Nguyen et al. [Glob. Planet. Change, 2008]

# Application to 20<sup>th</sup> century tide gauge analysis



A note on estimating ice loss over the 20<sup>th</sup> century and beyond.

Next class: ICE!

# Application to 20<sup>th</sup> century tide gauge analysis

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- Next slides on the current state of the art in tide gauge analysis [Hay et al., Nature, 2015] are from Dr. Carling Hay (Harvard University).

# Estimating Sea Level

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- Combine observations with models of the underlying physics of sea-level change.
- “Fingerprint” tide gauge records to estimate the individual contributions to 20<sup>th</sup> century sea-level change.

**Extract global information from sparse records.**

**Multi-Model Kalman Smoother**



# Estimating Sea Level

---

## **Multi-Model Kalman Smoother (KS)**

- Both algorithms are Bayesian in nature.
- Naturally accommodate measurements with data gaps.
- Allow the estimation of sea level at sites with and without observations.
- Compute GMSL by first estimating the equivalent global mean value of the individual contributions from their unique temporal-spatial fingerprints.
- The resulting uncertainties in our estimates of global mean sea level reflect the data sparsity over time.

# Kalman Filter

R. E. KALMAN

Research Institute for Advanced Study,<sup>2</sup>  
Baltimore, Md.

## A New Approach to Linear Filtering and Prediction Problems<sup>1</sup>

*The classical filtering and prediction problem is re-examined using the Bode-Shannon representation of random processes and the "state transition" method of analysis of dynamic systems. New results are:*

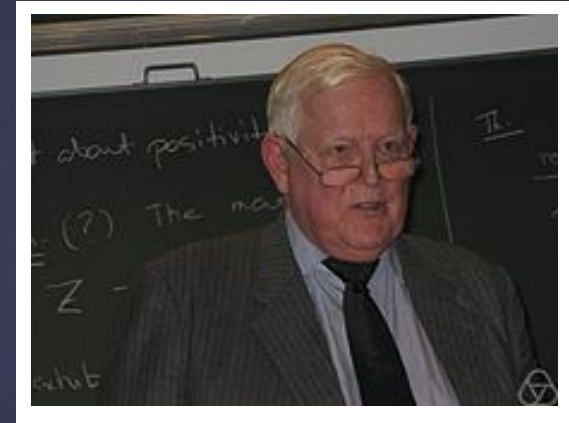
(1) *The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite-memory filters.*

(2) *A nonlinear difference (or differential) equation is derived for the covariance matrix of the optimal estimation error. From the solution of this equation the coefficients of the difference (or differential) equation of the optimal linear filter are obtained without further calculations.*

(3) *The filtering problem is shown to be the dual of the noise-free regulator problem. The new method developed here is applied to two well-known problems, confirming and extending earlier results.*

*The discussion is largely self-contained and proceeds from first principles; basic concepts of the theory of random processes are reviewed in the Appendix.*

Transactions of the ASME—Journal of Basic Engineering, 82 (Series D): 35-45. Copyright © 1960 by ASME



It's a method of predicting the future state of a system based on the previous states.

trajectory estimation for  
the Apollo program

space shuttle navigation  
systems

weather forecasting

ballistic missile  
navigation systems



# Kalman Filter

---

It's a method of predicting the future state of a system based on the previous states.

Iteratively performs a least squares analysis whenever observations are available, and in the absence of observations relies on the model dynamics to compute the best estimate of state variables.



# Kalman Filter

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It's a method of predicting the future state of a system based on the previous states.

Iteratively performs a least squares analysis whenever observations are available, and in the absence of observations relies on the model dynamics to compute the best estimate of state variables.

The state is a description of all the parameters we will need to describe the current system.

State vector at time  $k$ :

$$\vec{x}_k = \begin{bmatrix} \vec{S}_k & \vec{\beta}_k \end{bmatrix}^T$$

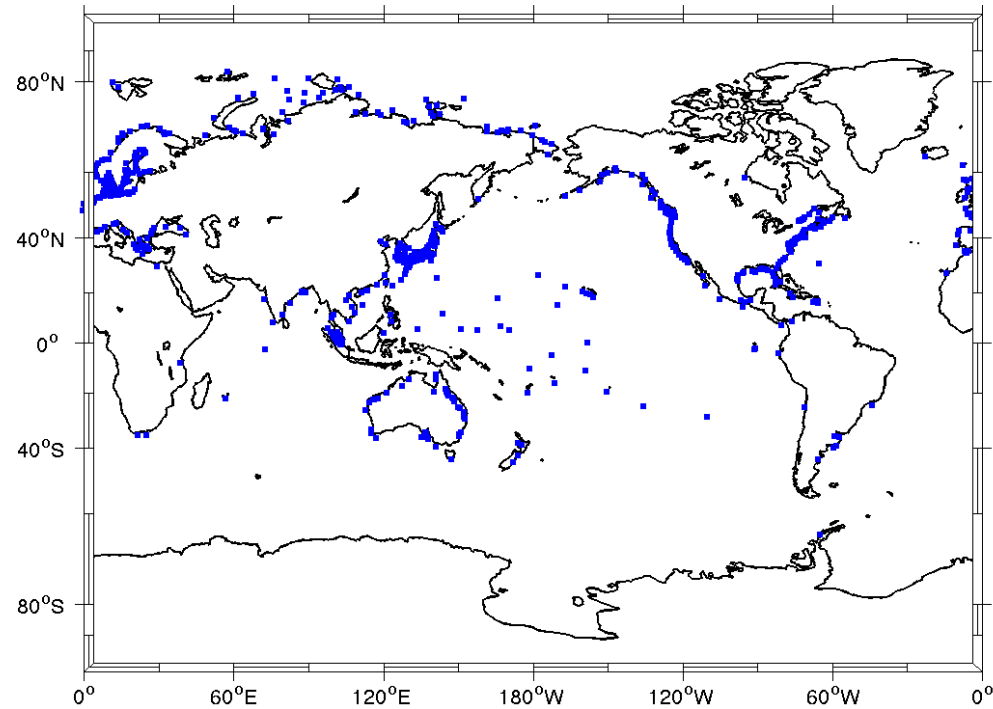
$\vec{S}_k$  = sea level at each tide gauge site

$\vec{\beta}_k$  = vector of ice sheet and mountain glacier melt rates

# Kalman Filter

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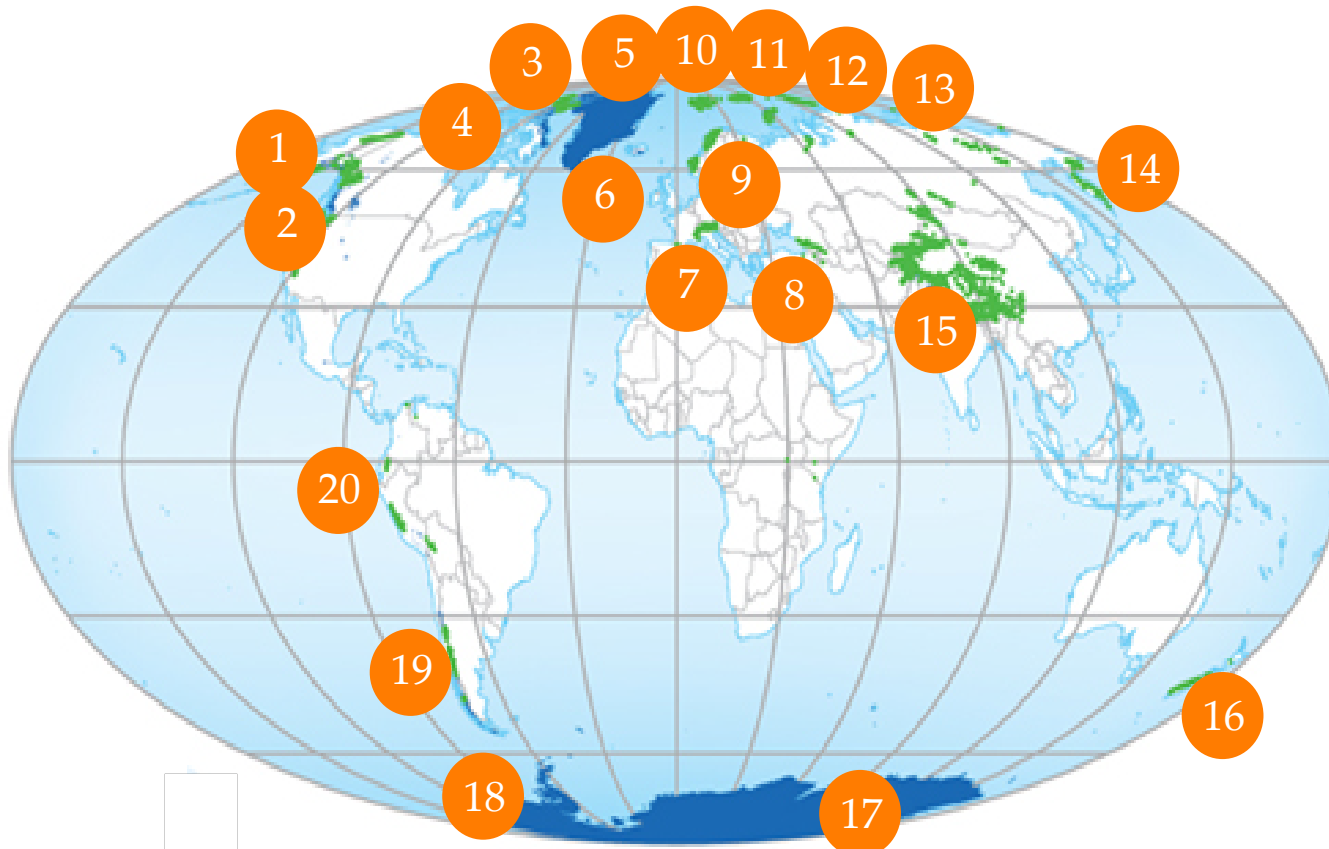
**sea level at each tide gauge site**



622 tide gauges

# Kalman Filter

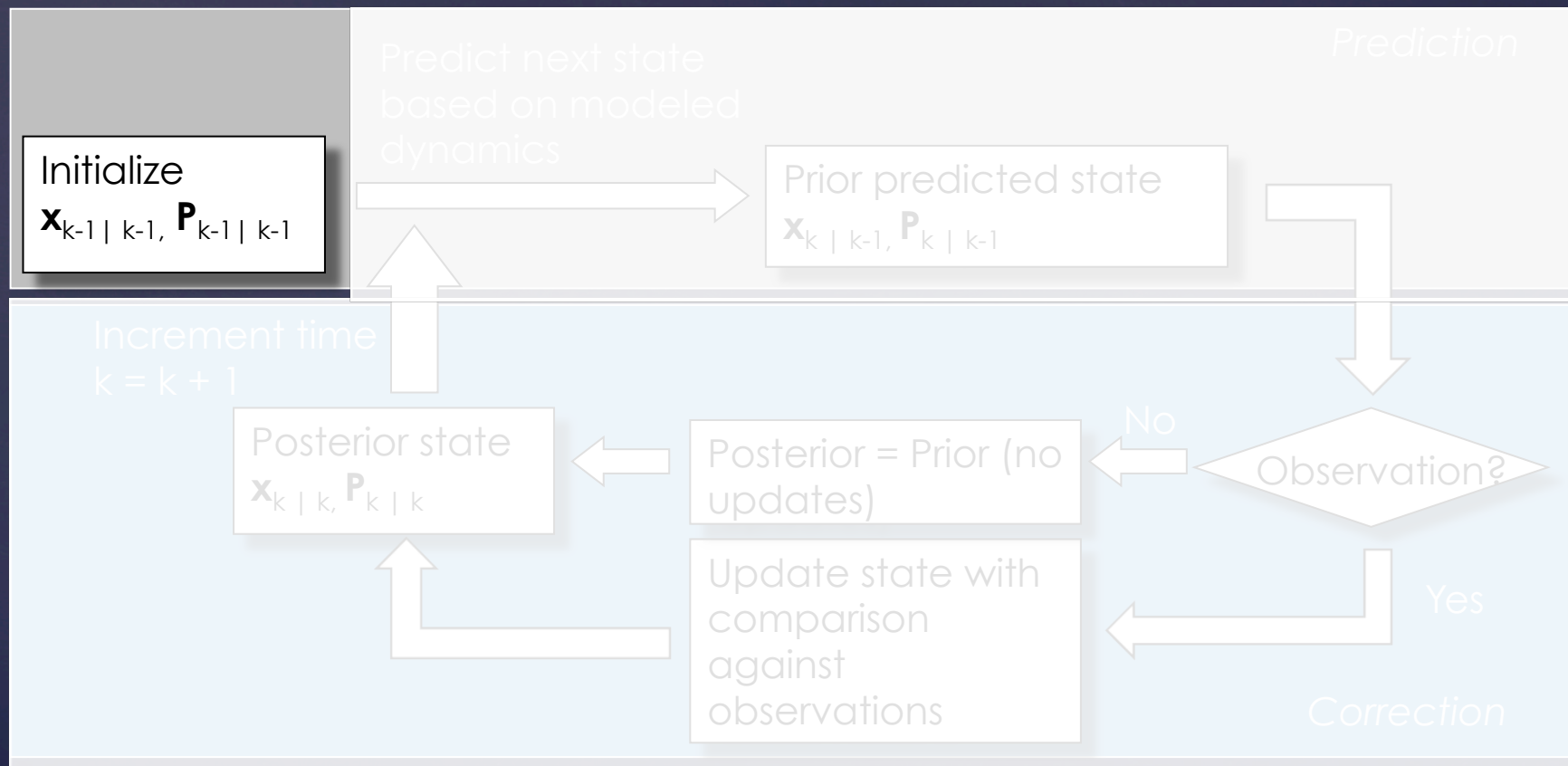
## ice sheet and mountain glacier melt rates



20 ice sheets and mountain glaciers  
and a globally uniform term

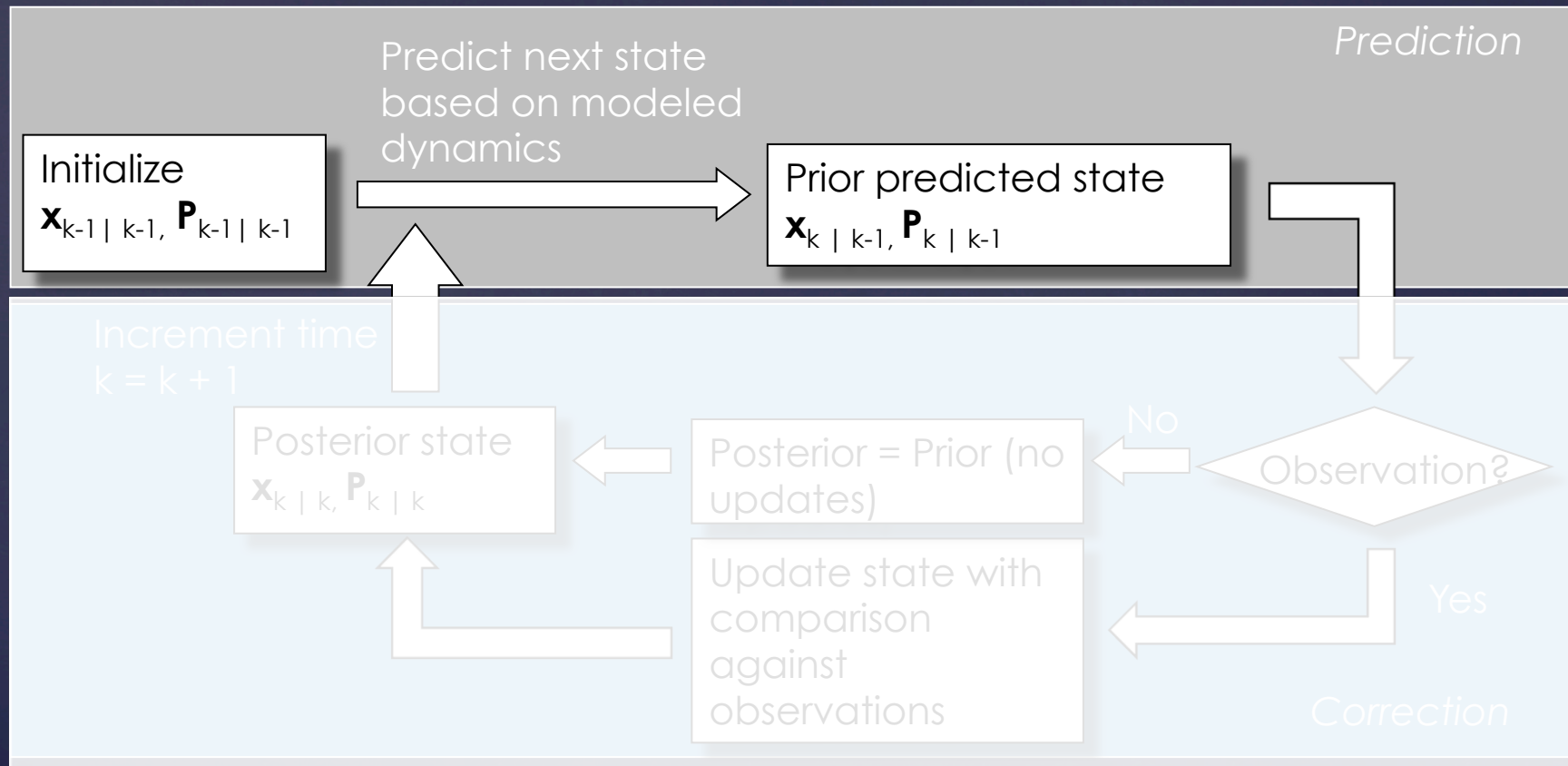


# Kalman Filter



Initialize our sea level and melt rates at the first time step (1900).

# Kalman Filter



At each time step, we use a model to describe how we think the system behaves.

# Kalman Filter

## PREDICTION STEP

At each tide gauge:

$$\text{sea level}_k = \text{sea level}_{k-1} + \text{additional melt water}_{k-1} + \text{GIA} + \text{ocean dynamics} + \text{noise}$$

Prescribed GIA & ocean models

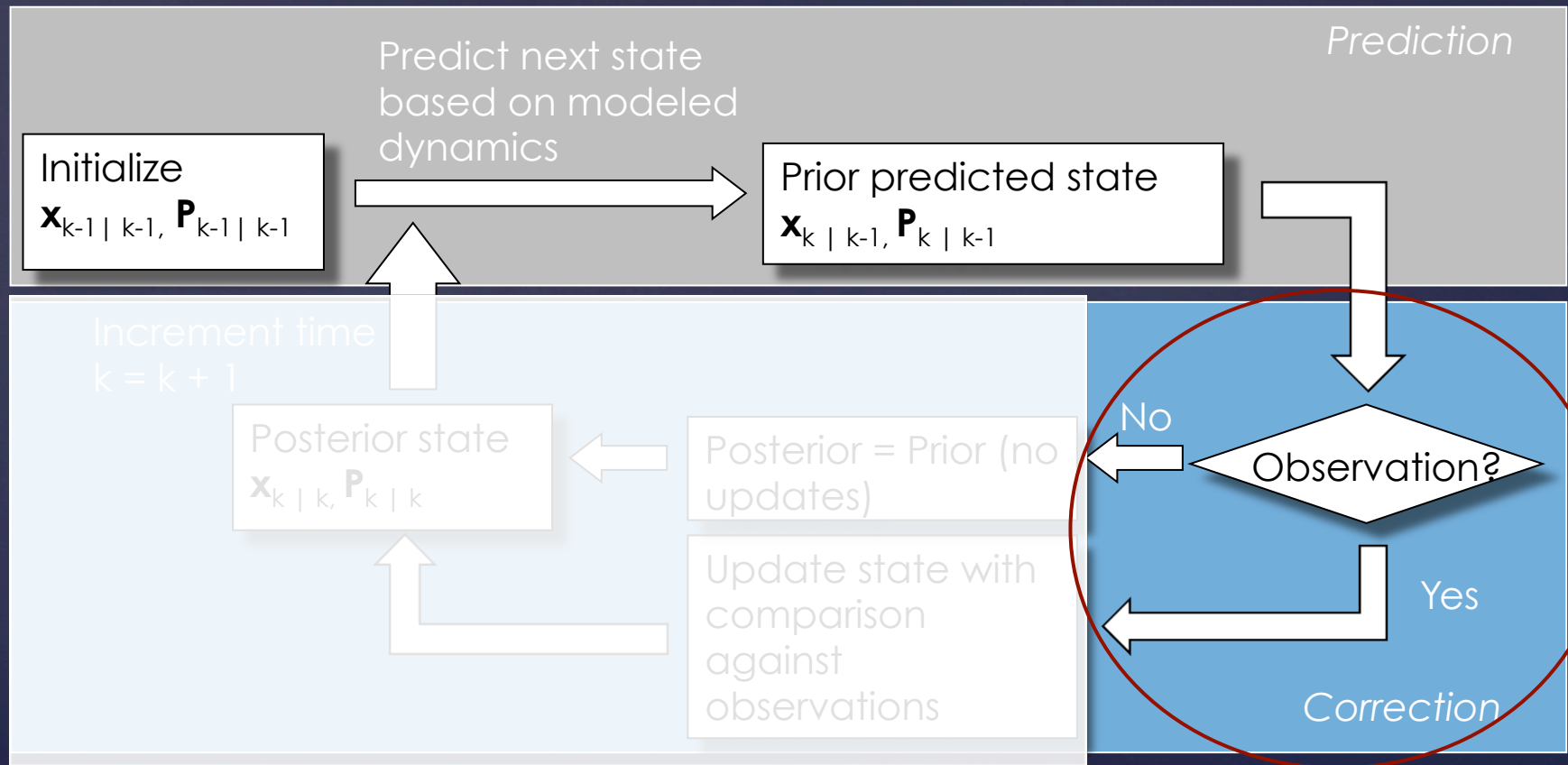
$$\text{additional melt water}_{k-1} = \sum_{\text{land ice}} \text{normalized fingerprints} \times \text{melt rate}_{k-1}$$

Normalized sea-level fingerprints

Each ice sheet and mountain glacier is modeled as an AR(1) process:

$$\text{melt rate}_k = \rho \times \text{melt rate}_{k-1} + \text{noise}$$

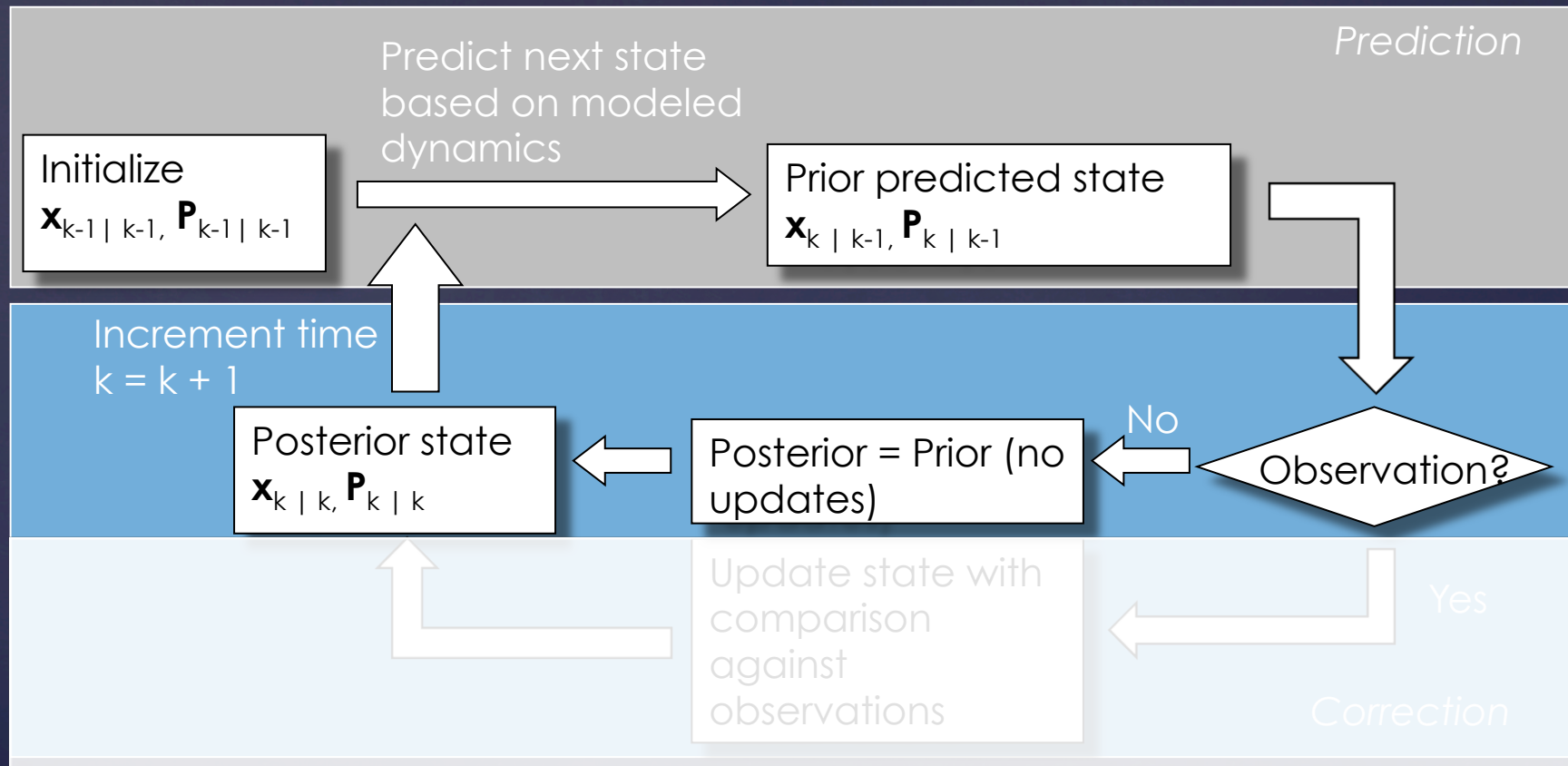
# Kalman Filter



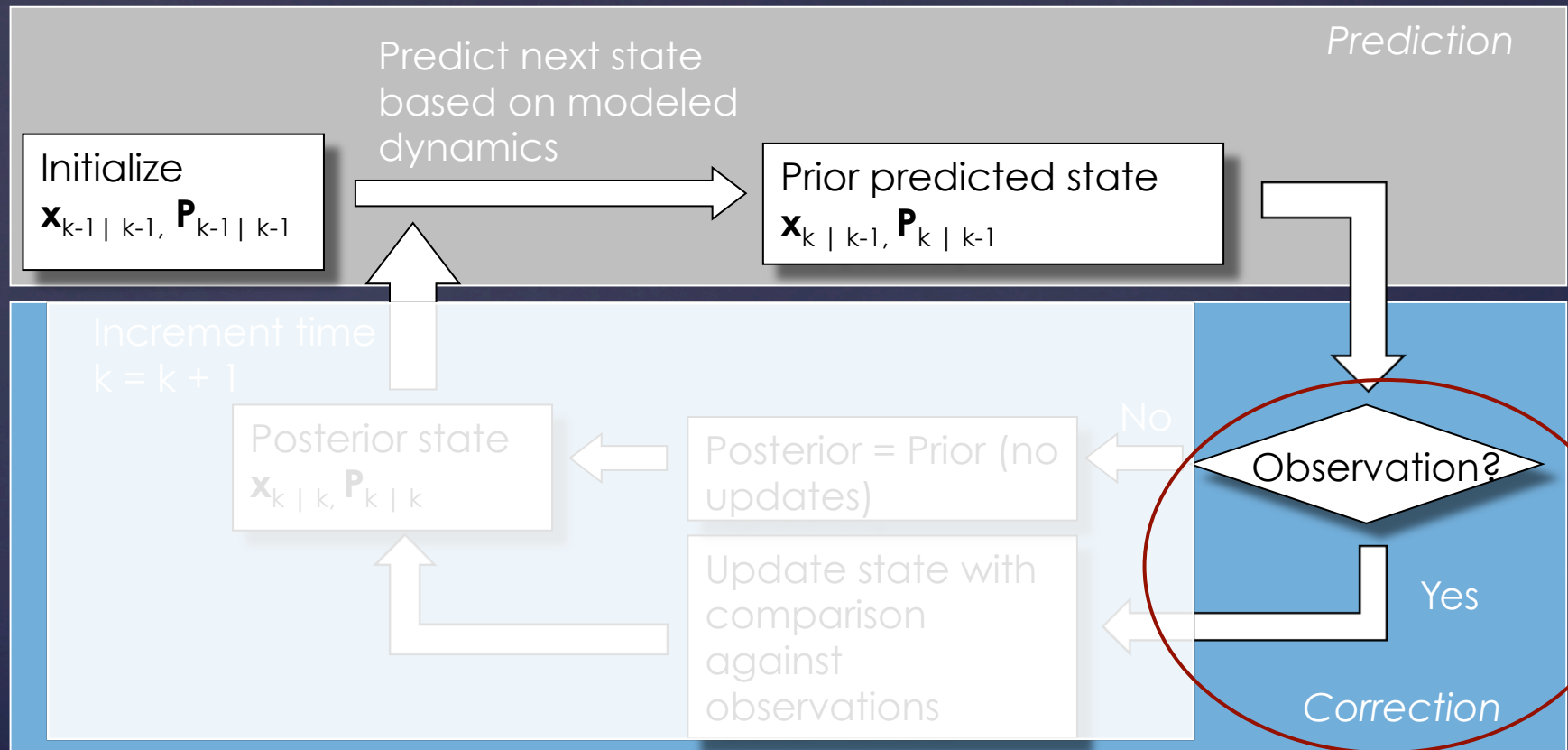


# Kalman Filter

## Option 1: No observations

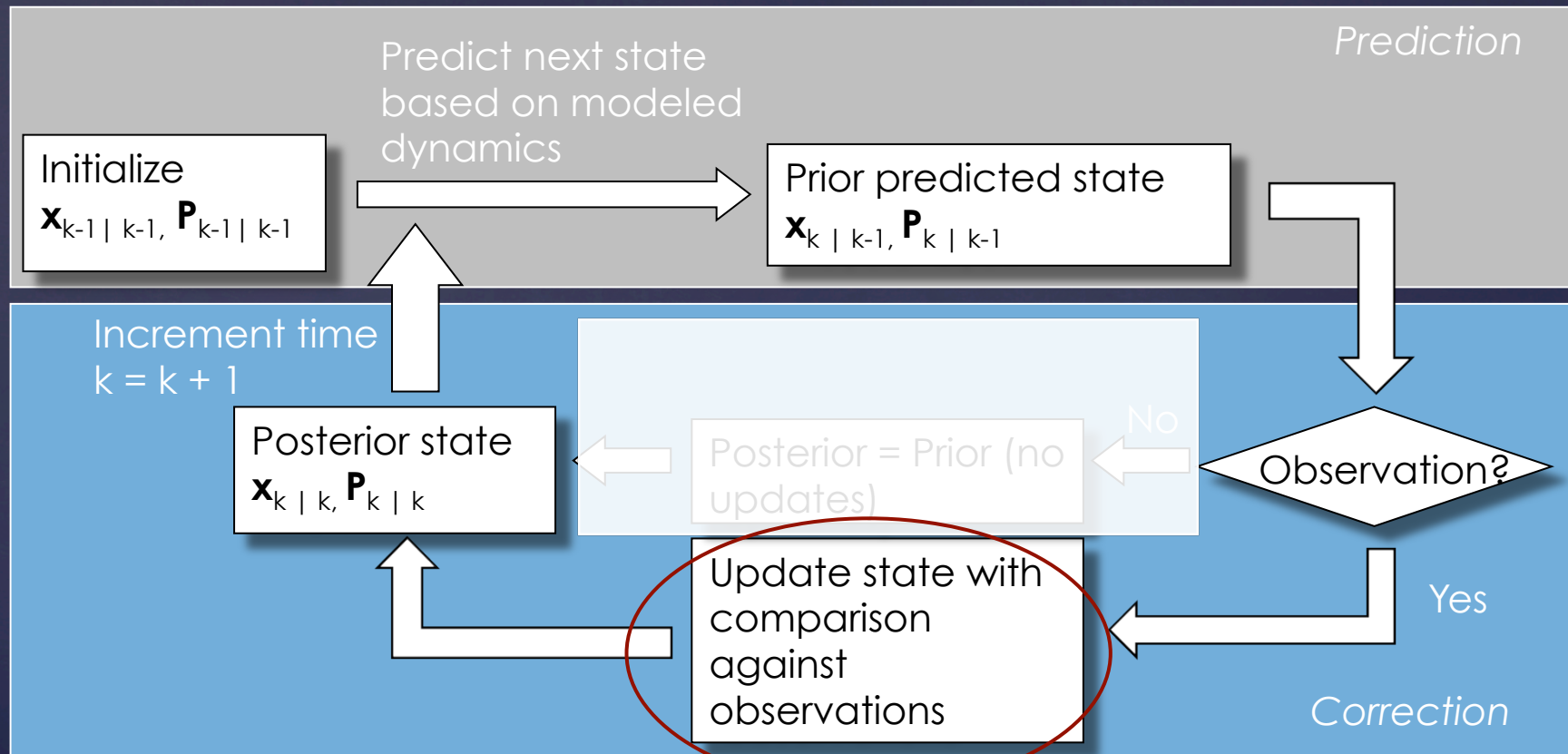


# Kalman Filter



# Kalman Filter

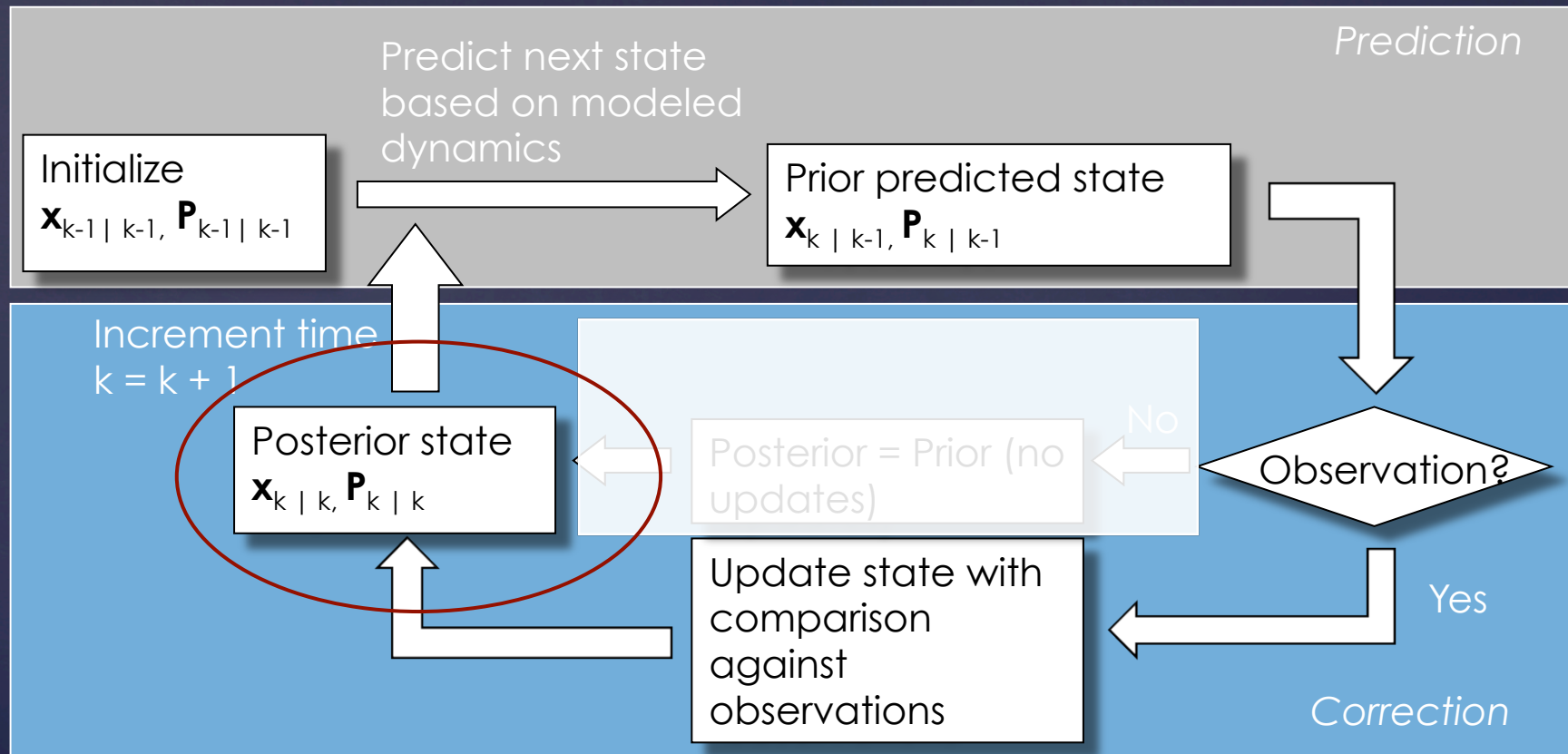
## Correction Step



When we get new data, our state vector (sea level and melt rates) should change slightly to refine our current model.

# Kalman Filter

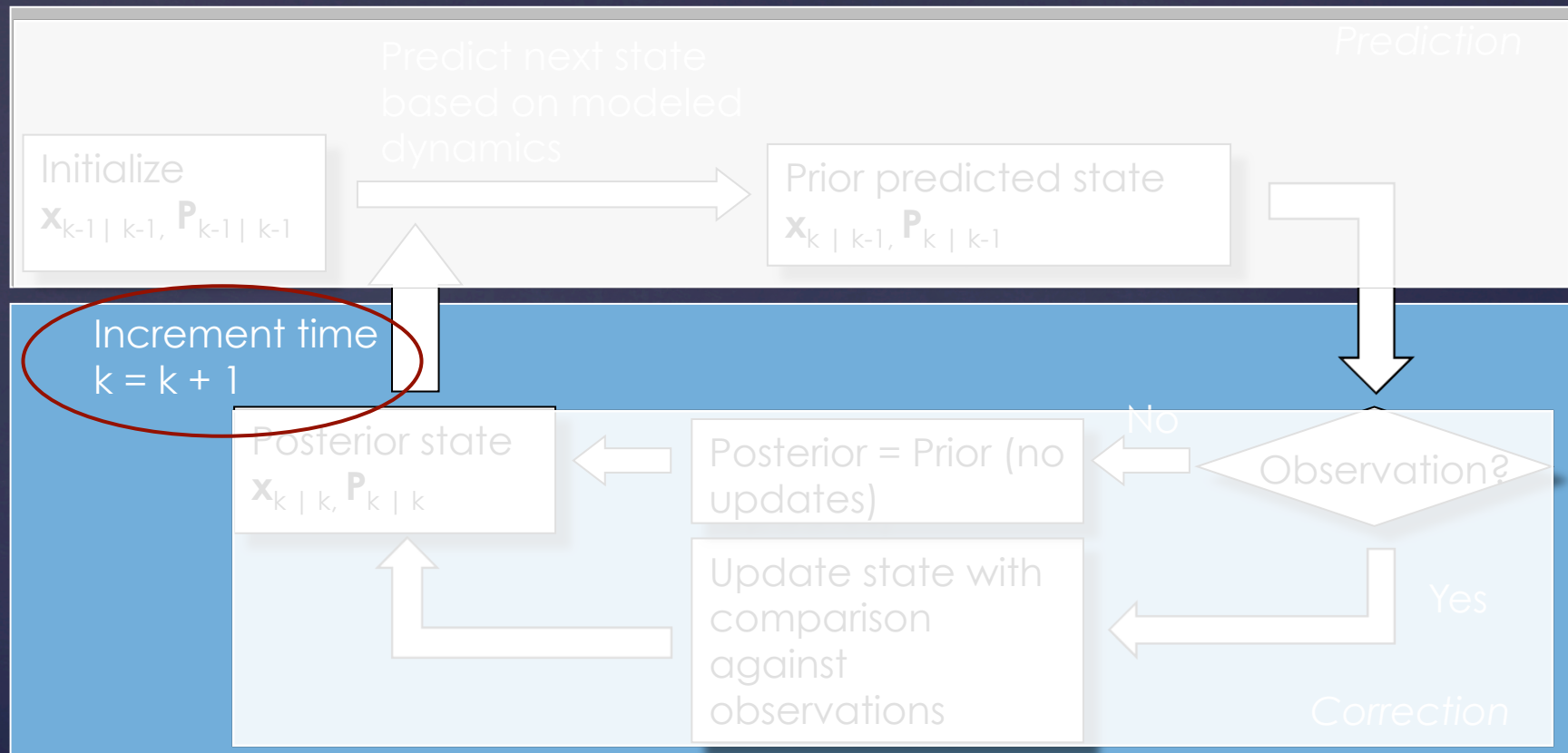
## Correction Step



The result is a new state estimate that lies between the predicted and measured state, and has a better estimated uncertainty than either alone.

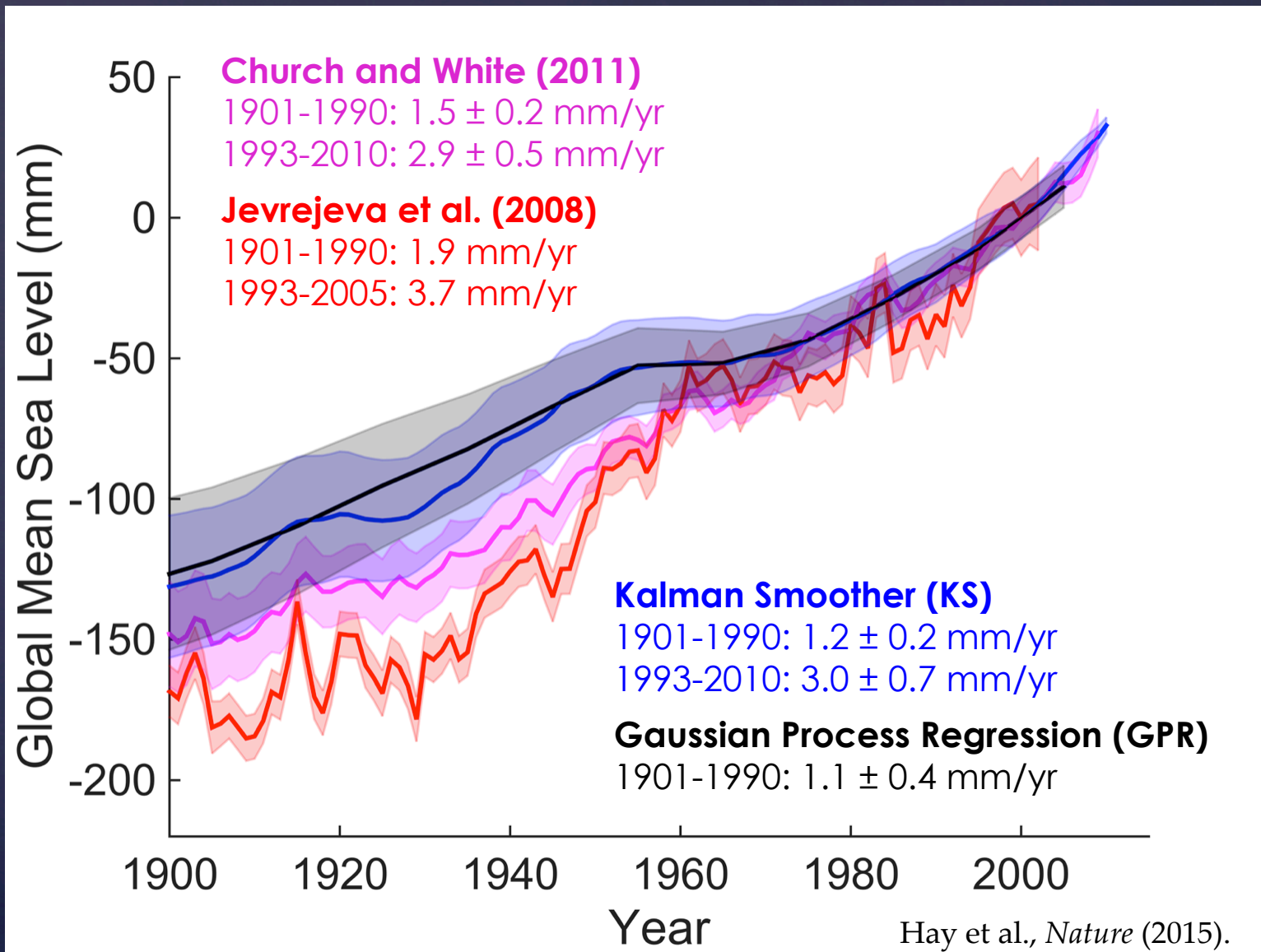


# Kalman Filter



Step forward in time to make of a new prediction on the evolution of the state

# Global Mean Sea Level



# Outline

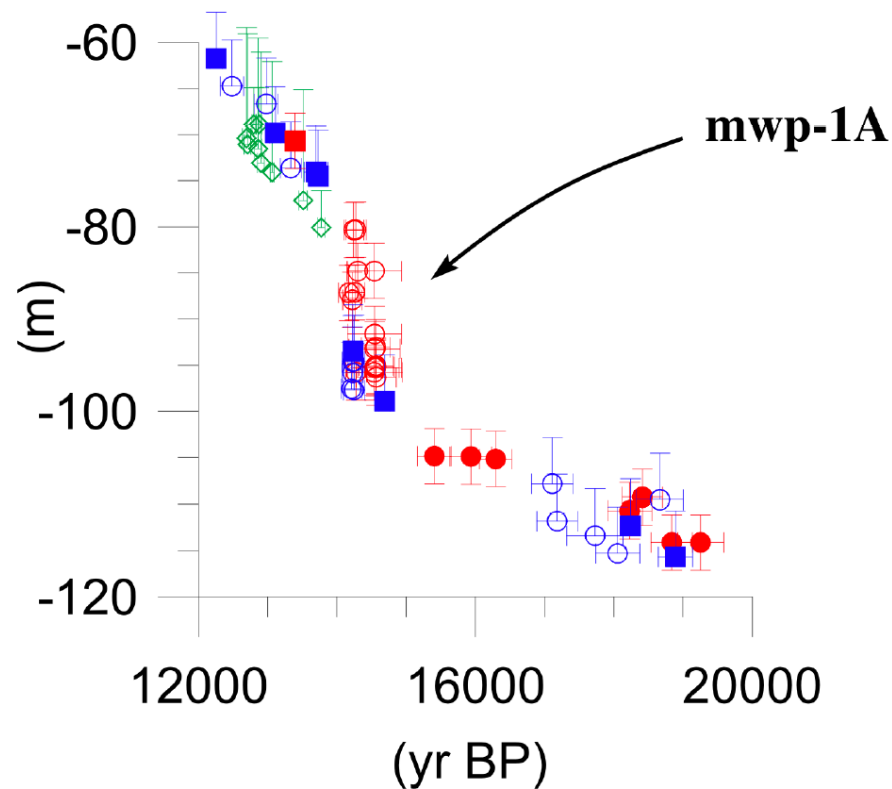
---

This Class: Sea Level Change Continued...

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# Meltwater Pulse 1A

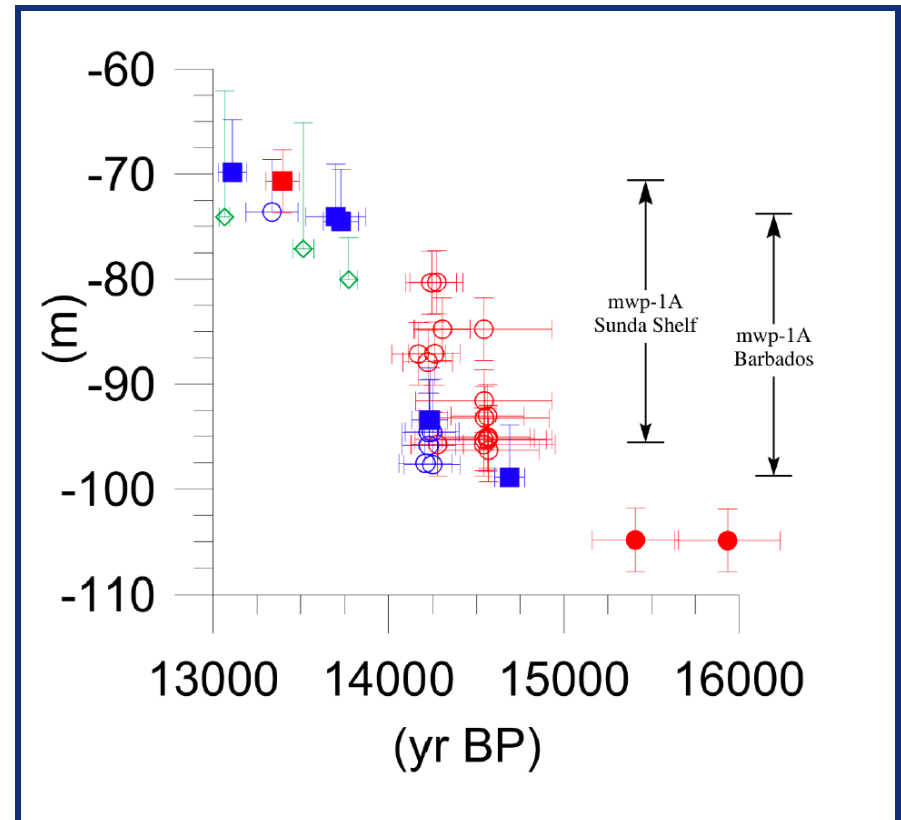
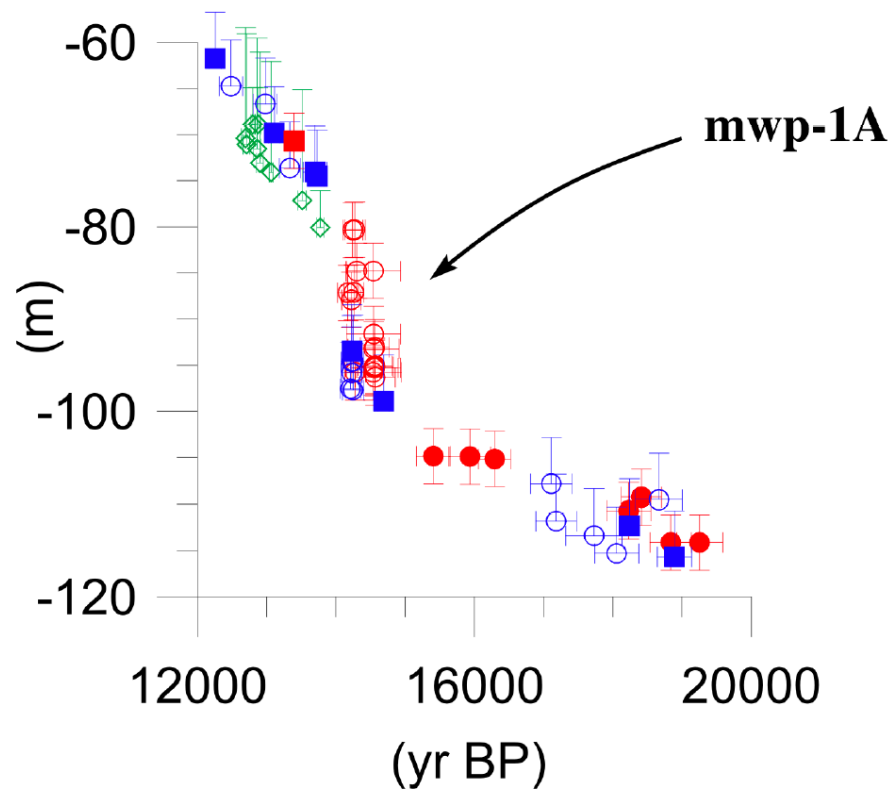


~ 15 meters of sea-level rise is < 350 years (Deschamps et al. 2012)

Source(s) of mwp-1A???

- Barbados (Cal.C14 Corals Fairbanks, 1989; Bard et al., 1993)
- Barbados (U/Th Corals Bard et al., 1990, 1993)
- Sunda Shelf (Cal.C14 Mangrove Hanebuth et al., 2000)
- Sunda Shelf (Cal.C14 Non-mangrove Hanebuth et al., 2000)
- Sunda Shelf (Cal.C14 Non-mangrove Hanebuth et al., 2000)
- ◇ Tahiti (U/Th Corals Bard et al., 1996)

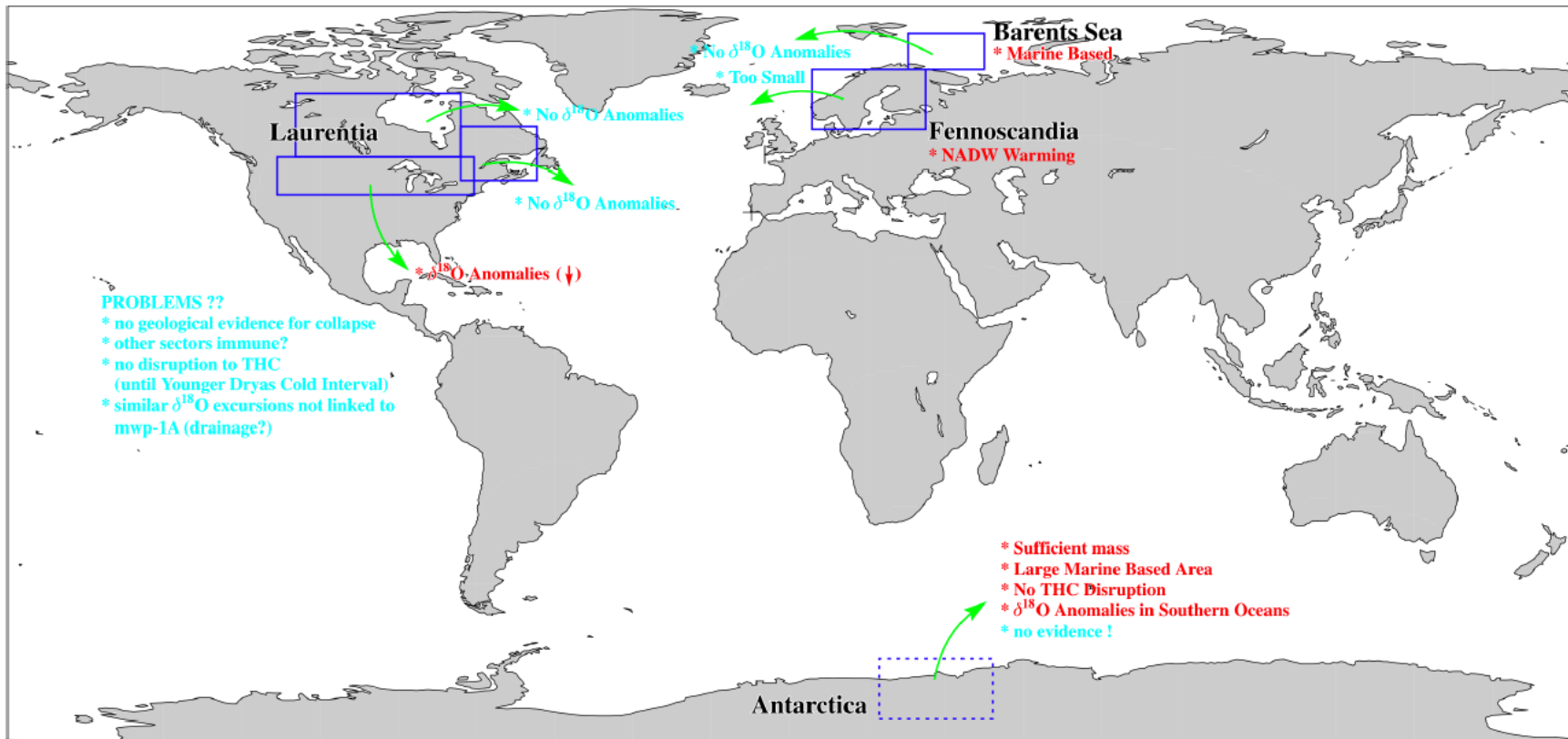
# Meltwater Pulse 1A



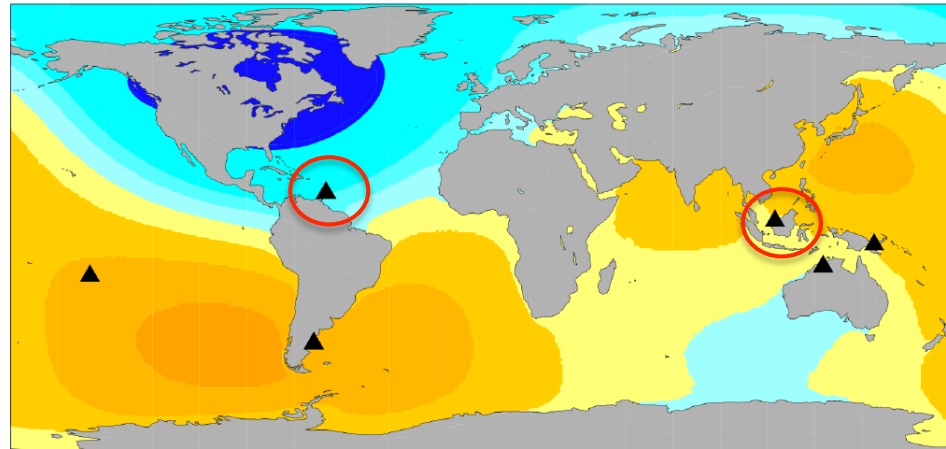
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- Sunda Shelf (Cal.C14 Non-mangrove Hanebuth et al., 2000)
- ◇ Tahiti (U/Th Corals Bard et al., 1996)

# Meltwater Pulse 1A

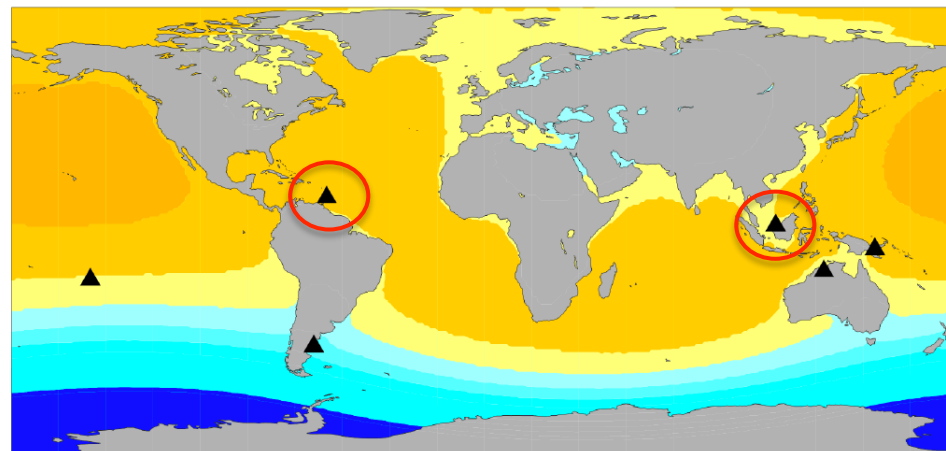
## Possible melt scenarios:



# Meltwater Pulse 1A



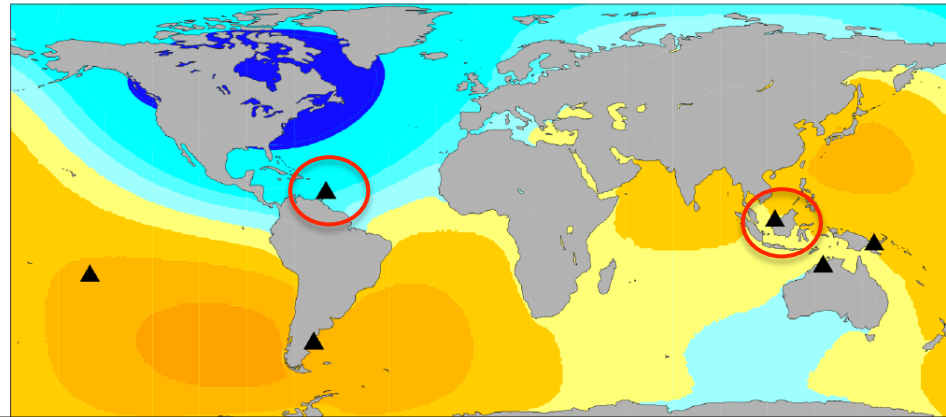
**Southern  
Laurentian  
Source**



**West  
Antarctic  
Source**

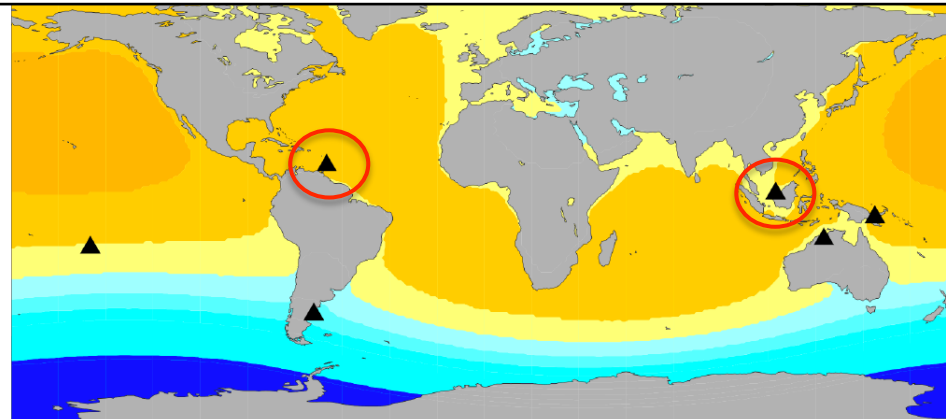


# Meltwater Pulse 1A



**Southern  
Laurentian  
Source**

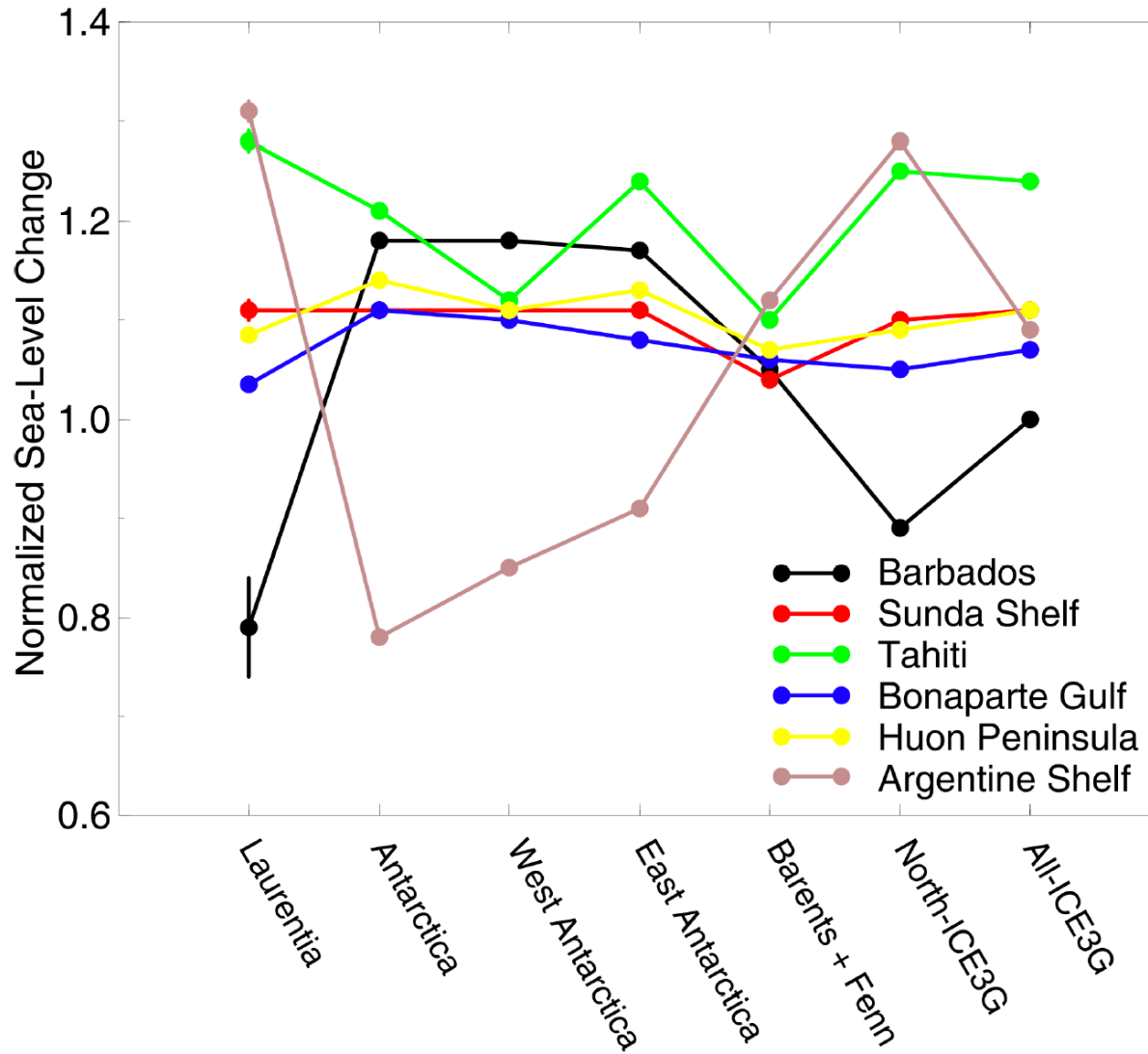
... but Antarctic ice sheet modeling studies suggest that Antarctica contributed ~4-8 meters over the whole deglaciation. (e.g. Whitehouse et al., 2012, Pollard & DeConto, 2009, Gomez et al., 2013)



**West  
Antarctic  
Source**



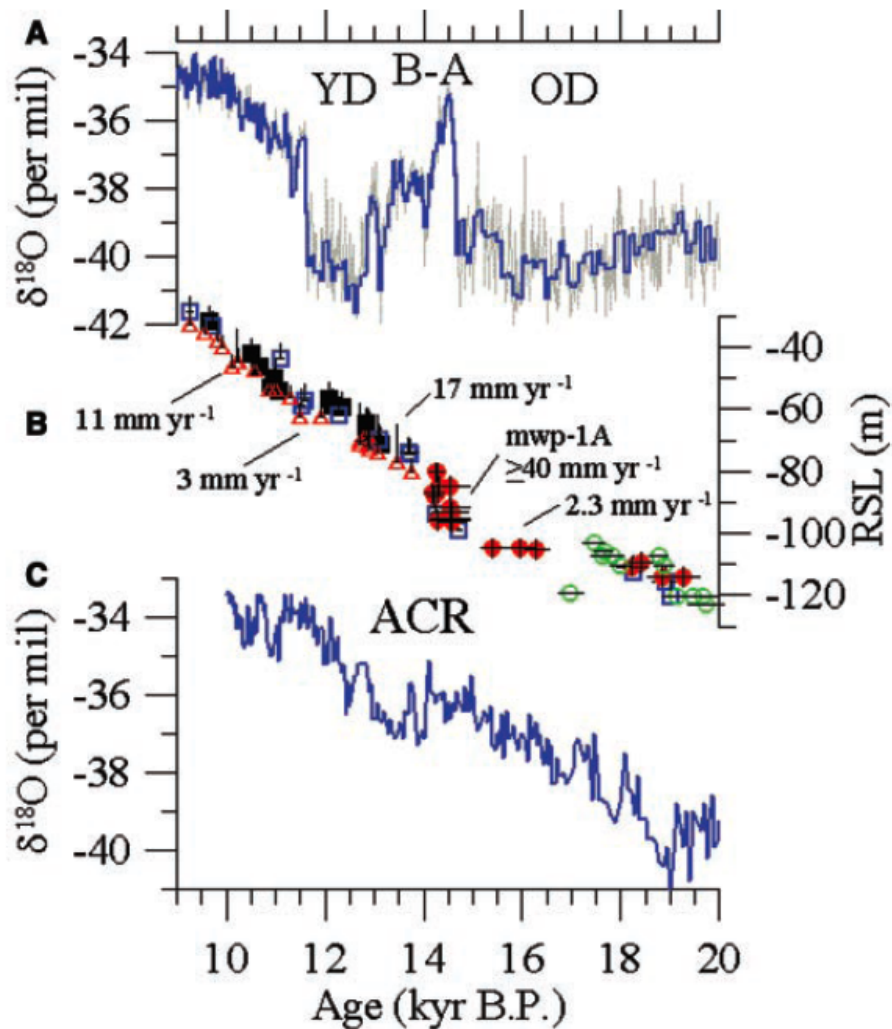
# Meltwater Pulse 1A



# Meltwater Pulse 1A

## Climate implications?

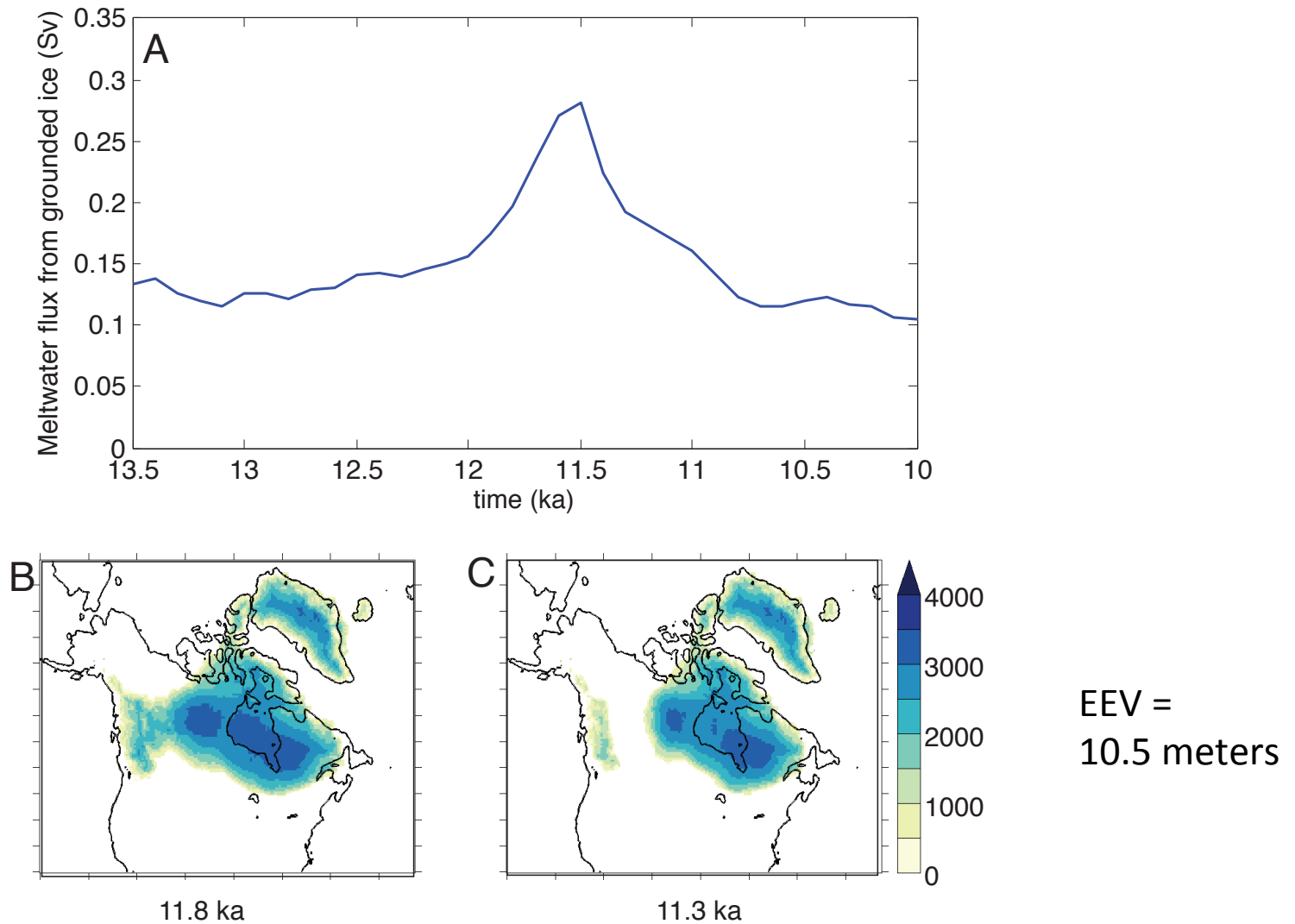
**Fig. 5.** Climate and sea-level records spanning the last deglaciation. **(A)** The Greenland Ice Sheet Project 2 (GISP2) oxygen isotope record (40, 41). OD is the Oldest Dryas cold period, and YD is the Younger Dryas cold period. **(B)** Relative sea-level (RSL) records from far-field sites. Also shown are average rates of sea-level rise for the periods 19 to 14.6, 14.6 to 14.1, 14.1 to 12.9, 12.9 to 11.6, and 11.6 to 6 kyr B.P. Data are from Bonaparte Gulf (green open circles) (42), Barbados U/Th dated corals (open blue squares) (5), Sunda Shelf (9), Tahiti (open red triangles) (5), and New Guinea (closed black squares) (43). **(C)** The Byrd ice-core oxygen isotope record on the GISP2 time scale (34). ACR is the Antarctic Cold Reversal.





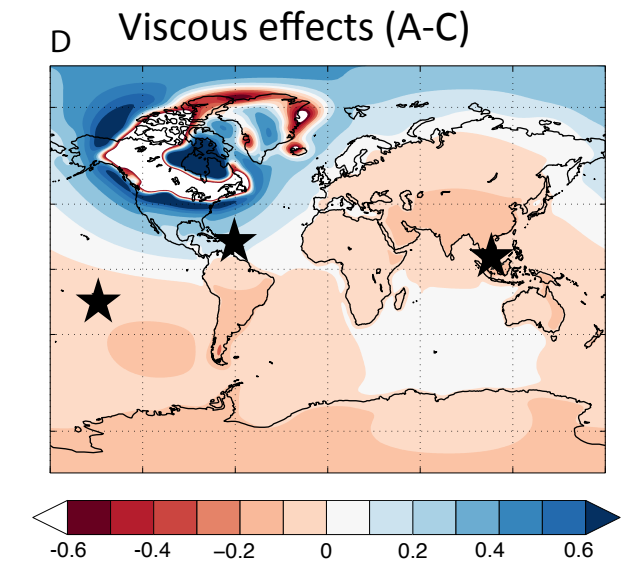
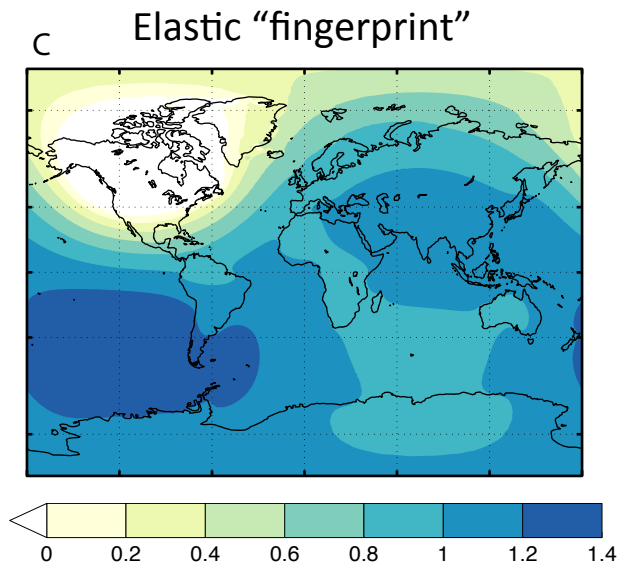
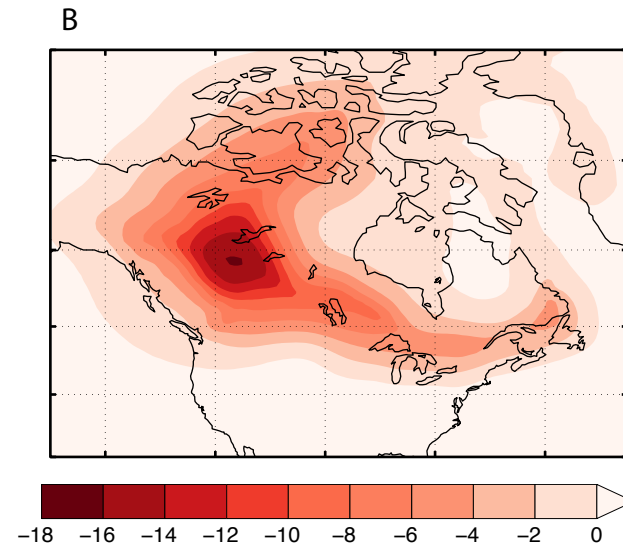
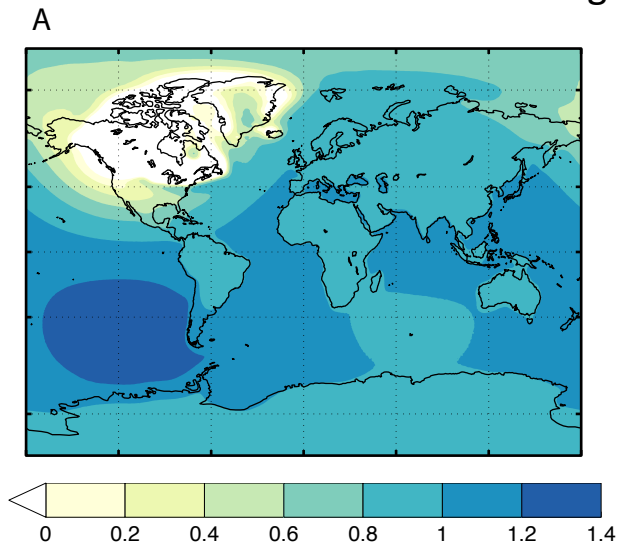
# Meltwater Pulse 1A

Saddle Collapse Associated with Meltwater Pulse 1A in the Gregoire et al. (2012) model

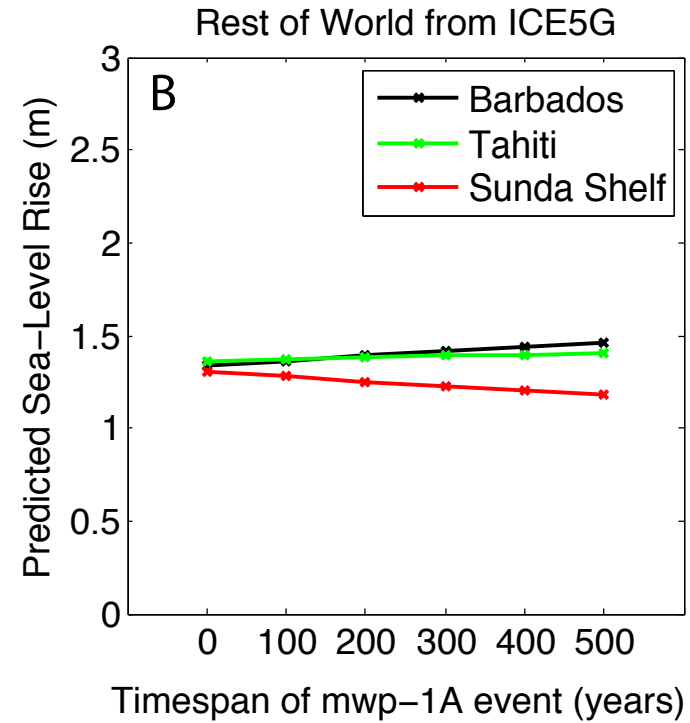
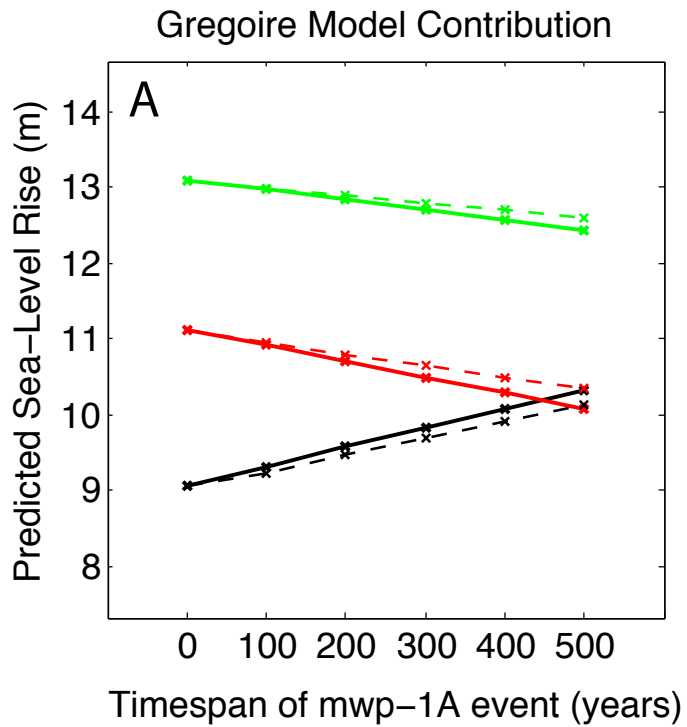


# Meltwater Pulse 1A

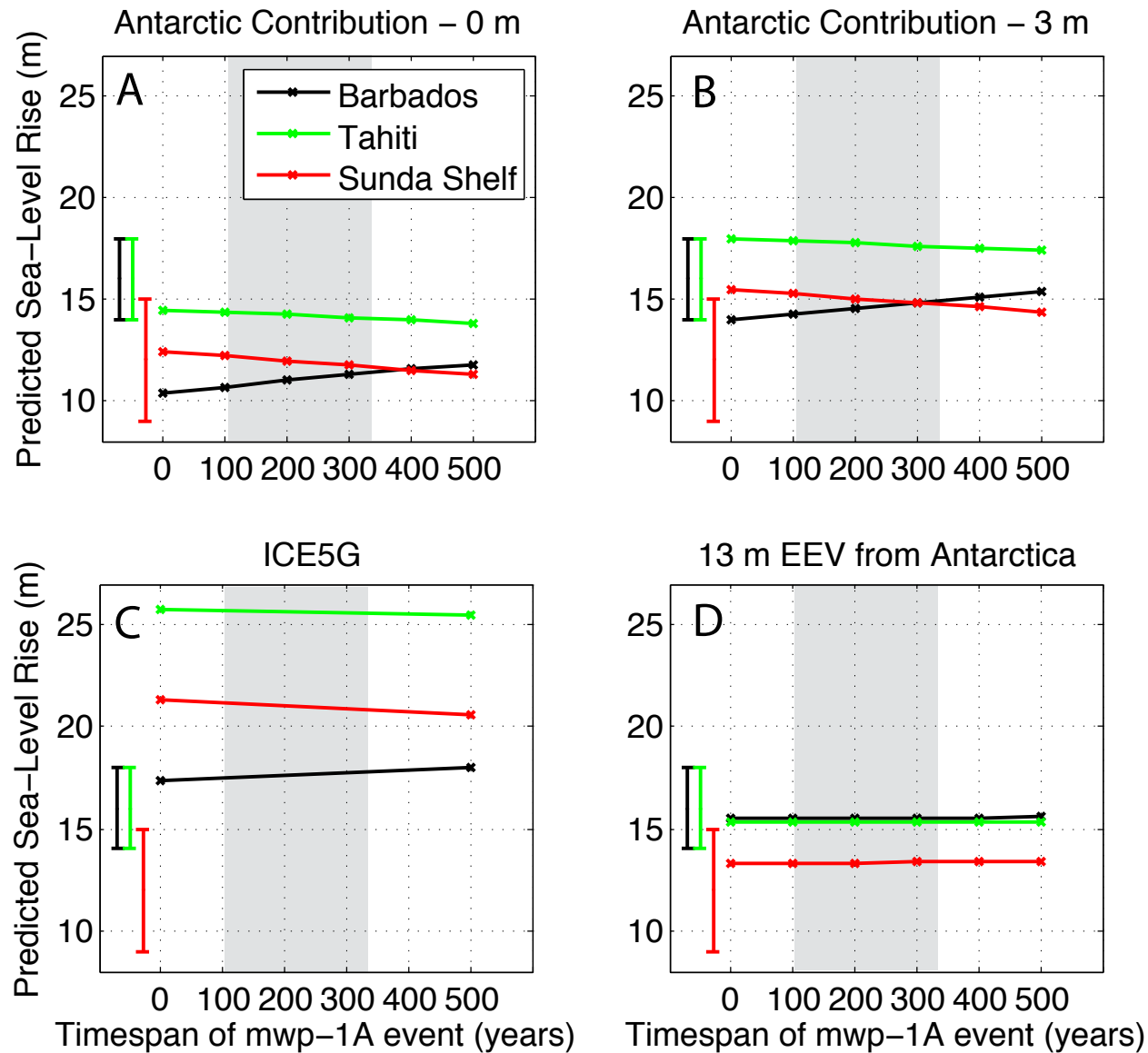
Normalized sea-level change across the 500 yr mwp-1A time window  
Calculated using a viscoelastic Earth model



# Meltwater Pulse 1A



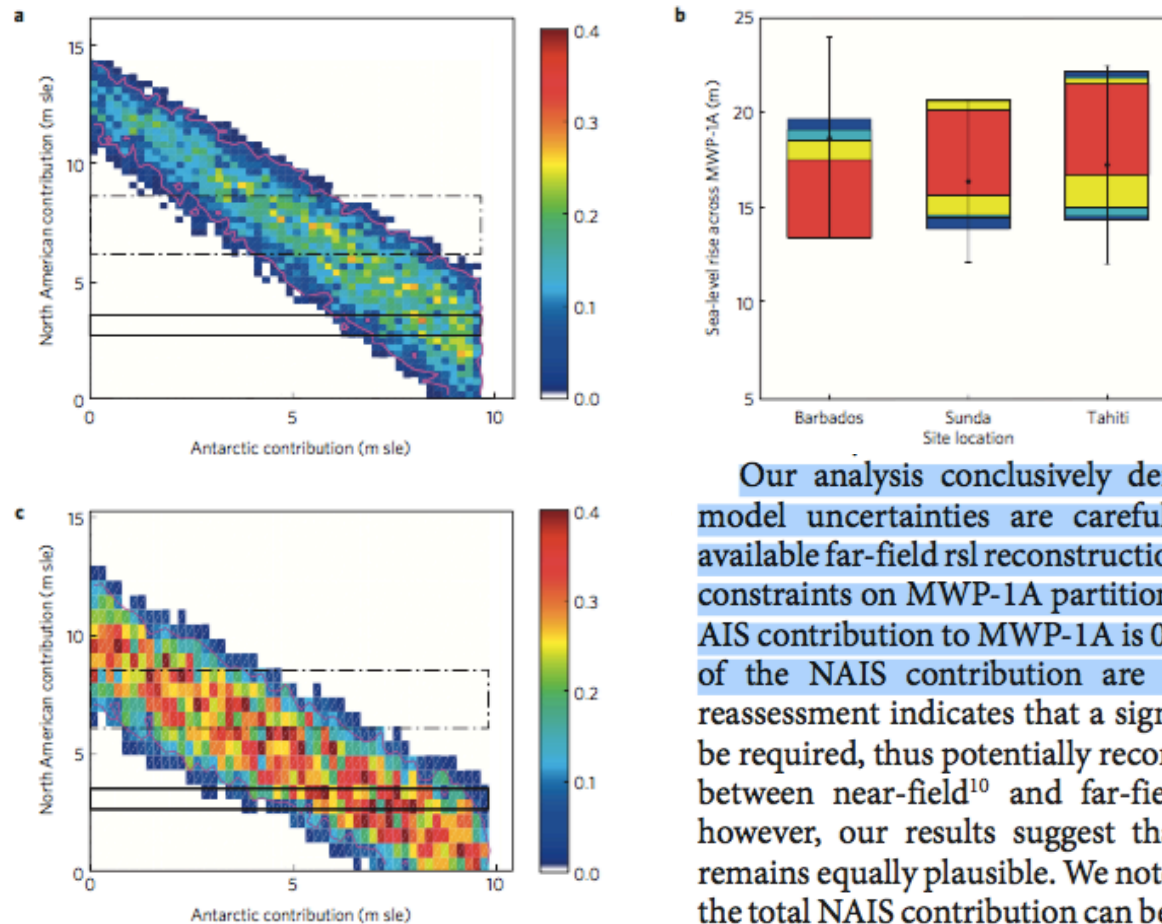
# Meltwater Pulse 1A



# Meltwater Pulse 1A

LETTERS

NATURE GEOSCIENCE DOI: 10.1038/NGEO2616



**Figure 3 | Posterior distribution of NAIS and AIS sea-level contributions conditioned on far-field MWP-1A amplitude estimates<sup>1–3</sup> (a) and for our revised amplitude estimates (c). The magenta contour indicates the central 95% credible range. The black outlines indicate near-field evidence: 2.8–3.7 m sle (solid line; ref. 11) and 6.4–9.0 m sle (dashed-dotted line). Thin vertical bars show the considered far-field sites corresponding to a and c, respectively. Cyan, yellow and red bars show the 95% credible range (minimum to maximum) is represented by the dark blue bars. Note that the model correlation is not visible.**

Liu et al. (2015)

Note: there exists a large body of literature on the climate and ice dynamics associated with MWP-1A

Our analysis conclusively demonstrates that, when data and model uncertainties are carefully accounted for, the presently available far-field rsl reconstructions do not provide tightly bounded constraints on MWP-1A partitioning: specifically, the 95% credible AIS contribution to MWP-1A is 0–10.0 m sle when recent estimates of the NAIS contribution are considered<sup>11,12</sup>. Accordingly, our reassessment indicates that a significant AIS contribution may not be required, thus potentially reconciling the apparent inconsistency between near-field<sup>10</sup> and far-field evidence. At the same time, however, our results suggest that a dominant AIS contribution remains equally plausible. We note that any future improvements on the total NAIS contribution can be directly applied to our AIS–NAIS partitioning diagram (Fig. 3c) and anticipate that the approach taken here will provide the means to further constrain the source regions of MWP-1A as more geologic evidence becomes available. At present, uncertainty in the source distribution of MWP-1A remains a primary limitation in our understanding of the causes and consequences of this extreme event.

# Outline

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1. Sea level change and GIA on ice age timescales.
2. An Example Calculations
3. Applications
  1. Short timescale modern: 20<sup>th</sup> Century Tide Gauge Analysis
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  3. GIA: Archaeological evidence for recent acceleration in sea level rise (Holocene – last 2 ky)
  4. Ice age timescale: Sea Level during the Last Interglacial (~125 ky ago)

# Archaeological evidence for recent acceleration in sea level rise

## Sea level in Roman time in the Central Mediterranean and implications for recent change

Kurt Lambeck<sup>a,\*</sup>, Marco Anzidei<sup>b</sup>, Fabrizio Antonioli<sup>c</sup>,  
Alessandra Benini<sup>d</sup>, Alessandra Esposito<sup>b</sup>

<sup>a</sup>Research School of Earth Sciences, The Australian National University, Canberra 0200, Australia

<sup>b</sup>Istituto Nazionale di Geofisica e Vulcanologia, Via di Vigna Murata 605, 00143 Rome, Italy

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Earth and Planetary Science Letters 224 (2004) 563–575

Fish tanks (Piscinae) (generally carved directly in rock) used at the end of 2nd century and early 1st century BC

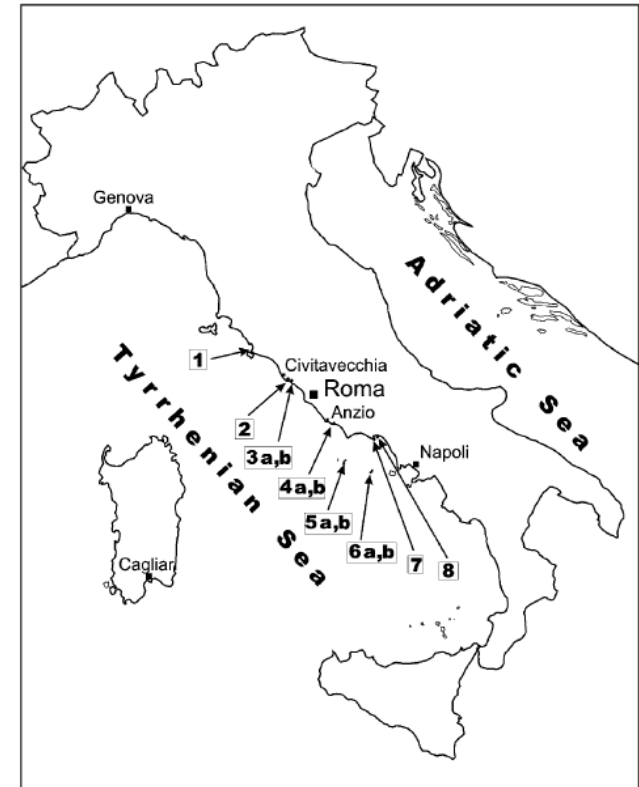


Fig. 1. Location map of the Roman epoch piscinae and tide-gauge sites along the central Tyrrhenian coast of Italy: (1) Santa Liberata, (2) Punta della Vipera, (3a) Santa Marinella Odescalchi, (3b) Santa Marinella Le Grottae, (4a) La Banca, (4b) Torre Astura, (5a,b) Ponza (outdoor and indoor fish tank), (6a,b) Ventotene harbour and fish tanks, (7) Serapo and (8) Sarinola. The four tide gauge sites are located at Genoa, Civitavecchia, Naples and Cagliari.



# Archaeological evidence for recent acceleration in sea level rise

B



A

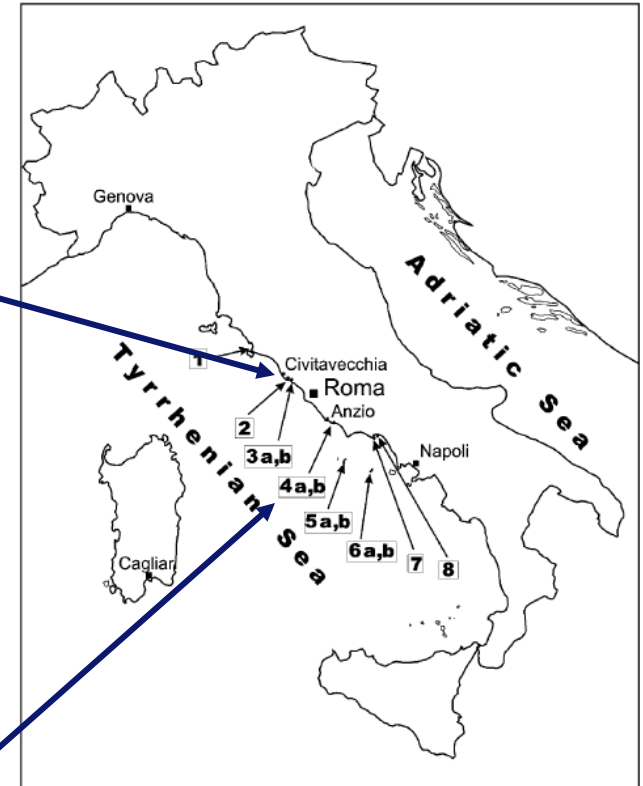


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# Archaeological evidence for recent acceleration in sea level rise

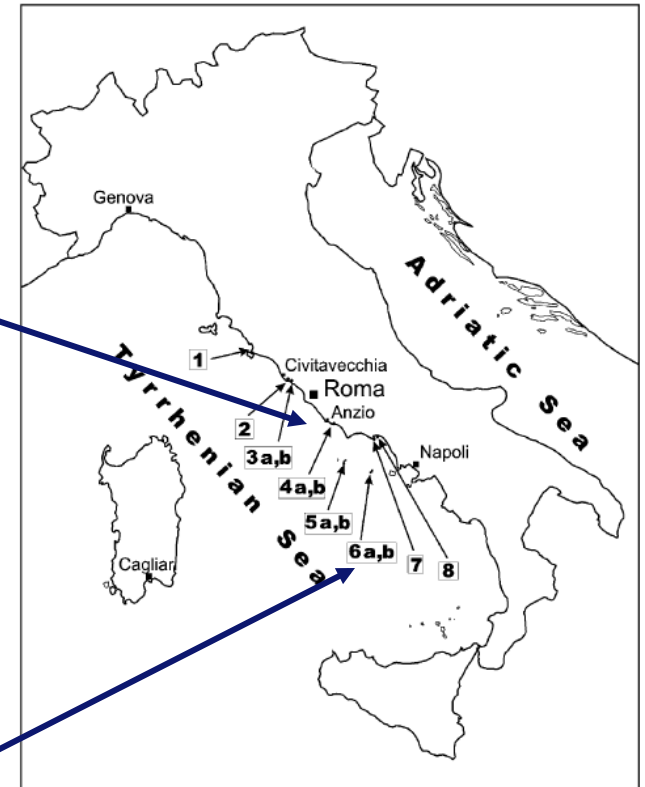
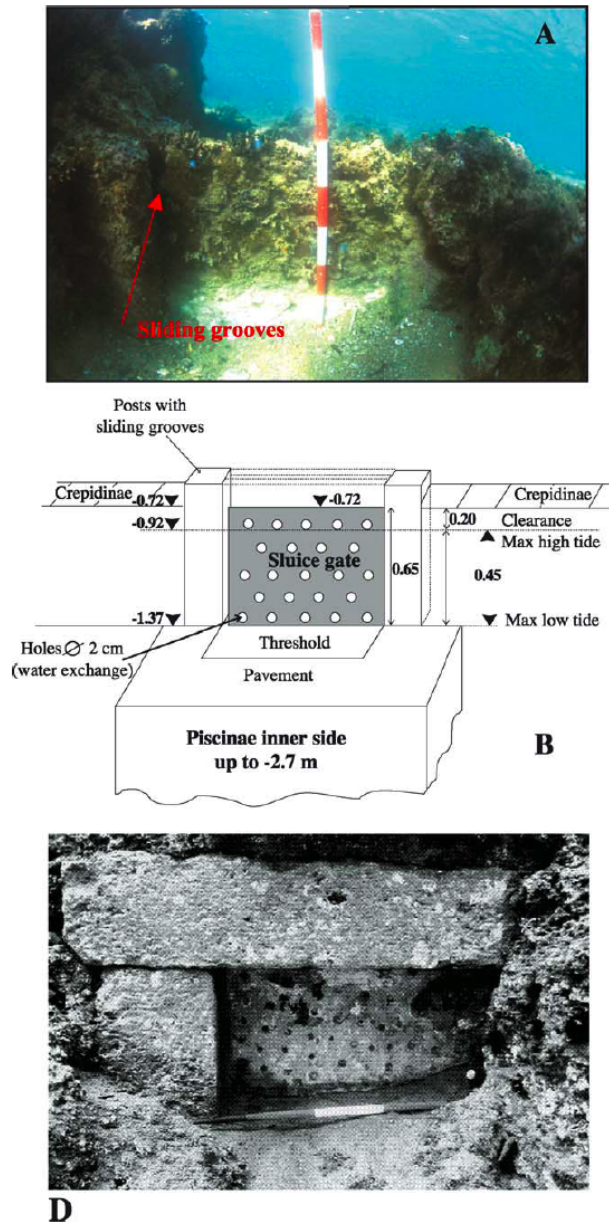
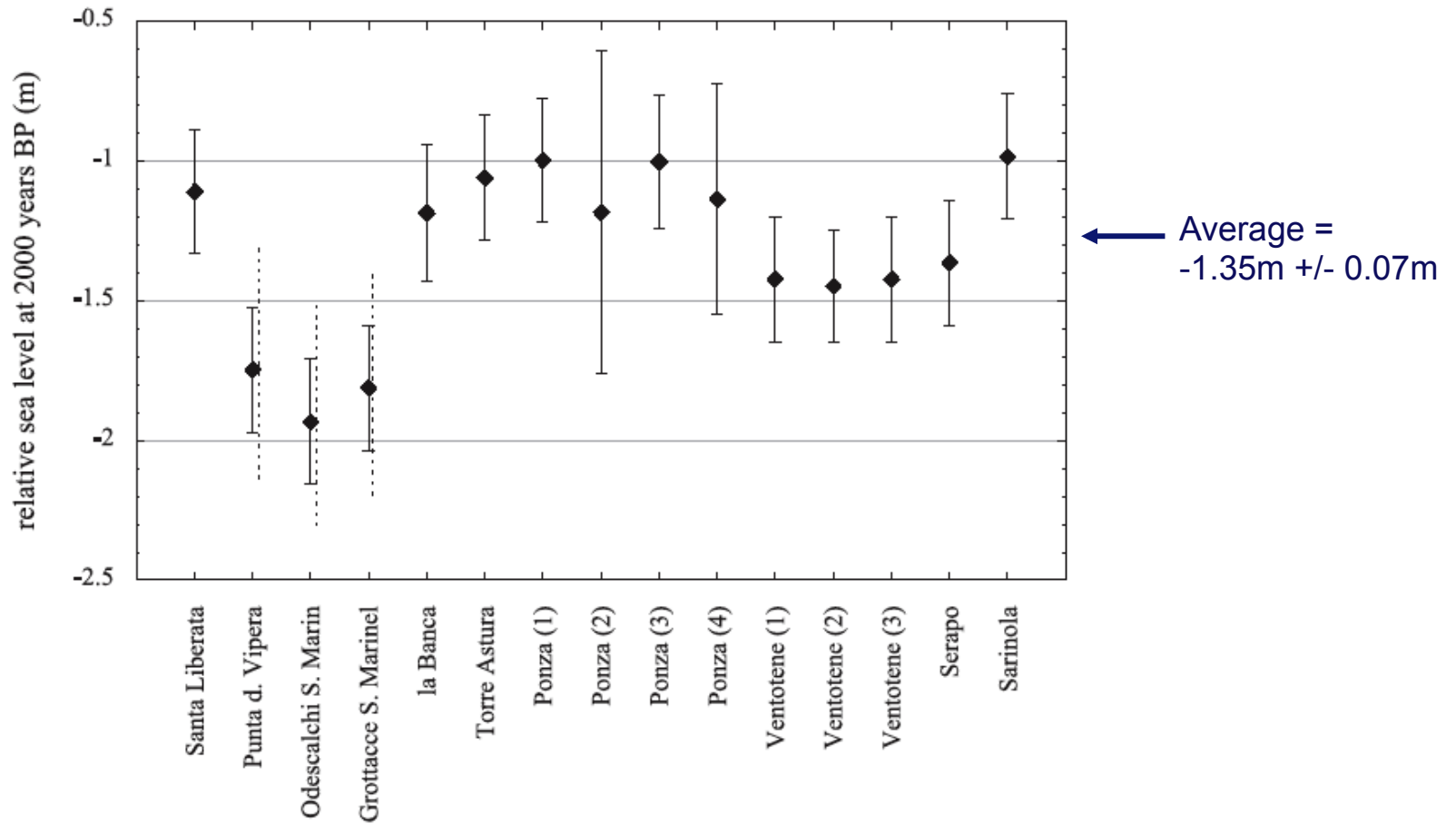


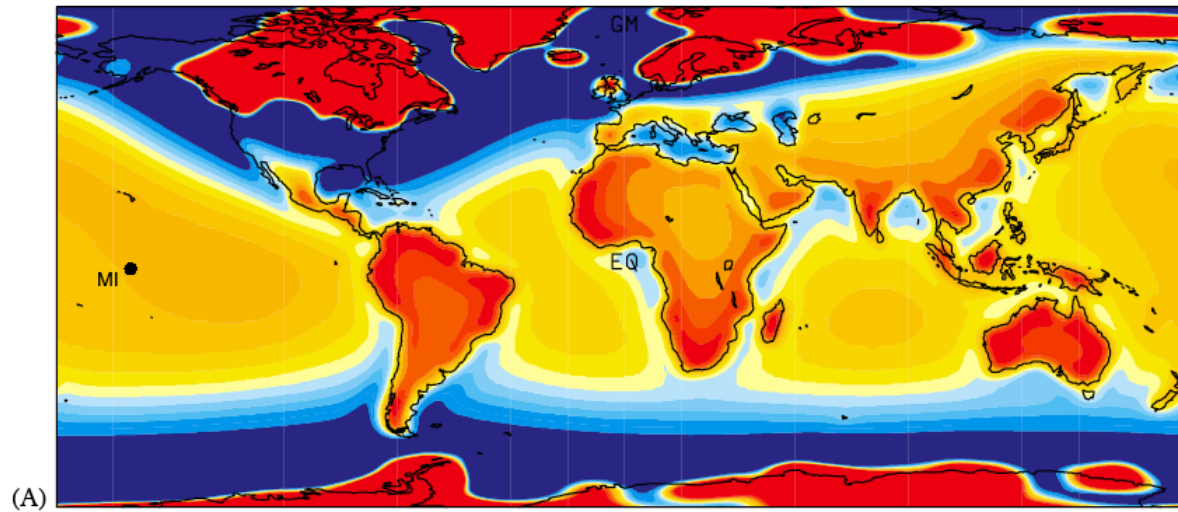
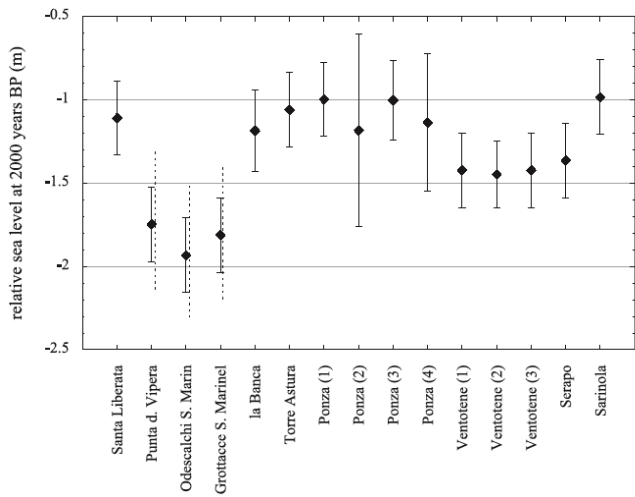
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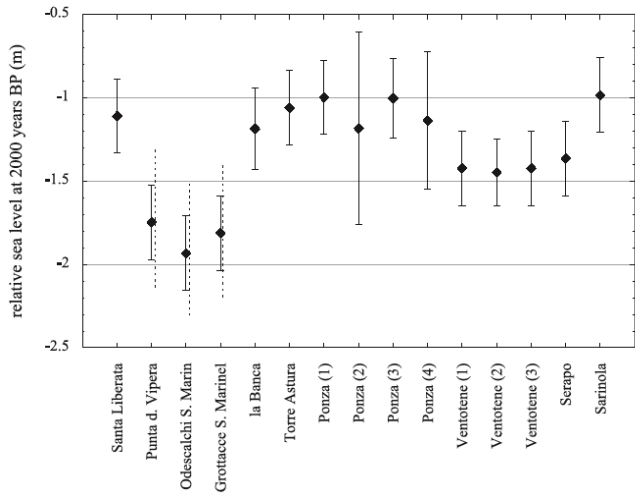


Corrected for vertical tectonic movement

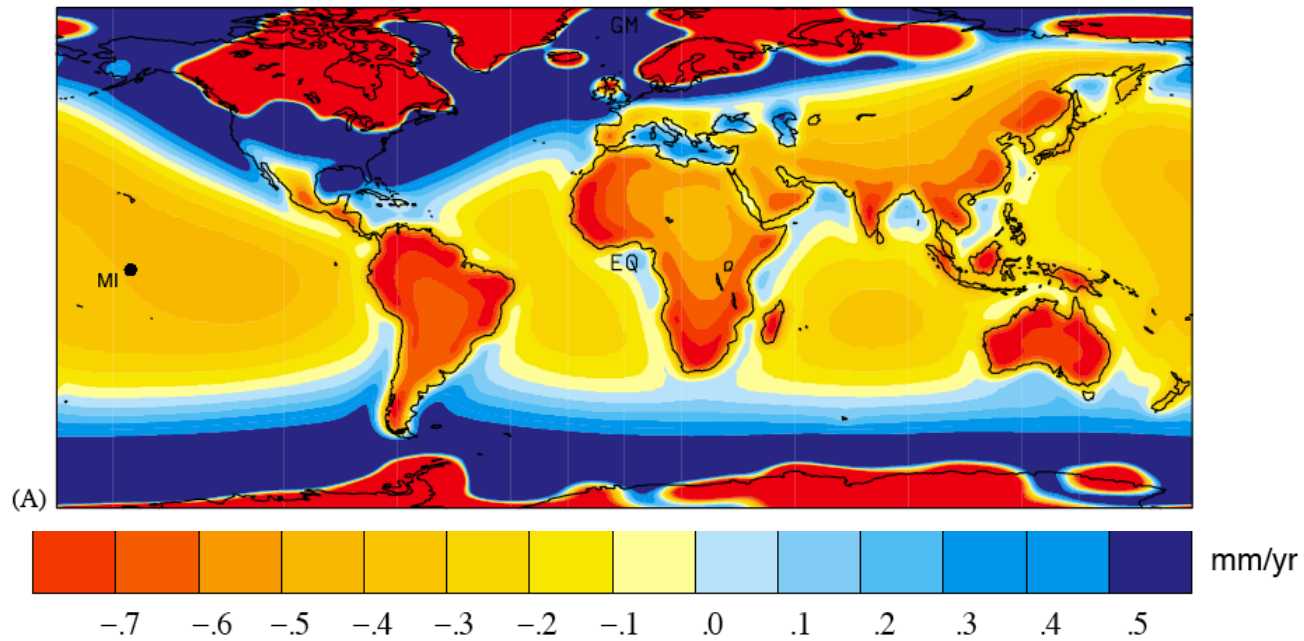
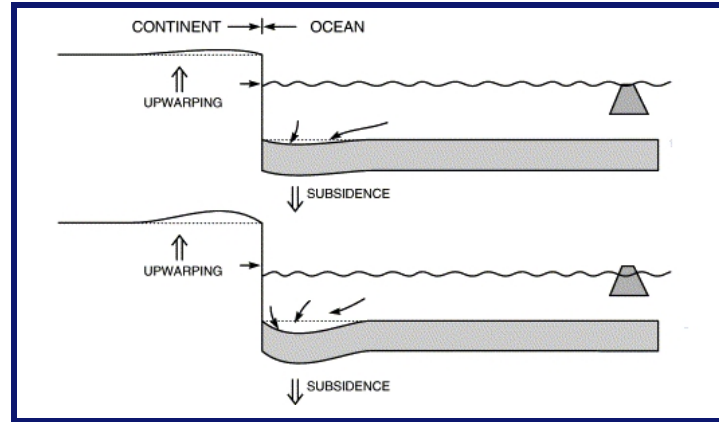
# Archaeological evidence for recent acceleration in sea level rise



# Archaeological evidence for recent acceleration in sea level rise



Dominated by continental levering!



# Archaeological evidence for recent acceleration in sea level rise

## Sea level in Roman time in the Central Mediterranean and implications for recent change

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$-1.37 \pm 0.07\text{m}$   $\longrightarrow$   $-0.13 \pm 0.09\text{m}$

Relative sea-level of 2000  
year old fish tanks



Correction for ice age effects

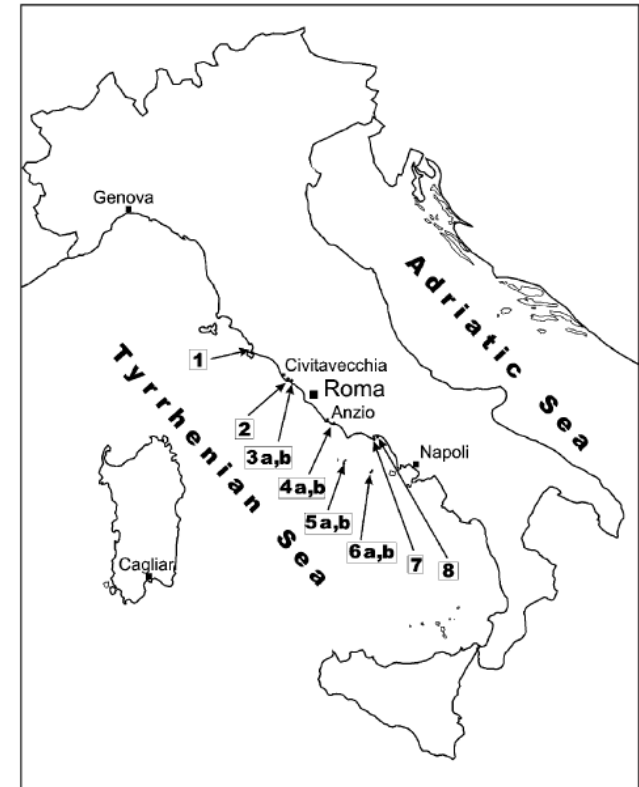


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-1.37 +/- 0.07m



-0.13 +/- 0.09m

Conclude: little mass  
change of polar ice  
sheets in last 2000yrs!

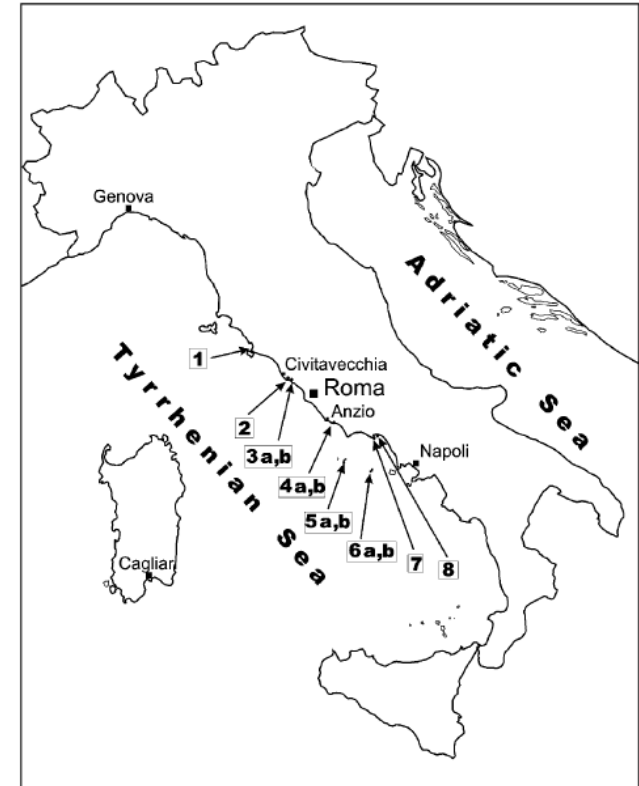


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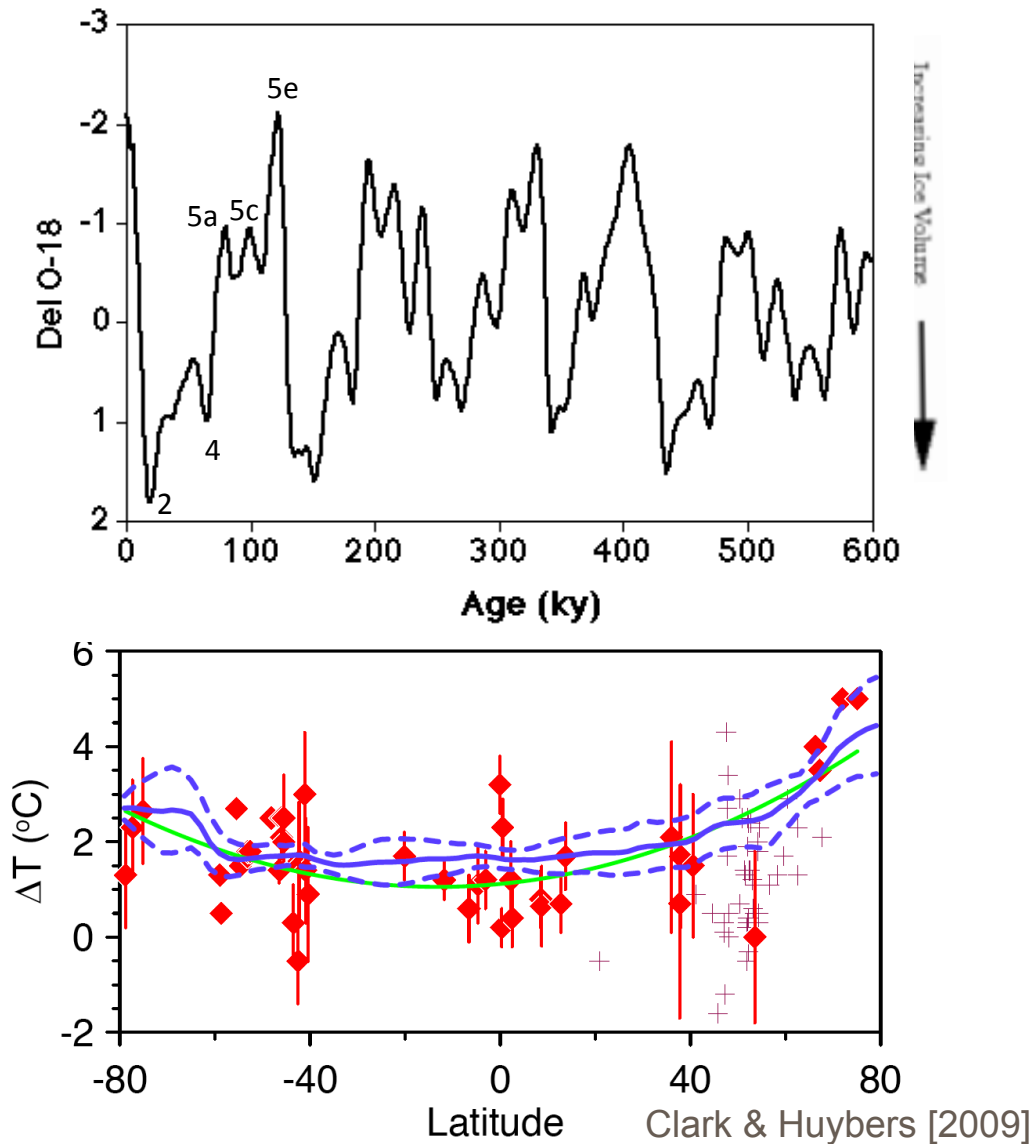
# Outline

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This Class: Sea Level Change Continued...

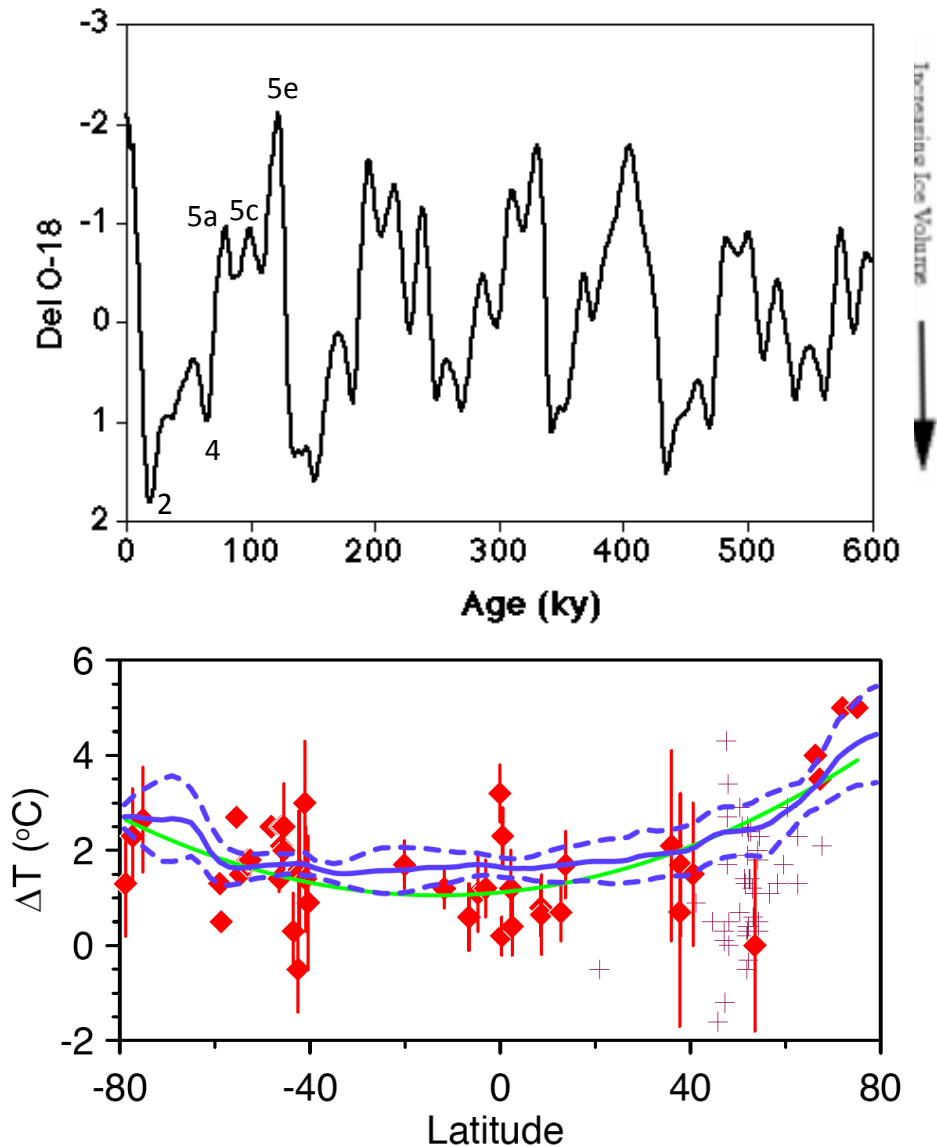
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# Ice Age Sea Level: The Last Interglacial



- Marine Isotope Stage 5e (or the Eemian stage)
- ~125 kyr B.P.
- Polar temperatures were 3-5° higher than present (consistent with 1-2° of global warming)
- Current greenhouse gas concentrations are sufficient to raise global temperatures 1.4-3.2°
- Thus, LIG may be a good analogue for reasonable global warming scenarios

# Ice Age Sea Level: The Last Interglacial



Interglacial outcrop Exmouth, W. Australia, *courtesy Bill Thompson (WHOI)*

Local LIG sea level markers ~4-6 m above present sea-level. What was globally averaged sea level at LIG?

## ARTICLES

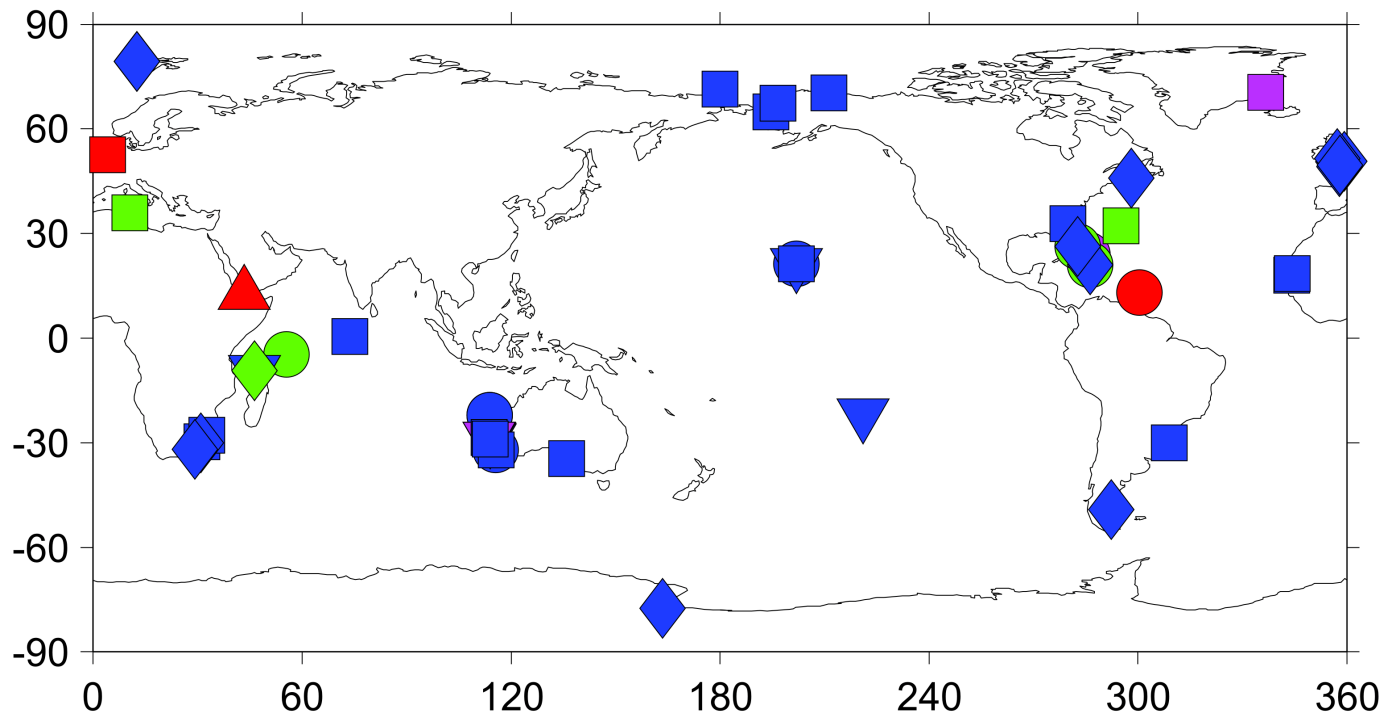
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# Probabilistic assessment of sea level during the last interglacial stage

Robert E. Kopp<sup>1,2</sup>, Frederik J. Simons<sup>1</sup>, Jerry X. Mitrovica<sup>3</sup>, Adam C. Maloof<sup>1</sup> & Michael Oppenheimer<sup>1,2</sup>

With polar temperatures  $\sim 3\text{--}5\text{ }^{\circ}\text{C}$  warmer than today, the last interglacial stage ( $\sim 125\text{ kyr}$  ago) serves as a partial analogue for  $1\text{--}2\text{ }^{\circ}\text{C}$  global warming scenarios. Geological records from several sites indicate that local sea levels during the last interglacial were higher than today, but because local sea levels differ from global sea level, accurately reconstructing past global sea level requires an integrated analysis of globally distributed data sets. Here we present an extensive compilation of local sea level indicators and a statistical approach for estimating global sea level, local sea levels, ice sheet volumes and their associated uncertainties. We find a 95% probability that global sea level peaked at least 6.6 m higher than today during the last interglacial; it is likely (67% probability) to have exceeded 8.0 m but is unlikely (33% probability) to have exceeded 9.4 m. When global sea level was close to its current level ( $\geq -10\text{ m}$ ), the millennial average rate of global sea level rise is very likely to have exceeded  $5.6\text{ m kyr}^{-1}$  but is unlikely to have exceeded  $9.2\text{ m kyr}^{-1}$ . Our analysis extends previous last interglacial sea level studies by integrating literature observations within a probabilistic framework that accounts for the physics of sea level change. The results highlight the long-term vulnerability of ice sheets to even relatively low levels of sustained global warming.

# Ice Age Sea Level: The Last Interglacial

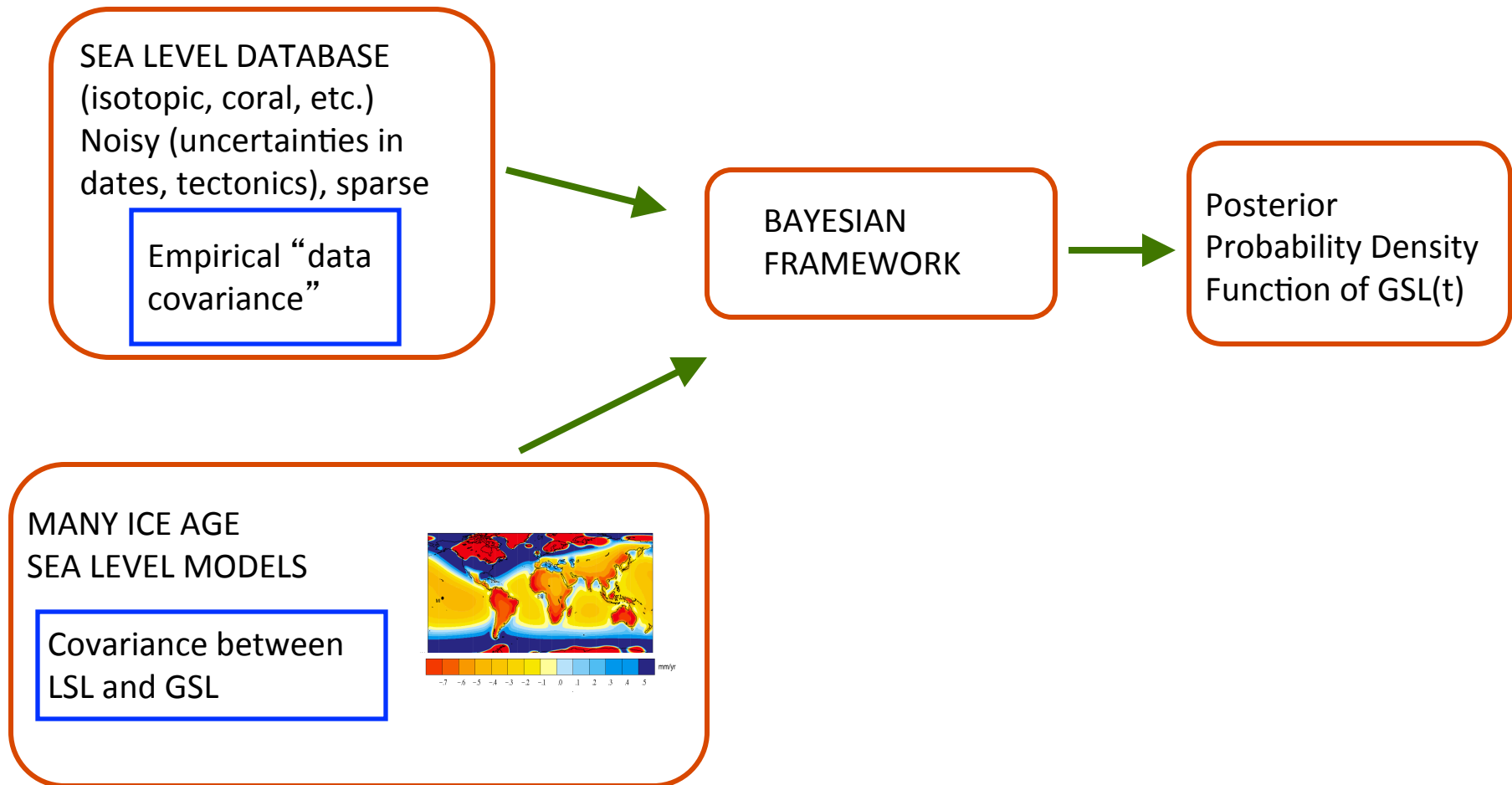


**Figure 1 | Sites with at least one sea level observation in our database.** The symbol shapes reflect the nature of the indicators (upward triangles, isotopic; circles, reef terraces; downward triangles, coral biofacies; squares, sedimentary facies and non-coral biofacies; diamonds, erosional). The colours reflect the number of observations at a site (blue, 1; green, 2; magenta, 3; red, 4 or more).

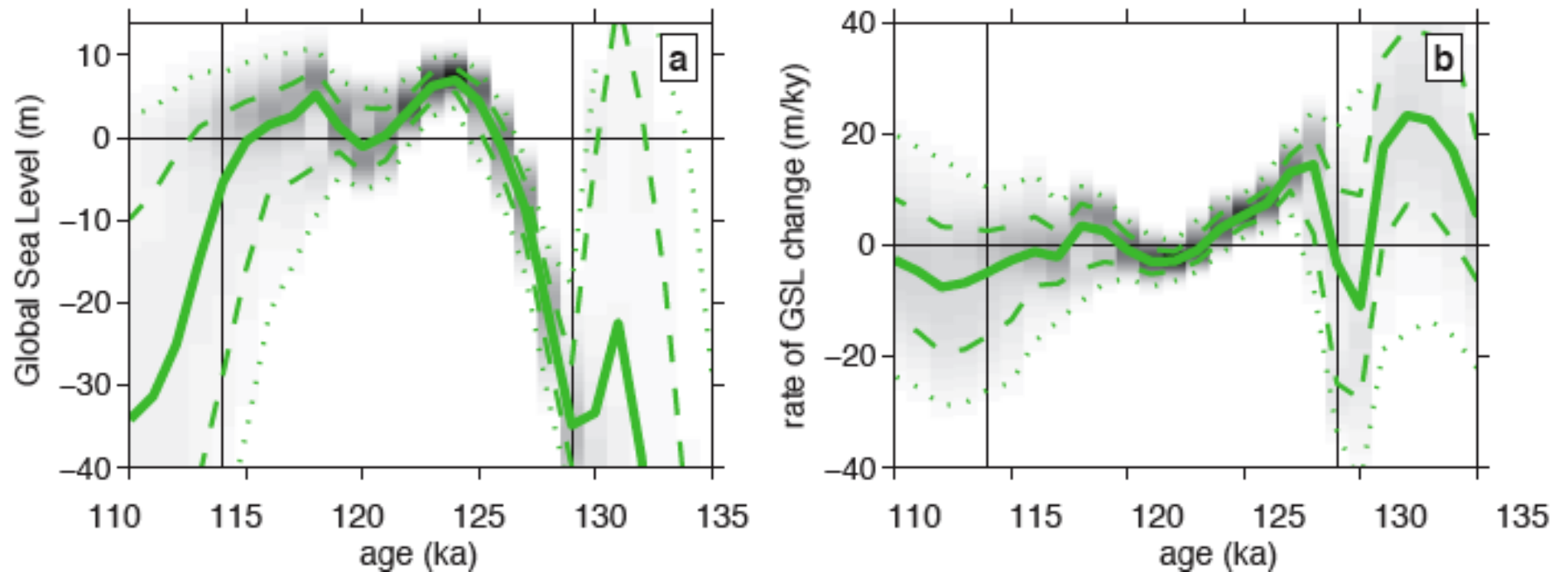
# Ice Age Sea Level: The Last Interglacial

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## Statistical Method (Complicated)



# Ice Age Sea Level: The Last Interglacial

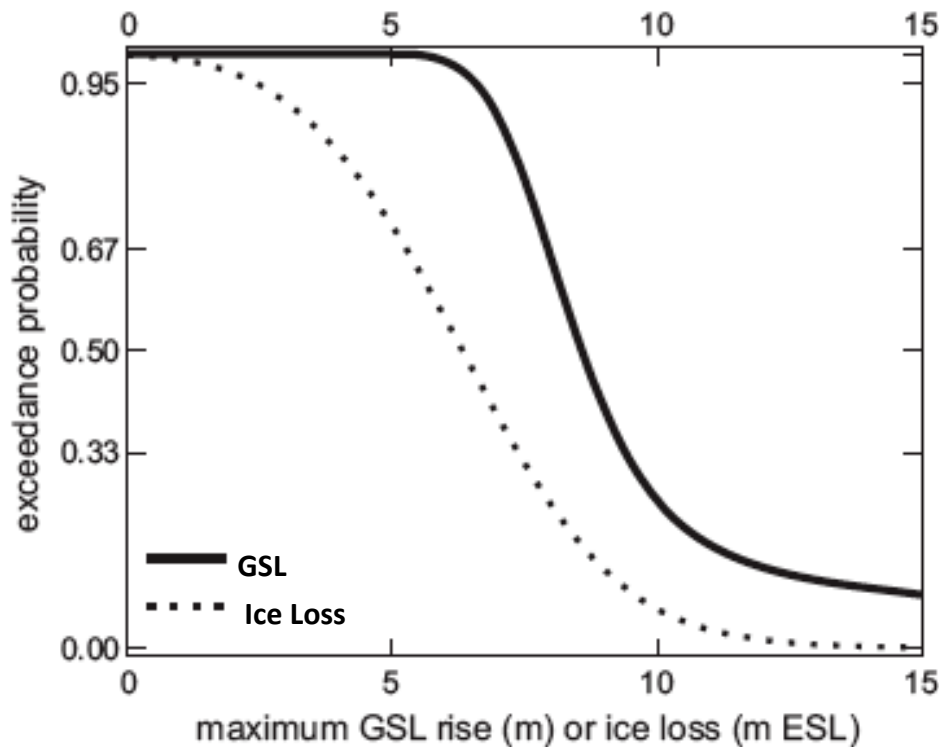


Posterior Probability Densities

Use these to set up hypothesis tests and confidence intervals



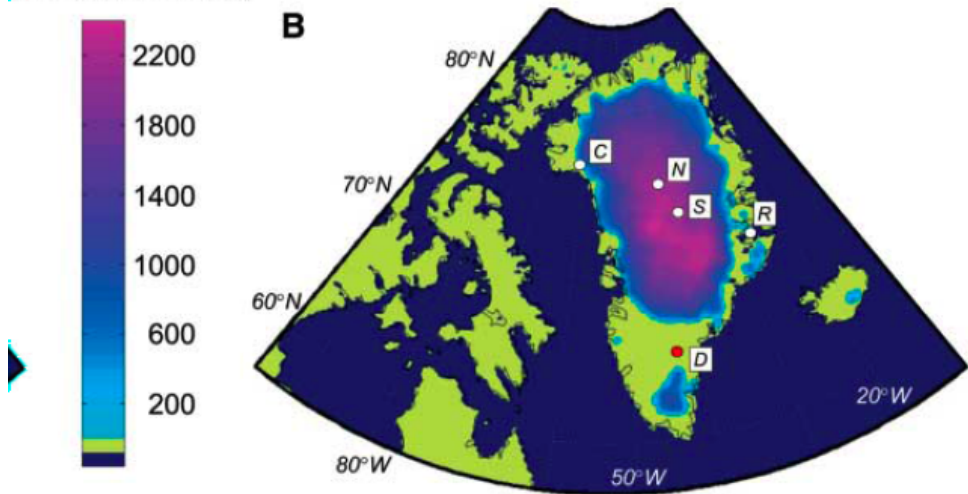
# Ice Age Sea Level: The Last Interglacial



- 95% likely that globally averaged sea level at LIG peaked > 6.6 m above present level (67% likely that it exceeded 8.0 m; only 33% likely that it exceeded 9.4 m)
- 95% likely that both Antarctica and Greenland ice loss at LIG exceeded 2.5 m (equivalent sea level units) relative to present day (not necessarily at the same time)

# Ice Age Sea Level: The Last Interglacial

Ice thickness (m)



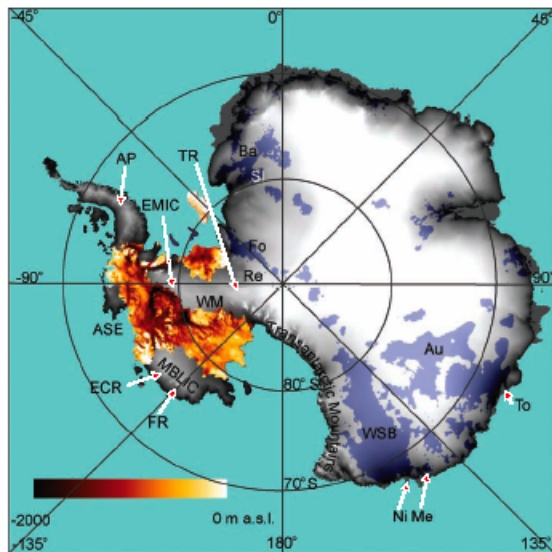
## The Greenland Ice Sheet

- Climate models (Otto-Bliesner et al., *Science*, 2006) suggest a maximum ice loss in the GIS and circum-Arctic ice fields at LIG = 3.4 m GSLR.

Thermal expansion ~ 1 m GSLR

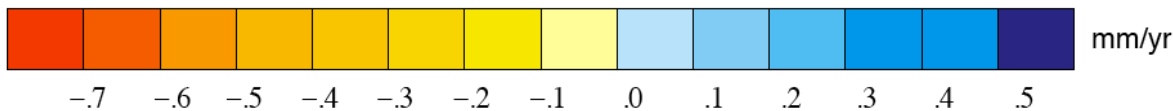
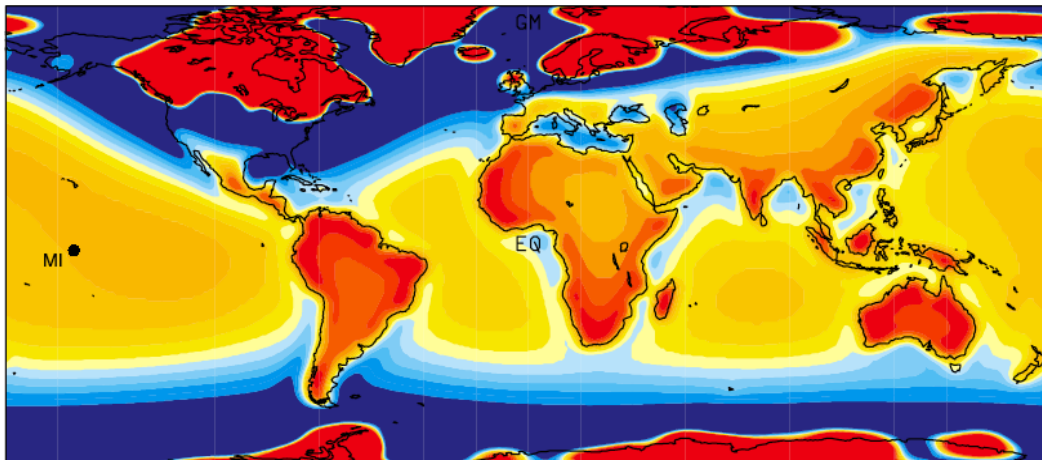
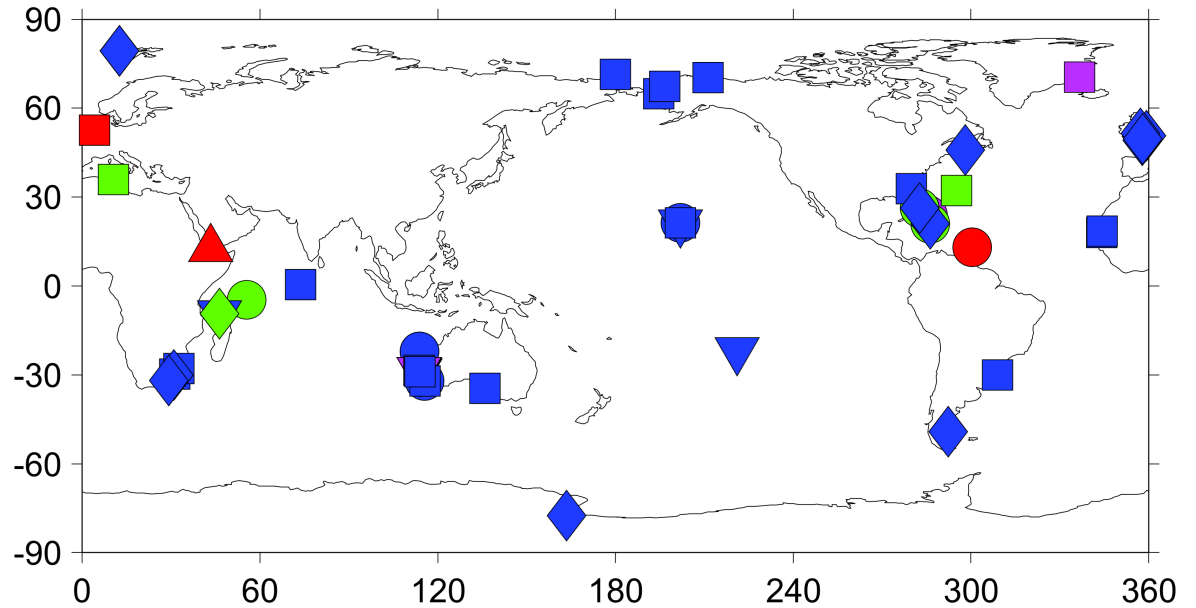
## The West Antarctic Ice Sheet

- Collapse of marine-sectors = 3.2m GSLR (Bamber et al., *Science*, 2009)



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Physics of this result?

Mean prediction of sea-level change at these sites (weighted by number of data points)?

---

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