1 Heat and temperature within the Earth

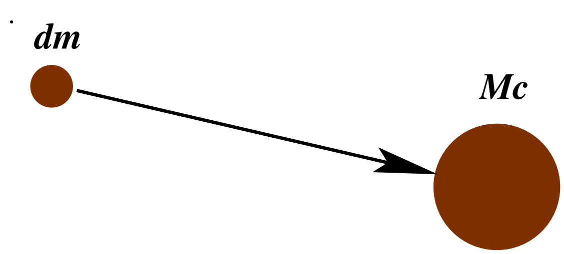
Several scenarios are arguable for the original accretion of the Earth. It may have accreted rapidly and so retained much of the gravitational potential energy as internal heat. It may have accreted slowly enough to have radiated most of the heat of gravitational potential into space and so have been relatively cool in its interior. If so, its interior must have heated quite quickly as a result of radioactive decays. If Earth's accretion arose shortly after supernoval explosions had formed the dust cloud from which the proto-solar nebula condensed, the cloud would have contained substantial quantities of short-lived radionuclei that would have quickly warmed the interior. In any event the heat flow through the surface of the Earth into space attests to a very warm interior at the present time.

1.1 Gravitational energy retained as heat in a condensing planet or the Sun

Recall that von Helmholtz recognized that the gravitational energy contained within the Sun could account for its shining for between $\mathbf{20}$ and $\mathbf{40 \times 10^6}$ years.

How can we estimate this energy? Let's do the physics and calculate the heat equivalence of the gravitational accretion energy for the Earth.

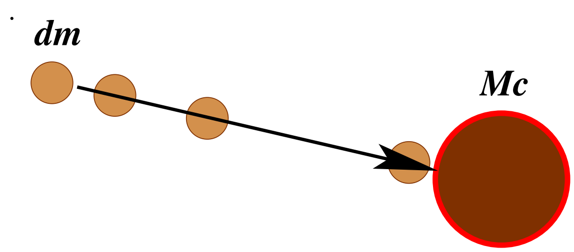
- Starting from an extended and "absolutely" cold (i.e. $\boldsymbol{0K}$) cloud...
- Somewhere a small mass $\boldsymbol{M_c}$ of radius $\boldsymbol{r}$ assembles, perhaps under electrostatic or magnetic forces.
- The volume of our centre, presume a sphere, is then, $\boldsymbol{V_c = 4/3\pi r^3}$ and its density, $\boldsymbol{\rho}$ is such that $\boldsymbol{M_c = \rho V_c}$.
- Now, suppose that at some great distance $\boldsymbol{R_{start}}$ a small element of mass, $\boldsymbol{dm}$, is waiting to fall in upon this gravitating centre.



The small difference in ***potential energy*** of this small mass in the gravitational field of our central mass from that which it would have on the surface at $\boldsymbol{r}$ is determined by ***Newton's Law of Gravity***[1] as

$$dE = dm \cdot (GM_c/r - GM_c/R_{start}).$$

- For large enough R_{start}, it doesn't make much difference to this form if we assume $R_{start} = \infty$. And, then, the difference in potential energy of our small element of mass is
$$dE = dm \cdot GM_c/r.$$
- Now if our small element of mass were to fall in towards our condensation centre, it would ***accelerate*** gaining ***velocity*** and ***kinetic energy*** of motion equal to its continuing loss of potential energy[2].
- Eventually, it would, moving perhaps very fast, hit our central mass centre and release all of its kinetic energy in heat.



- The little condensation centre increases slightly in volume $dV_{layer} = 4\pi r^2 dr$ and if this layer density is just like that of our initial mass centre, the relationship between these elements is
$$dm = \rho\, dV_{layer} = 4\pi\rho r^2 dr,$$
and remembering that $M_c = 4/3\pi r^3 \cdot \rho$, the energy contributed by the infalling dm is
$$dE = 4\pi\rho r^2 dr \ \cdot \ 4/3\pi\, r^3\, \rho\, G/r = G\frac{16}{3}\rho^2\pi^2\, r^4\, dr.$$
- Now we have a mathematical description of how much energy is contributed to a mass centre of radius r when a small amount of material of density ρ falls in from infinite distance.

[1] $G = 6.67 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$ is the universal Cavendish gravitational constant.
[2] This is consequent to the ***Law of Conservation of Energy***.

- All we have to do to find the total energy accumulated in the accretion is to add up all the contributions of the small infalling masses – We ***integrate*** our functional relationship.

- We start our summation or integration a radius $r = 0$ and continue to add up contributions until we come to the full radius $r = R_p$ of our planet.

$$E = \int_{r=0}^{r=R_p} \frac{16G\pi^2\rho^2}{3} r^4 dr.$$

- Those who know some calculus might be able to do this simple integration to obtain

$$E = \left. \frac{16G\pi^2\rho^2\, r^5}{15} \right|_{r=r_0}^{r=R_p}$$

or

$$E = \frac{16}{15}\pi^2\rho^2 R_p^5 G.$$

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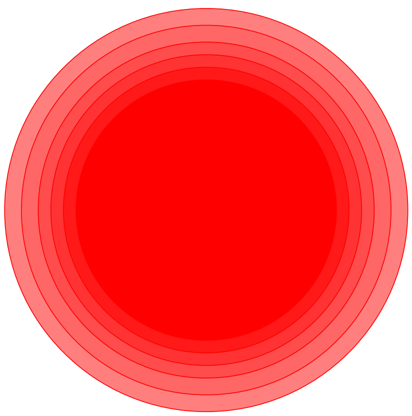


Layer–by–layer, the Earth forms.... hotter and hotter it becomes as more and more gravitational energy is converted to heat!

- Note that the total mass of our now-condensed planet, $M_p = \frac{4}{3}\pi\rho\, R_p^3$, so

$$E = \frac{3}{5}\frac{GM_p^2}{R_p}.$$

For the Earth:

$$\begin{aligned} M_\oplus &= 5.97 \times 10^{24} kg, \\ R_\oplus &= 6.371 \times 10^6 m; \end{aligned}$$

.... the total energy, measured in ***joules*** ($1J = 1kg \cdot m^2 \cdot s^{-2}$) accumulated in the condensation of the Earth: $E = 2.24 \times 10^{32} J$.

For the Sun:

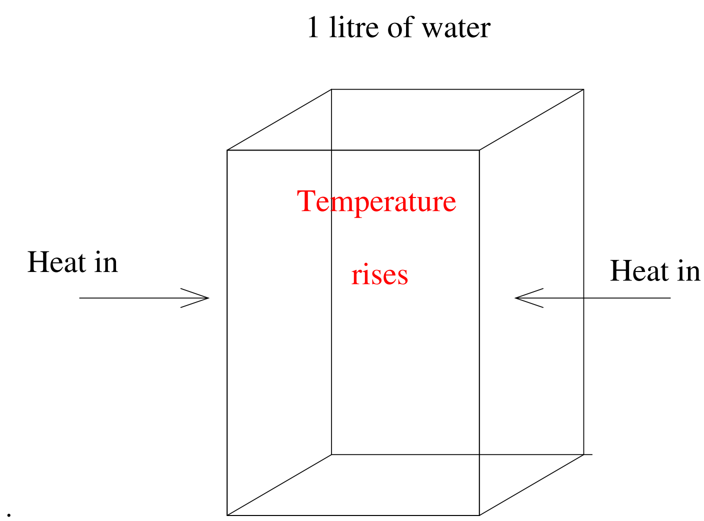
$$\begin{aligned} M_\odot &= 1.99 \times 10^{30} kg, \\ R_\odot &= 6.96 \times 10^{8} m; \end{aligned}$$

...... I leave it to those of you who are itching to do some little physics. Following the arguments offered next, you might be able to estimate the original *average temperature* of the Sun.[3]

Well, now for our nascent Earth, we have a lot of energy largely contained as ***heat***... How hot might the Earth have been, originally? It depends on how well the Earth can hold heat.

1.2 Internal temperature of the condensing Earth

The temperature of a material which internally holds energy in the form of heat depends upon its ***heat capacity***, its ability to hold heat. Water is very efficient, one of the most efficient of all materials, for holding heat.



- The ***calorie*** is a measure of quantity of heat. If we were to introduce $1000\, calorie$ into $1\, litre$ or water at 4^oC, we would increase the temperature of the water to 5^oC. The heat capacity of this litre of water is then $C_H = 1000\, cal/litre/\,K$. As $1\, l$ of water at 4^oC has a mass of $1\, kg$, $C_H = 1000\, cal \cdot kg^{-1} \cdot K^{-1} = 1\, cal/g/^oC$. As $1\, cal = 4.180\, J$, it then requires $4180\, J$ of energy in the form of heat to raise the temperature of $1\, kg$ of cold water through 1^oC. The heat capacity of water in liquid form doesn't depend very strongly on temperature: $C_H \approx 4.180 \times 10^3 J \cdot kg^{-1} \cdot K^{-1}$.

[3] By my calculation $\approx 114\,000\,000 K$, easily hot enough to start the nuclear fires.

- The ***Calorie***...... We all know that weight watchers watch Calories. In food measurements of energy equivalents, the Calorie used is really $\boldsymbol{1\,kcal = 1000\,cal}$. The energy equivalent in 1 pat of butter is about $\boldsymbol{100\,kJ}$ which if efficiently metabolized or burnt could raise the temperature of $\boldsymbol{1\,kg}$ of body mass, mostly water, by $\boldsymbol{24^oC}$ or the body of a young woman of $\boldsymbol{48\,kg}$ by $\boldsymbol{0.5^oC}$. Fats are obviously rich hoards of chemical energy.

- The heat capacity of rocks....
  The heat capacity of rock and metals is quite a lot less than that of water. Typically, rocks show $\boldsymbol{C_{H_{rock}} \approx 10^3 J \cdot kg^{-1} \cdot K^{-1}.}$

- Now we can calculate something of a temperature for the nascent Earth...
  Considering that all of the mass of the Earth is made of rock-like materials or metals, what would be the temperature of the Earth if all the gravitational energy of condensation were retained? It is easy to show that, if the Earth had retained all of this energy at condensation, its temperature would have started at a ridiculously high $\boldsymbol{37\,000^oC}$!

- We now think that the interior of the proto-Earth was actually quite cool....
  $\boldsymbol{37\,000K}$ is $\boldsymbol{6.5}\times$ hotter than the surface of the Sun. All Earth materials would have vapourized and all the atoms and molecules dissociated into a plasma. We actually know that the Earth must have condensed quite cool.

- What is wrong with the physics in our simple argument?
  We have not considered that heat could be lost, through radiation out into a cold empty space, during condensation. By current modelling of Earth formation, it seems that more than 95% of the gravitational energy of condensation would be re-radiated into space.

  - The average temperature of the early proto-Earth was probably not much more than about $\boldsymbol{1100^oC}$ and certainly very much lower than the ridiculous value, $\boldsymbol{37\,000^oC}$.

  We now have good evidence that the Earth condensed cool and has subsequently heated up. What heated it up?

1.3 The accretion and differentiation of Earth

Out of a cloud of dust and gas that comprised all of the elements known to exist today in our Solar System except for promethium, the terrestrial planets condensed from planetesmal fragments in orbits about a proto-sun about $\boldsymbol{4.6 \times 10^9}$ years ago. Most of the hydrogen and helium and other light volatiles like carbon and neon from that primordial cloud condensed into the central Sun whose nuclear fires started to burn in its core.

The composition of the asteroidal and planetesmal fragments that condensed into the Earth closely approximated an olivine stoichiometry with a $\boldsymbol{Mg : Fe}$-ratio of **9**.

Elemental abundances in the Solar System by atom-relative

Element	Atomic number	Atomic weight	Abundance (Urey, 1950)
Hydrogen	**1**	**1**	**400 000 000**
Helium	**2**	**4**	**31 000 000**
Oxygen	**8**	**16**	**215 000**
Neon	**10**	**20**	**86 000**
Nitrogen	**7**	**14**	**66 000**
Carbon	**6**	**12**	**35 000**
Silicon	**14**	**28**	**10 000**
Magnesium	**12**	**24**	**9 100**
Iron	**26**	**56**	**6 000**
Sulfur	**16**	**32**	**3 750**
Argon	**18**	**40**	**1 500**
Aluminum	**13**	**27**	**950**
Calcium	**20**	**40**	**490**
Sodium	**11**	**23**	**440**
Nickel	**28**	**59**	**270**
Phosphorus	**15**	**31**	**100**
Chlorine	**17**	**35**	**90**
Chromium	**24**	**52**	**78**
Manganese	**25**	**55**	**69**
Potassium	**19**	**39**	**32**
Titanium	**22**	**48**	**24**
Cobalt	**27**	**59**	**18**
Fluorine	**9**	**19**	**16**

In the table above, the abundances take into account the preponderant mass of the Sun in our Solar System.

The planets and especially the smaller inner terrestrial planets did not gravitationally hold onto the hydrogen and helium but as their contribution to the overall mass of the Solar System is so small, their lacking in these light elements has little bearing on overall abundances.

1.3.1 On the abundances....on Earth?

Several lines of argument bring us to know that the Earth contains (by mass rather than atom-number):

- $\boldsymbol{Fe}$: $\sim$ **35%**
- $\boldsymbol{O}$: $\sim$ **30%**
- $\boldsymbol{Si}$: $\sim$ **15%**
- $\boldsymbol{Mg}$: $\sim$ **13%**

If we were to assemble these four elements in just this abundance, melt them together and then let them cool and crystallize at relatively low pressure, we would form the mineral *olivine*: $[\boldsymbol{Mg, Fe}]\boldsymbol{SiO_4}$. In the nebular cloud formed about the protosun, such temperatures and pressures are thought to have existed at distances from Mercury out to the asteroids.

1.3.2 The accretion of a terrestrial planet – Earth

During a few tens of thousands of years preceding the birth of our Earth, $\mathbf{4.567 \times 10^9}$ years ago, material out of the primordial dust cloud was being attracted to a gravitational centre in orbit about the already brightening proto-Sun. Materials first coalesced chemically into minerals like olivine or water ice where temperatures were low enough and these, then under physical forces formed into *planetesmals* which bombarded the ever growing Earth. It is thought to have taken less about 10 million years and perhaps as little as a few hundred thousand years for most of Earth's mass to have been assembled. During this process of accretion, much of the heat of bombardment derived from the gravitational potential energy was reradiated into space resulting in a body that was probably not extremely hot or molten throughout. Late in this relentless bombardment, one last large object, perhaps the size of Mars, crashed into the proto-Earth and splashed up an enormous volume of material which itself coalesced in orbit about the Earth and formed our Moon. The energy from this "***Big Whack***" left the outer regions, perhaps to a depth of 1000km, molten. When did this happen? We are quite sure that it happened somewhat before $\mathbf{4.4 \times 10^9}$ years ago (the famous Jack Hills zircons[4] have not been melted since then!) and probably before $\mathbf{4.42 \times 10^9}$ years ago (the recent measurement of the ages of the oldest rocks returned from the Moon).

[4] The Earliest Piece of the Earth

What do we know of the Earth at the time of the Big Whack? The Earth was already partially ***differentiated***.

- Overall, the Moon – we know from its density – contains much *less* $\boldsymbol{Fe}$ than does Earth: little of the deep iron core was spashed from the Earth.
- The surface rocks of the Moon contain *more* $\boldsymbol{Fe}$ than surface rocks of Earth: the Earth has further differentiated since that catastrophic collision.
- The age of the oldest rocks on the Moon are about $\mathbf{40 \times 10^6}$ years older than the oldest minerals found on Earth and about $\mathbf{200 \times 10^6}$ years older than the oldest rock masses found on Earth – the recently famous ***faux-amphibolites*** from the Porpoise Cove area of Northern Quebec[5] which were discovered by Jonathan O'Neil, Ph.D. student in McGill's own Department of Earth and Planetary Sciences.

  Why? How?

  - The small Moon solidified quickly after the collision...
  - The surface of the Earth remained largely molten – and possibly originally to a depth of hundreds of kilometres – for another $\mathbf{200+}$ million years.

The greater amount of iron in lunar surface rocks tells us something about the degree of ***differentiation***[6] of Earth that had already happened by the time of the collision. The geochemistry of these rocks is our best model for that of the outer regions of Earth 4.44 billion years ago.

Cold (slow) accretion model

While there remains a healthy argument concerning the early condition of the Earth and especially as to whether or not it had ever undergone a general melting, one classical model leads us to a ***cold accretion***. According to this model, most promoted by Hanks and Andersen starting in the 1970s, the original temperature profile within the Earth as the bombardment of condensing materials came to an end didn't exceed $\mathbf{2000°}\boldsymbol{C}$ anywhere. For a cool accretion, the Earth could not have assembled so quickly that the heat of formation could not largely be lost through radiation. Today, the deep interior of the Earth is very much hotter than $\mathbf{2000°}\boldsymbol{C}$. Temperature at the centre of the Earth's solid iron core is demonstrably at least $\mathbf{6000}^{\boldsymbol{o}}\boldsymbol{C}$. Even magmas erupting from Hawaïan volcanoes show temperatures exceeding $\mathbf{1500}\boldsymbol{K}$. If accretion was cool, the Earth has heated up internally since.

If the Earth started out cold how has it heated up since its initial formation?

[5] O'Neil, J., Francis, D.F. and .... Science article

[6] Differentiation: the denser elements and minerals fall toward the centre of the Earth and the lighter elements and minerals rise towards the surface.

Two processes are surely implicated: the "***big whack***", the late collision with a large, Mars-sized planetesmal that splashed the Moon into orbit about Earth and ***radioactive decay***.

- Aluminum is abundant on Earth. One isotope of aluminum, $^{26}\boldsymbol{Al}$, would have been relatively abundant if the condensing supernoval explosion cloud of debris had not long lingered before condensation began.
  - $^{26}\boldsymbol{Al}$ decays to $^{26}\boldsymbol{Mg}$ by emitting a $\boldsymbol{\beta^+}$ particle with a half-life of only $\boldsymbol{7.3 \times 10^5\, yr}$.
  - Enough of this best-candidate isotope condensed into the Earth to have produced the sufficient heat distributed throughout the Earth to have already started the physical differentiation during the period of accretion.
  - Also, many other short-lived isotopes of the lighter elements must have also condensed with the cloud and so contributed to a rather rapid heating of the Earth.
  - If the cloud had been produced by a supernoval explosion, it is possible that it contained quite a lot of $^{60}\boldsymbol{Fe}$ which again decays to $^{60}\boldsymbol{Ni}$, the second most abundant isotope of nickel on Earth, in a short, $\boldsymbol{3 \times 10^5\, yr}$ half-life.
  - As well, if the cloud condensed soon enough after a supernoval explosion – and stellar and planetary formation[7] seems to be happening today in the young ***Crab*** and ***Orion Nebulae*** which are understood to be supernoval remnants – highly radioactive and fissionable transuranic elements such as einsteinium, $\boldsymbol{Es}$, fermium, $\boldsymbol{Fm}$, europium, $\boldsymbol{Em}$, and californium, $\boldsymbol{Cf}$, may have still existed in quantity and their decay could have produced prodigious amounts of heat within the newly condensed Earth.
  - None of these original radioisotopes are naturally occuring in measurable quantities today. Now, substantial quantities of only uranium, $\boldsymbol{U}$, thorium, $\boldsymbol{Th}$, and potassium $^{40}\boldsymbol{K}$ are contributing to the planetary heating and these are largely concentrated in the outer rocky crust and upper mantle of the Earth though recent research has shown that $^{40}\boldsymbol{K}$ could alloy with iron and so might exist in the inner core. These radionuclei contribute significantly to the internal heating of the planet at present. The now-high internal temperature of the planet still partially derives from the very early radioactive heating of the planet by the short-lived radioisotopes but most of the present internal heat and consequently high internal temperature, especially of the mantle, is due to the continuing decay of $\boldsymbol{U}$, $\boldsymbol{Th}$, and $^{40}\boldsymbol{K}$. The decay of $^{40}\boldsymbol{K}$ into $^{40}\boldsymbol{Ca}$ and $^{40}\boldsymbol{Ar}$ now contributes most to

[7] http://hubblesite.org/discoveries/10th/vault/in-depth/search.shtml

the continuing internal heating of the Earth's mantle. Still, all the early heat of original accretion has not yet been entirely radiated away from the Earth through its surface into cold space.

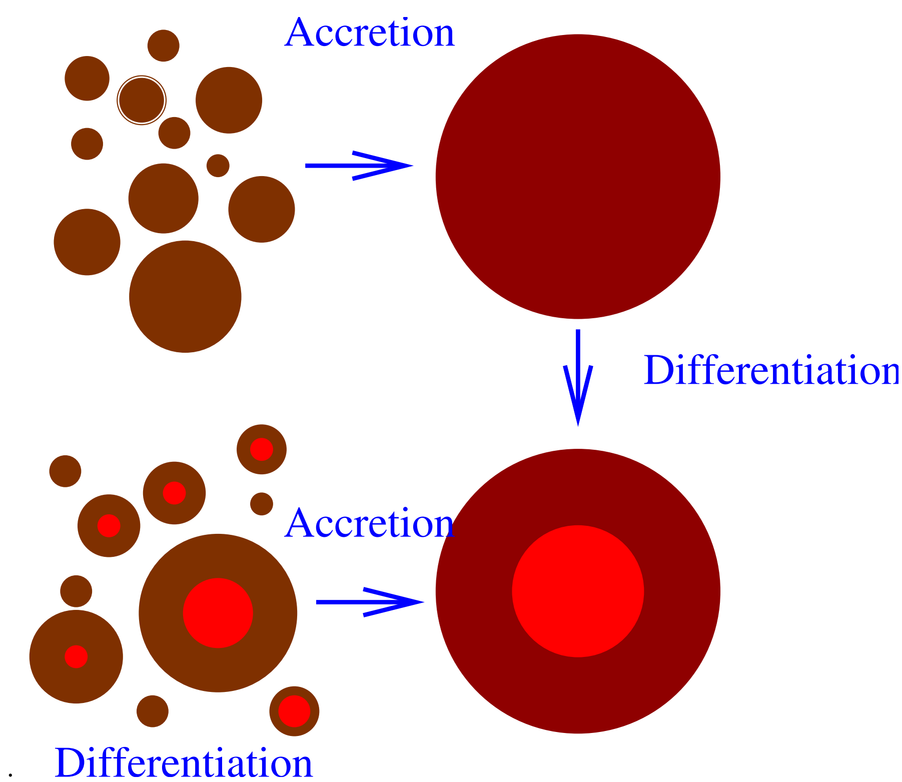
- Importantly, now, the deepest interior heating derives from the continuing geochemical differentiation and from the release of the latent heat of fusion of iron as the Earth's inner core slowly freezes. As it has been shown that potassium can alloy with iron at very high temperatures and pressures, $^{40}\boldsymbol{K}$ could also be contributing to the heating of the deepest interior regions of the planet.

Other arguments...

One alternative argument for the early stages of differentiation and iron melting in the upper mantle relates to the formation of the Moon. Collision with a Mars-sized object reasonably accounts for the formation of the Moon, for the Earth's rather high angular rotation rate and, perhaps, for its inclined rotation axis. Had such a collision occured, as argued above, it must have occured very early on in the Earth's history and certainly before ***4.4Ga***[8] for we have minerals of that age on Earth and they could never have been since melted. Such a collision would have caused global and deep melting of the mantle of the Earth... the Earth would have been awash in a deep "***magma ocean***" of molten rock from which the melted iron would sink to depth. Recall that the Moon seems deficient in iron relative to the Earth. If radioactive heating of cold-accreted Earth had already taken place and so started the migration of iron towards the Earth's core, this would account for a lower abundance of iron on the Moon which would have formed of material from the colliding object and from only the outer regions of the Earth. Of course, it could well be that the colliding object was poorer in iron than the Earth and so diluted the iron abundance of the splashed-up mix of material. It could well be that both bodies were already differentiated with an iron core and that mostly mantle layer materials from both objects contributed to the splash which condensed into the iron deficient Moon. Probably, the iron core of the impactor assembled into the inner Earth.

There are several alternative scenarios that are arguable – the one presented in detail is that many planetologist see as most reasonable.

[8]We often use Ga, "giga-ans", to describe a billion years past.



Two models for accretion: (top) homogeneous, (bottom) planetesmal differention.

A second model for planetary formation argues that even relatively small planetestmals that assembled to form the Earth and terrestrial planets had, themselves, already differentiated out iron from the essentially *olivine* mineral matrix.

A third scenario follows from a very rapid accretion of Earth and, presumably, the other terrestrial planets. In a sufficiently rapid accretion, the outer regions of the neo-planet could remain essentially molten, perhaps to depths of hundreds or even a thousand kilometres in the case of Earth. In this molten magma ocean, metallic iron could have separated and sunk away to depth as a consequence of its high density. The unknown chemical oxidation state of the condensing materials becomes an important factor is this latter scenario for if the ocean were sufficiently oxidizing, the iron would have remained combined as $\boldsymbol{Fe_2O_3}$ which may not have easily and quickly sunk to depth.

Whether the iron was originally relatively homogeneously distributed within the proto-Earth or was already assembled and separated in the planetesmals or quickly melted away into the deep interior of the proto-Earth is now a matter of intense debate among planetologists. When we better understand the environment and condition of the solar nebula, we may be able to differentiate between these scenarios.

The environment in which the Solar System formed is still under debate. A cold, slow accretion model for the formation of planets would require a rather stable environment such as that of the ***Tauri Auriga molecular cloud***[9]. Recently, a more violent and chaotic environment, such as that in the Orion[10] or Crab[11] Nebulae, has been argued as being more probable for the formation of our Solar System. The violent environment could account for the rather small distance to the edge of *our* Kuiper belt and for the fact of Uranus and Neptune having much less $\boldsymbol{H}$ and $\boldsymbol{He}$ than do Jupiter and Saturn. In an environment with many supernoval explosions, the outer regions of the proto-planetary Solar System could be stripped away. Moreover, the violent environment can account for many details of the cosmochemistry of meteorites. It could explain a heating for the possible partial differentiation of iron in olivine minerals before accretion.

The Crab Nebula is the remnant of a supernoval explosion that was observed and documented by Chinese astronomers in 1054. A rapidly rotating ***neutron star*** or ***pulsar*** exists within the nebular debris cloud; this fact tells us that the explosion was that of a massive-star supernova as the white-dwarf supernoval explosions leave no cores behind.

1.4 Geochemical differentiation of an Earth-like planet

Many scenarios have been proposed for the earliest condition of the Earth. Whatever its early state, the Earth quickly separated out a deep iron core.

- **Scenario 1:** It may have been substantially heated through one or many post-accretion collisions. Alternately, if the accretion of the Earth happened sufficiently quickly – over say, a few 10s of thousands of years – the Earth would have retained much of its gravitational energy of accretion.
  - Large enough collisions or sufficiently quick accretion causes a melt of a thick outer layer of the newly accreted Earth.
  - The olivine-like composition of this layer separates
  - The iron-rich composition of olivine melts at a lower temperature than the magnesium-rich component. The iron-rich partition stays in the liquid state to lower temperatures as the melt cools and the iron leaches out.
  - The lighter elements like $\boldsymbol{Ca}$, $\boldsymbol{Na}$, $\boldsymbol{K}$, $\boldsymbol{Al}$ float in the melt and form the minerals of the crust of the Earth.

[9] http://hubblesite.org/newscenter/newsdesk/archive/releases/2000/32/

[10] http://hubblesite.org/newscenter/newsdesk/archive/releases/1995/49/image/b

[11] http://hubblesite.org/newscenter/newsdesk/archive/releases/2000/15/

- **Scenario 2:** The Earth may have assembled from already iron-differentiated planetesmals.
  - The dense iron sinks into the deep Earth during the process of accretion.
  - The lighter fractions rise toward the surface and the least dense of the minerals are left behind in the upper mantle to form the early crust.
- **Scenario 3:** Homogeneous accretion (an early model that still "works")
  - An initial geochemical differentiation of the Earth from a cool, relatively homogeneous accretion would require that the Earth accreted relatively slowly – over a few million years.
  - The subsequent differentiation process was rapid enough that the Earth had already differentiated its core before the collision of a Mars-sized object with Earth splashed up the Moon, perhaps within less than **100** million years of the accretion.
  - Following this scenario, we presume that the Earth accreted with an internal temperature probably not exceeding about $\mathbf{2000}\boldsymbol{K}$ anywhere.
  - Surface temperature: $\approx \mathbf{270}\boldsymbol{K}$ in equilibrium with our cool station in the Solar System.
  - Temperature at depth: $\approx$*adiabatic* rising to perhaps $\mathbf{2000}\boldsymbol{K}$ at depth.
  - Homogeneous composition...

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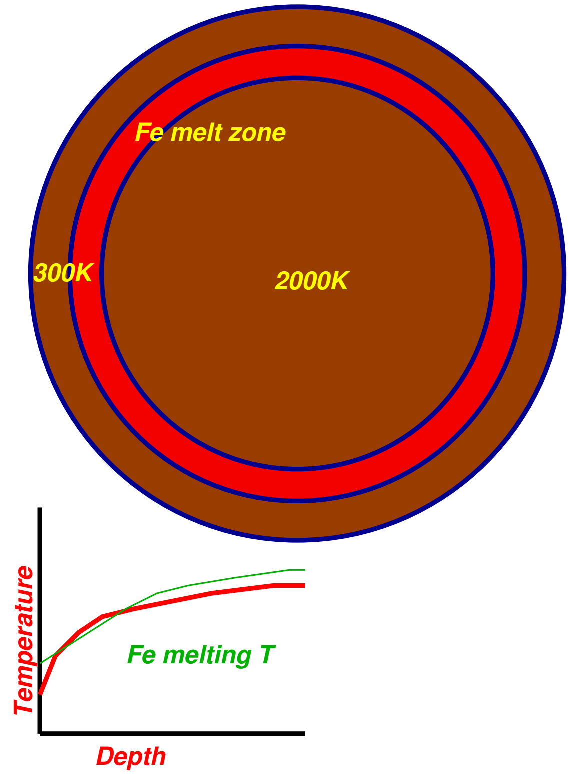


Iron melts in a shallow zone when it is warmed beyond the melting point for the containing pressure.

- Subsequent heating: as the relatively short-lived radionuclei within decayed, the Earth heats up.... until...
- At some depth (perhaps $\mathbf{200 - 300}\boldsymbol{km}$), the pressure-equilibrium melting temperature of iron is exceeded and a layer of liquid iron begins to form...
- Iron, being very dense and now liquid, would tend to work its way deeper into the Earth releasing gravitational potential energy.
- Now, heat derives from Earth's continuing geochemical differentiation and from the release of the latent heat of fusion of iron as the Earth's inner core slowly freezes as well as decay of radionuclei and other possible nuclear processes.

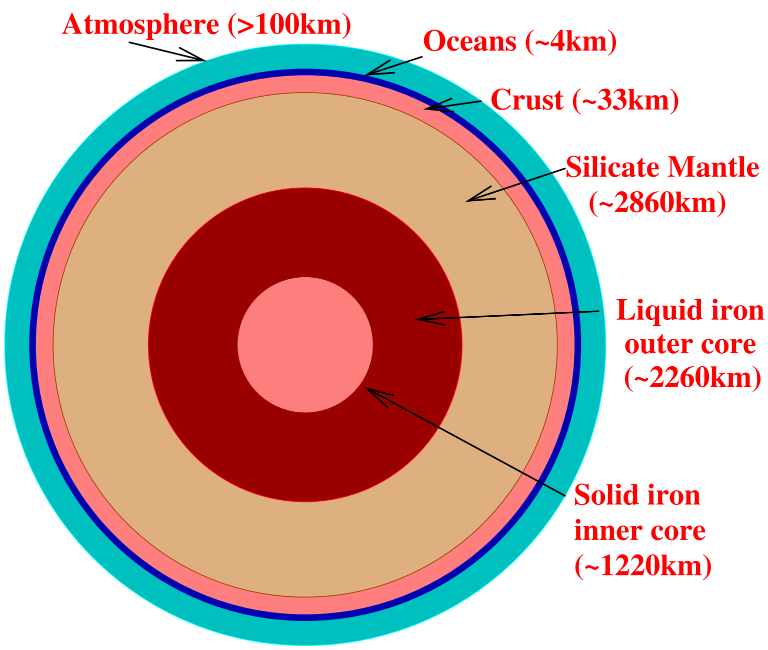
1.4.1 The warming Earth and iron melting

The early Earth contained many short-lived radioisotopes that decayed so quickly and released so much heat that the Earth continued to warm up throughout. The dense iron continued to work its way down into the deeper Earth forming an iron core with only a few million years and displacing the lighter minerals – largely $\boldsymbol{SiO_4^{4-}}$ and

O^{2-} combined with Ca^{++}, Mg^{++}, Na^{+}, K^{+}, Al^{3+} and some Fe^{++} or Fe^{3+} – and lighter elements which then rise to shallower depths. ***Silicates*** are minerals combining metal ***cations*** (eg. Fe^{2+} or Mg^{2+}) with the SiO_4^{4-} ***anion***, as in, for example, ***olivine*** $[Fe, Mg]_2SiO_4$ or with SiO_3^{2-} as in, for example, ***perovskite***, $[Fe, Mg]SiO_3$. ***Oxides*** are minerals combining metals such as Fe^{2+} or Mg^{2+} with O^{2-} as in, for example, ***magnesiowustite***: $[Fe, Mg]O$. At depth in the mantle, olivine separates into magnesiowustite and perovskite. Very deep in the mantle, perovskite probably undergoes a chemistry-preserving ***phase change*** into ***post-perovskite***[12].

Within a few million years the Earth had largely differentiated though the mineralogical characteristics of the contemporary crust would not be established until some later remarkable events occurred.

1.4.2 The contemporary Earth



A model of the contemporary Earth

Overlain by a thin crust of ***granitic***[13] and ***basaltic***[14] rocks, the greatest part of the Earth's volume comprises its *silicate* ***mantle***. The chemical composition of the mantle is like that of an iron-depleted olivine $\approx Mg_{1.8}Fe_{0.2}SiO_4$. By mass, its

[12] Kei Hirose, The Missing Ingredient
[13] On granites: http://en.wikipedia.org/wiki/Granite#Mineralogy
[14] On basalts: http://en.wikipedia.org/wiki/Basalt#Petrology

elemental composition is approximately: 44.8% $\boldsymbol{O}$, 22.8% $\boldsymbol{Mg}$, 21.5% $\boldsymbol{Si}$ and only 5.8% $\boldsymbol{Fe}$! The remaining 0.3% of its mass is thought to be comprised of $\boldsymbol{Ca}$, $\boldsymbol{Al}$, and $\boldsymbol{Na}$, in that order, with perhaps some $\boldsymbol{S}$ and $\boldsymbol{C}$ and 0.03% $\boldsymbol{K}$ which provides the major radioactive heat source within the body of the Earth.

About half way from the surface to the centre, we come to the ***core*** which is largely composed of iron.

The ***outer core*** is a liquid mix of $\boldsymbol{Fe}$ with traces of $\boldsymbol{Ni}$ and $\boldsymbol{Co}$ along with, probably, some $\boldsymbol{S}$ and $\boldsymbol{O}$ and perhaps $\boldsymbol{C}$. When observed over even short times, it flows easily. Resistance to liquid flow is measured by the ***viscosity*** of the fluid; the Earth's outer core has been variously estimated to have a viscosity of $\boldsymbol{\eta \approx 0.01 - 10^5\, Pa \cdot s.}$ Water, for comparison, has a viscosity of $\boldsymbol{\eta \approx 0.001\, Pa \cdot s}$, liquid mercury, $\boldsymbol{\eta \approx 0.0015\, Pa \cdot s}$. The outer core's viscosity is low enough that over periods of seconds to hours, it shows no measurable rigidity; rigidity is a mechanical property that distinguishes solids from fluids.

At depth, the mixture forms into a solid ***inner core*** which is probably almost pure $\boldsymbol{Fe}$. The inner core is a near spherical ball, about **2500** km in diameter. Over periods from seconds to months or years, the Earth's inner core does express significant rigidity. Over much longer periods, it may, however respond to stresses (distributed forces) like a very high viscosity fluid. Buffet has estimated the viscosity of the inner core on very long time scales to be about $\boldsymbol{\eta \approx 5 \times 10^{16}\, Pa \cdot s.}$ The inner core may flow like a fluid on timescales of hundreds of years.

The Earth continues to differentiate itself both geophysically (density) and geochemically (mineralogy). How did the early and how does the continuing differentiation arise? Heat!

The major source of interior heat at present

For most materials, compression brings them to solidfy. That is, for most materials, the higher the pressure, the higher the temperature at which they solidify or freeze. Water is an exception[15]. Water freezes (or melts) at lower temperatures as pressure increases. Water as ice can be melted by applying pressure. Iron, for example, and contrarily, can be frozen by applying pressure.

Within the Earth, pressure increases from the pressure of the atmosphere at the surface, $\boldsymbol{1\, bar = 101.3\, kPa}$ to about $\boldsymbol{360\, GPa = 3.6\, Mbar}$ at its centre[16]. Deep within the Earth, iron is freezing onto the inner core at a temperature of about $\boldsymbol{4500^o C}$; at the surface, iron would remain liquid at this temperature and at any

[15] Phase diagram for water:
http://www.uni-frankfurt.de/%7Escherers/blogging/AdventsKalenderPlots/water/water.jpg

[16] $\boldsymbol{1\, Pa}$ or $\boldsymbol{1\, pascal}$ is equivalent to $\boldsymbol{1\, N \cdot m^{-2}}$. $\boldsymbol{1\, N}$ or $\boldsymbol{1\, newton = 1\, kg \cdot m \cdot s^{-2}}$ is a measure of *force* approximately equivalent to the weight of sandwich. A pressure of $\boldsymbol{1\, Pa}$ is equal to the pressure exerted on ones body by a single thin bedsheet.

temperature above $\mathbf{1600^oC}$. The interior temperature, even in the upper regions of the Earth, is now well above the melting temperature of $\boldsymbol{Fe}$. The pressure-freezing of iron releases its ***latent heat of fusion***. This source of heat accounts for, perhaps, 5% to 10% of the heat flowing from Earth's interior.

The outer core of the Earth is a fluid of iron, nickel and cobalt mixed with lighter elements. As pure iron or an iron-nickel-cobalt mix freezes onto the inner-core, the dense metals are depleted in the outer core, lowering its density. The corresponding density differentiation releases gravitational potential energy of these sinking metals as heat. This probably accounts for as much or even more release of heat than does the direct latent heat of fusion of iron freezing onto the inner core.

The major radioactive elements, $\boldsymbol{U}$, $\boldsymbol{Th}$ and $\boldsymbol{K}$, probably still account for a major source of internal heating but there remains a problem that wherever deep materials come to the surface, the amounts of these radionuclei in the issuing ***magmas*** (*molten rock*) are very much less than that required to account for the heat flow from the interior. J. Marvin Herndon[17] speculates that the required additional heat is now being produced in a small ***U-Th fission reactor*** deep within the frozen inner core. The existence of such a reactor could be, in principle, recognized via the ratio of helium isotopes, $\boldsymbol{^3He/^4He}$, issuing from the planet's interior.

1.4.3 Geophysical-geochemical differentiation and the formation of Earth's core

The warming and melting iron in the upper regions of the eartly Earth sinks into the depths, displacing lighter materials towards the surface... this is *differentiation*!

Iron in differentiation

- The early Earth lost most of its ***volatiles***: $\boldsymbol{H}$, $\boldsymbol{He}$, $\boldsymbol{Ne}$, etc.
- Iron (35% of Earth's mass), silicon, aluminum and magnesium were retained because of their density and/or lack of volatility.
- Oxygen (30% of Earth's mass) was retained largely bound with silicon in silicates ($\boldsymbol{SiO_4^{4-}}$).
- These elements along with calcium, sodium and potassium are the most common elements making up rocks and the Earth.
- $\boldsymbol{Fe}$ was probably quite evenly distributed through nebula from which the planetesmals that formed the body of the proto-Earth. A train of partitioning processes has taken iron to the deepest interior of the Earth and other planets.

[17] A nuclear reactor in Earth's core?: http://nuclearplanet.com/index.html

As the iron reaches the deep interior of the Earth pressures begin to compress it into a solid. Whether or not the iron is solid at any particular depth depends upon the pressure (its depth) and the local temperature. Increasing pressure and/or decreasing temperature solidify iron. When the pressure-temperature conditions at the centre of the Earth are amenable, a solid *inner core* forms surrounded by a still liquid *outer core*, both largely composed of iron. As the Earth cools, the inner core grows larger and larger. The pressure-freezing of the iron onto the inner core releases heat – the *latent heat of fusion*. This heat raises the temperature of the liquid iron outer core and helps to maintain its liquid state. Our magnetic field derives from the differentiated core.

The Geodynamo in the Earth's core

Heat flows from hot towards cold. The heat so-released at the inner core boundary seeks to flow towards the surface. As it flows towards the surface through the liquid outer core, an organized convective motion of the core's fluids is initiated.

- The outer core fluid, being largely iron, is conductive of electricity.
- If a conductor is moved through a ***magnetic field***, an ***electrical current*** is generated in the conductor: ***Faraday's Law***.
- Current flows in closed loops and a loop of current generates a magnetic field: ***Ampere's Law***.
- We have a process in which a magnetic field induces a current which produces a magnetic field – a feedback loop!
- The Earth's spin helps to align the new field with the original field, thus maintaining the global magnetic field of the Earth. This is the only conceivable way the Earth's magnetic field could arise.
- Convective motion of the outer-core fluids forces the geodynamo's feedback process. The convection is powered by the escape of the heat generated in the differentiation and freezing of the inner core and the decay or fission of possible radioactive isotopes within the solid core. A $\boldsymbol{U - Th}$ fission reactor in the core as speculated by Herndon could produce sufficient heat to drive the convection. Alternately, recent research has shown that potassium can be alloyed into the crystalline structure of the inner core. Even with only traces ($\boldsymbol{\sim 300ppm}$) of potassium so-alloyed, the decay of $\boldsymbol{^{40}K}$ to $\boldsymbol{^{40}Ar}$ could provide sufficient power ($\boldsymbol{\sim 1TW}$ is required[18]) to maintain the Earth's magnetic field for eons without the inner-core having completely solidified in the process.

[18] $\boldsymbol{1TW = 10^{15}}$ watts

- This ***geodynamo*** is the source of the Earth's magnetic field.

As we shall learn later, magnetic fields can be frozen into a rock as it cools through the ***curie temperature***. Magnetic fields can also be frozen into mineral crystals as they crystallize from hot geothermal fluids. Some of the oldest rocks and minerals known on Earth show frozen-in magnetic fields.

The geodynamo had surely started by $\boldsymbol{3.5\,Ga}$ because mineral crystals found in Komati, South Africa show remnant magnetic fields. This proves that Earth had, grosso-modo, already geophysically and geochemically differentiated by that time.

The Moon again... the "Big Whack" scenario

Somewhere preceding about $\boldsymbol{4.4\,Ga}$ and probably following quite a lot of geochemical differentiation during which much of the iron must have already assembled at depth, Earth was impacted by a Mars-sized body. The collision splashed up material from the outer shells of the Earth and from these materials and whatever came from the collider, the Moon formed:

- The Moon is less rich in iron than is Earth.

  Argument: lower density and $\boldsymbol{Fe}$ is the only abundant dense element.

  Implications?

  - At the time of lunar formation, the Earth had already partially differentiated with much of its $\boldsymbol{Fe}$ already in the core and so not splashed up from Earth's outer layers.

    Argument: that zircons, $\boldsymbol{ZrSiO_4}$, formed at almost $\boldsymbol{4.4\,Ga}$ tell us that the Earth's surface had begun to cool from the general melting of the outer regions (perhaps to $\boldsymbol{1000\,km}$ depth). Zircons melt at $\boldsymbol{1859^oC}$; they can also "dissolve" into acidic aqueous solutions rich in flourine or chlorine as well as into several magmatic melts. We know, then, that these zircons have not faced such destructive conditions in the past 4.4 Ga. Zircons can entrap traces of $\boldsymbol{Ti}$ (titanium) and the quantity and isotopic composition of the $\boldsymbol{Ti}$ entrapped can tell us something about the temperature conditions that existed at the time of their crystallization. These oldest zircons formed under relatively cool, aqueous conditions: the Earth's surface was already cool and wet by $\boldsymbol{4.4 \times 10^9}$ years ago!

  - The magma ocean would have geochemically differentiated with the least dense materials floating to the surface forming ***continental cratons***, the earliest masses of rock.

    Argument: we have continental cratonic materials by $\boldsymbol{4.03Ga}$... the Acasta Gneiss.

1.4.4 Mantle, crust, the continents and oceans

As heat from the Earth flowed towards the Earth's cool surface, a process of convection began in the silicate surrounding the iron core. This silicate mantle now extends from a depth of about $\boldsymbol{3000\,km}$ almost to the surface. The very lightest materials which first froze out of the magma ocean, also silicates, formed the Earth's overlying crust.

- The *granitic* or granite-like crust formed the continents and
- the denser *basaltic* or basalt-like crust formed the ocean basins.

The denser basalt floated deeper on the still-*plastic* mantle and the cooling magma ocean and gave room for the waters of the oceans to in-fill.

1.4.5 The waters of the oceans and the atmosphere

Oceans

Where the ocean water came from is a matter of some continuing debate.

- possibly (and at least partially) through outgassing of hydrogen and oxygen from the Earth's interior during differentiation.
- probably, also, cometary bombardment brought vast amounts of water to the Earth's surface in its early history.

But then why is there no ocean on the Moon, Mercury, Venus or Mars as they should have been similarly bombarded?

- gravity was not great enough on the Moon or Mars to hold the water onto its surface against its vapourization.
- Mercury and Venus were just too hot for the water to remain on the surface and it largely escaped into space.

It seems that Mars did have surface waters and perhaps even large oceans which slowly evaporated into space because its gravitational force was insufficient to contain it. Presently, Mars holds vast quantities of water in its polar glaciers which are almost $\boldsymbol{4\,km}$ deep in places and probably as permafrost throughout the ***regolith*** regolith. It is estimated that there is enough water on Mars to cover its entire surface to a depth of about $\boldsymbol{20\,m}$.

Water ice has been detected in permanently shaded craters on Mercury and water droplets are known to exist in the very high ($+50km$) atmosphere of Venus. There may also be water in the regolith, the mineral soils, of the Moon.

Water is is an abundant molecule in the inner Solar System and ice an abundant mineral.

The present oceans of Earth are "***saline***[19]" with a content of dissolved salts of about $3.5\%m$ (by mass). The salts have been leached from the continents and released from the interior of the Earth by volcanism during the past $4Ga+$.

Atmosphere

The lighter volatile molecules and elements that do not condense into liquid at the temperature and pressure on the Earth's surface formed a thin atmosphere enveloping the planet. Presently, Earth's atmosphere is composed of:

- 78%v (by volume) N_2, from the primordial condensation and subsequent outgassing of the planet
- 21%v O_2, reduced from combination with C and in SiO_4^{4-} by lifeforms
- a little less than 1%v Ar, primordial and outgassed and derived from β-capture decay of ^{40}K
- a varying amount of water (H_2O) vapour evaporating from surface liquid water and transpiring from plant life (at saturation, 20^oC, $\sim 0.4\%v$ by volume)
- an ever-increasing component of CO_2 (presently 0.039%v = 392 ppmv[20])
- traces of NO_x and CH_4, both of which are strong infrared absorbers (greenhouse gases) like water vapour and CO_2.

The effect of the major $300K$ (temperature of Earth's surface) infrared radiation absorbers in our atmosphere (namely, H_2O, CO_2, CH_4, N_2O in order of importance) is to maintain the surface temperature about $35K$ warmer than it would be if none of these gases were present. The Earth would be hard frozen everywhere without the greenhouse effect trapping heat near the surface. There is evidence that the Earth was completely frozen, with tropical oceans frozen to depths of hundreds of metres, about 2.2 billion years ago, $2.2Ga$ (the ***Archean-Proterozoic*** boundary)[21] and only recovered because CO_2 levels in the atmosphere, fed by continuing volcanism eventually reached levels of $20\%v$ resulting in a super-greenhouse warming that melted all this ice in as little as a few hundred years. Such "***Snowball Earth***" glaciations seem to

[19] Salinity of oceans: http://www.marinebio.net/marinescience/02ocean/swcomposition.htm

[20] 1 ppmv = 1 part in 1 million by volume; current levels

[21] Snowball Earth: http://www.snowballearth.org/

have occurred again **710*Ma*** and **640*Ma*** just preceding the explosion of life that characterizes the ***Phanerozoic***. Recovering from each of these extreme glaciations, the diversity of plant and animal species on Earth increased spectacularly.

The internal dynamics of our Earth

The crust of continents and the ocean basins is dynamic. The continents move slowly across the surface of our planet and the floors of the oceans recirculate into the mantle under the process of mantle convection which is necessary to the continuing cooling of the Earth's interior. Shortly, we shall study the physics and later again the geochemical consequences of this convective process.

Next, however, we shall learn something about the paths, orbits, of the terrestrial planets about the Sun, about the orbits of the moons about their mother planets and what we can learn about planets by studying these orbits.

1.5 Geophysical processes in planetary differentiation

The differentiation of the terrestrial planets and asteroids and also of the gas giants depends upon both geophysical and geochemical processes. Properly, ***geophysics*** refers to the *physics of the Earth, i.e. "geo-"* and ***geochemistry*** to the *chemistry of the Earth*. Applied to the planets and Solar System and the Universe as a whole, the science of chemistry is sometimes better called ***cosmochemistry***. Presently we don't use a similar term to describe a physics generalized to the description of processes on the planets. Some authors do use terms like, for example, ***selenophysics*** to describe the *physics of the Moon*.

- The geophysical processes involved in differentiation depend largely upon differential densities of materials which might be either inherent or dependent upon their temperature.
  - Bouyant materials rise and dense materials sink... The temperature caused bouyancy of ***mantle*** materials and ***fluid core*** is largely due to the slow freezing of the iron inner core and the release of the ***latent heat of fusion***.

1.6 The internal structure of Earth, Moon and the terrestrial planets

We know from the rotational dynamics of Earth, Moon and Mars that their internal density increases rapidly towards their centres. We infer the same for Venus and

Mercury though we really haven't obtained accurate measures of their moments of inertia.

- The typical structure of a typical terrestrial planet (Earth as model) or our Moon comprises:
  - A thin outer crust of lighter silicates (largely granitic) where high standing and somewhat denser silicates (largely basaltic) where low standing.
  - A very deep mantle of silicates, ever denser with depth.
  - A core, largely composed of $\boldsymbol{Fe}$, $\boldsymbol{Ni}$ and some alloying lighter elements such as $\boldsymbol{O}$ and $\boldsymbol{S}$. The core may be frozen solid or, like Earth and Mercury, have an overlying melted shell.
  - If the planet's gravity is strong enough to hold volatiles against their evaporation into space, it may well have a substantial atmosphere. Earth has large oceans of liquid water.

  The crustal skin and mantle of the planet has little $\boldsymbol{Fe}$ and $\boldsymbol{Ni}$. The crust is especially enriched in $\boldsymbol{Ca}$, $\boldsymbol{Na}$, $\boldsymbol{K}$, $\boldsymbol{Al}$, $\boldsymbol{Si}$ and $\boldsymbol{O}$. The $\boldsymbol{Fe}$ and $\boldsymbol{Ni}$ has mostly sunk into the core. In the table below, one might note that overall, the Earth comprises about 15% by mass $\boldsymbol{Si}$ but the crustal abundance[22] is about 28% $\boldsymbol{Si}$; take care with the meaning of the normalization to $\boldsymbol{Si} = \mathbf{1}$.

Estimated relative abundances by mass

Element	Solar photosphere	Av. meteorite	Earth (whole)	Earth (crust)
H	**803**	***little***	***little***	**0.0050**
He	**201**	***little***	***little***	**< 0.0006**
O	**8.8**	**1.95**	**2.1****	**1.69**
C	**4.0**	−−	−−	**0.0034**
Si	**1**	**1**	**1**	**1**
Mg	**0.77**	**0.82**	**0.93**	**0.075**
Fe	**1.41**	**1.69**	**2.2**	**0.18**
S	**0.41**	**0.12**	**0.03***	**0.0019**
Al	**0.071**	**0.065**	**0.071**	**0.29**
Ni	**0.089**	**0.099**	**0.17**	**0.0007**
Ca	**0.072**	**0.082**	**0.096**	**0.13**
Na	**0.035**	**0.040**	**0.025**	**0.10**
K	**0.0045**	**0.0060**	**0.00012***	**0.093**

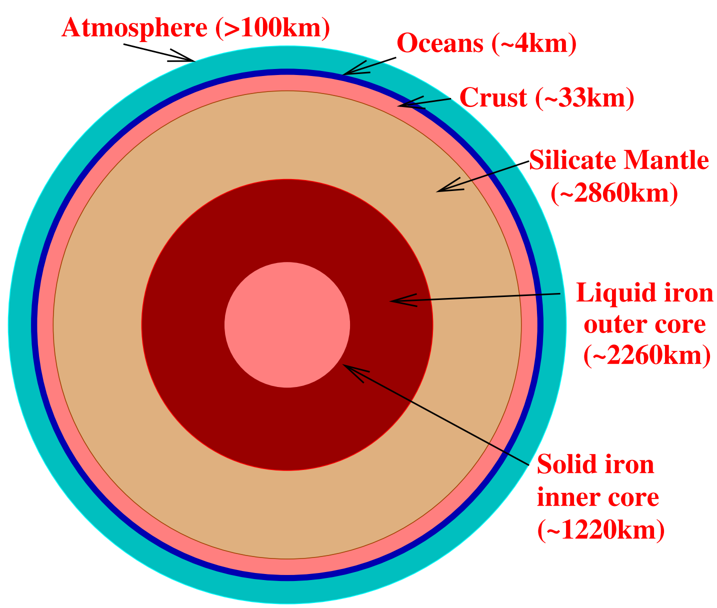
[22] Crustal abundance

*Notice, especially that S, sulfur, and K, potassium, seem to have significantly lower relative abundance on Earth than do other heavy elements in the list. It may well be that we do have more normal sulfur if most sulfur is alloyed with iron in the fluid outer core as is argued by many geophysicists and geochemists. We know that there must be some lighter alloying element in the core; sulfur, oxygen and carbon are the possible candidates.

**Oxygen, like hydrogen and helium, all being quite volatile, were not easily contained to Earth its gravitational pull during the early evolution.

Structurally, the Earth is now well ***differentiated***.

- The thin ***crust*** extends from the surface to an average depth of $\mathbf{33}\boldsymbol{km}$.
- Below the crust, the ***mantle*** which comprises $\approx \mathbf{85\%}$ of the volume of Earth extends to a depth of $\mathbf{2900}\boldsymbol{km}$ below the surface. It is composed mostly of silicates along with metal oxides, $\boldsymbol{MgO}$, $\boldsymbol{FeO}$, $\boldsymbol{Al_2O_3}$, $\boldsymbol{CaO}$, $\boldsymbol{Na_2O}$, in mineral compositions.
- Starting from the base of mantle at $\mathbf{2900}\boldsymbol{km}$ depth and extending to the centre of the Earth, this last $\mathbf{1/8}$ volume of Earth is composed of $\boldsymbol{Fe}$ $(\mathbf{90\%})$, $\boldsymbol{Ni}$ $(\mathbf{5\%})$ and a possible mix of $\boldsymbol{C}$, $\boldsymbol{Si}$, $\boldsymbol{O}$, $\boldsymbol{S}$ and $\boldsymbol{H}$ comprising $\mathbf{5\%}$ by mass. $\mathbf{35\%}$ of the total mass of Earth is in the core. The central, solid ***inner core*** is probably almost pure $\boldsymbol{Fe}$, perhaps even in a single crystal form; its radius is about $\mathbf{1290}\boldsymbol{km}$.



Earth among all planets has the highest ***density***, mass for its volume. Its high density is partially accounted for by the compression of overlying materials squeezing the deep iron core to high density. The iron core is somewhat less dense than one expects for the temperatures and pressures at depth so it must include somecomponent of lighter elements. The vigorous circulation of the fluid outer core layer produces Earth's strong magnetic field.

The crust of the Earth is differentiated laterally over the surface into continents and ocean basins.

- The continents are high-standing, relatively low-density ***granitic*** materials. Granitic means *granite-like* rock which is a rock type very rich in $\boldsymbol{SiO_2}$ or *quartz* and without *olivine* mineral, $\boldsymbol{(Mg, Fe)_2SiO_4}$. Quartz is a relatively low density mineral – it is the most common type of sand and is the chemical composition of ordinary glass.

- The ocean basins are low-standing (and water filled) higher density ***basaltic*** materials. Basaltic or *basalt-like* rocks have essentially no free quartz mineral components, their $\boldsymbol{Si}$ and $\boldsymbol{O}$, being largely contained in olivine.

- Whereas average, uncompressed granitic rocks have a density of about $\boldsymbol{2.5gm/cm^3 = 2500kg \cdot m^{-3}}$, basaltic rocks have a density of about $\boldsymbol{3.3gm/cm^3}$.
  - Basaltic rocks seem to "float deeply", to the depths of the ocean basins when assembled into large masses on Earth. The average depth of the ocean floor, underlain by basaltic rock, is about $\boldsymbol{-4.5km}$.
  - Granitic rocks float high above the oceans, forming the continents. The average elevation of the granitic continents is about $\boldsymbol{0.9km}$.

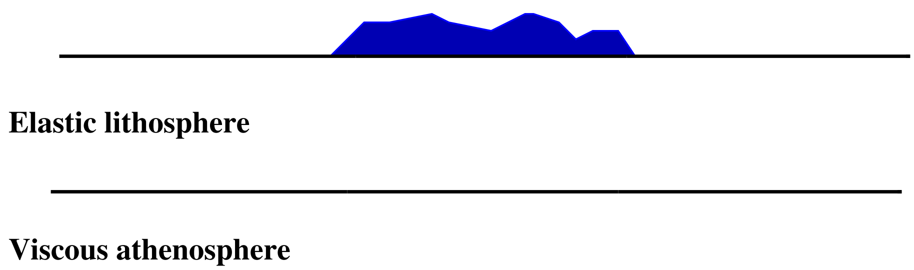
  How do we account for these large-scale differences in elevation?

1.6.1 Post-glacial rebound, isostatic adjustment, mantle viscosity

For centuries, tidal gauges along the coasts of Sweden and Finland in the Gulf of Bothnia and also along the states bordering the Baltic Sea, Latvia, Lithuania and Estonia, have been observed to show that these coasts are slowly rising relative to sea level. At the northern coast of the Gulf of Bothnia, tide gauges are showing land uplift rates of more than $\boldsymbol{1cm/year}$. In the last **5 000** years, this coast of Sweden and Finland has risen by more than $\boldsymbol{100m}$. What is causing this continuing rise of land? ***Post glacial rebound!***

- Until about **7 000** years ago, this region was overlain by Fennoscandian icesheet to a depth of more than $\boldsymbol{3km}$ above the Gulf of Bothnia.[23]

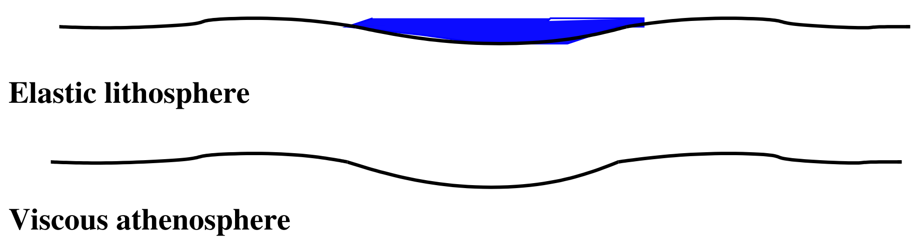
Glacial loading



- It had lain there for at least **30 000** years, depressing the Earth's lithosphere.

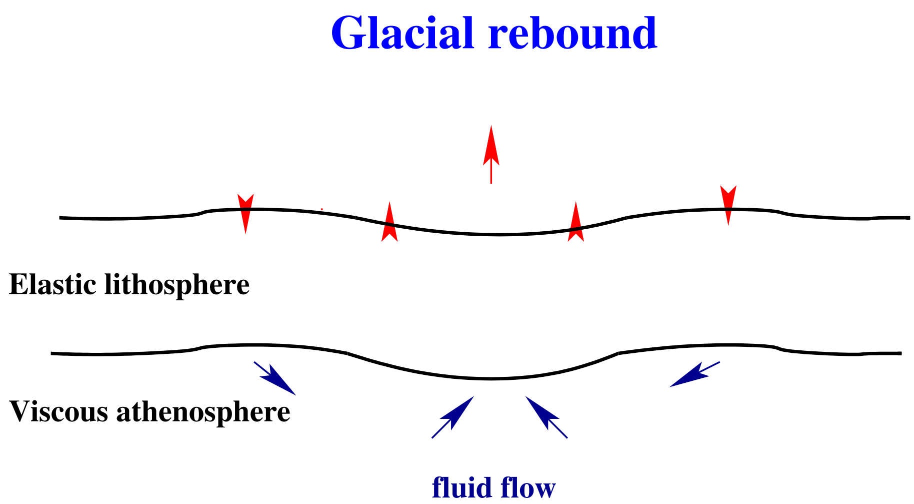
- Then about **10 000** years ago it began to melt quickly. The heavy load of the ice over time had depressed the underlying continent. The region had come into isostatic adjustment with the load.

Glacial melting



[23] Fennoscandian glaciation
...and rebound
Laurentian icesheet extent
...Laurentian uplift

- Relieved of the heavy ice load the whole continental region began to *rebound* – to bounce back up.



Glacial rebound is also observed at about the same rate, $1-2cm/yr$, over James and Hudson's Bay in Canada. The Laurentian icesheet was larger and deeper than that of Fennoscandia.18 000 years ago, it covered almost all of Canada. It had depressed the Earth's surface even deeper below average sea-level. It had largely melted by about $6\,500$ years ago and the land was rebounding. This is a very well studied phenomenon.

1.6.2 Viscosity of the fluid mantle

Seismic waves transit the mantle as though it were a very rigid elastic solid. Still on long time scales, the mantle looks like a viscous fluid. We might model the rheology of the mantle so as to accommodate its short-time-scale elasticity and its long-time-scale viscosity:

$$e_{ij} = \frac{1}{2\mu}\sigma_{ij} + \frac{1}{2\eta}\frac{\partial \sigma_{ij}}{\partial t}; \qquad i \neq j.$$

This equation describes a model rheology, a ***Maxwell solid***, that is often used in describing the mantle materials. η, ***viscosity***, measures the flow resistance to shears, the σ_{ij}. That we have restricted our view to shear stresses and shear strains is indicated in the equation by the $i \neq j$ clause. You might note that when $\mu \sim \eta/t$, the two terms become of equivalent scale. The value of μ that characterizes mantle rheology on seismic time scales ranges from about 100 to $300\,GPa$. What of the viscosity? If the "fluid" upon which these depressed continental areas were floating could have moved very quickly, it would have infilled under the lowered load quickly. The fluid, though, flows extremely slowly; it has a very high viscosity, that measure of its self-stickiness.

- Water has a viscosity of $\boldsymbol{\eta \approx 10^{-3} Pa \cdot s}$, maple syrup, $\boldsymbol{\eta \approx 2.5 Pa \cdot s}$ and red hot common glass, $\boldsymbol{\eta \approx 10^{12} Pa \cdot s}$.
- From the rate of rebound, we can calculate the viscosity of the slow moving, underlying mantle fluid!

  $\boldsymbol{\eta \approx 10^{20} - 10^{23} Pa \cdot s}$.
- It flows extremely slowly, so slowly in fact that we can't even recognize its fluid-like property over short times.
- Glass at normal temperatures is an amorphous solid – not crystalline. It behaves like a fluid.

  At room temperature $\boldsymbol{\eta \approx 10^{19} Pa \cdot s}$.
- The "fluid" mantle which underlies central Canada and Fennoscandia is even stickier than glass but, still, on time scales of hundreds or thousands of years, it looks like a fluid.
- On short time scales – and note that glass actually behaves like a very brittle solid when struck by a stone – the mantle is a very hard solid too! In fact, almost all regions of the mantle are "harder" and "stronger" than any known materials on the surface.
- For a time scale $\boldsymbol{t \sim \eta/\mu}$, the mantle materials move from looking like an elastic solid to looking like a flowing fluid. This ***"Maxwell time constant"*** ranges from a period of a few hours or days in the most plastic upper reaches of the mantle to about $\boldsymbol{100\, yr}$ in the deep mantle and, should we look to this rheological for the lithosphere, to $\boldsymbol{10s}$ to $\boldsymbol{100s}$ of thousands of years in the upper crustal regions. The crust and much of the lithosphere is *brittle* and fractures under shear stress; the mantle is viscous and flows. For magmas issuing from the Hawaiian volcanoes, the time constant is of the order of a second.

The fact of the fluid nature of the mantle allows for the ***convection*** process and the continuing geophysical differentiation of the planet.

1.7 Mantle convection

Geologists have known for more than 100 years that the continents move across the surface of the Earth.

Sir Francis Bacon in about 1620 recognized that the coastline of North America could be nicely fit against the coastlines of Europe and northern Africa. In 1799, **von Humboldt** noted that the same symmetry existed between the coastline of South America and that of central and southern Africa and that there were extensions of mountain ranges of South America on the African continent. It seemed as though the continents, separated by the Atlantic Ocean, had once been joined. This hypothesis was clearly stated by **A. Snider** in 1858 in France. In the early 1900s, the hypothesis took on some geological popularity as **F. Taylor**, **H.D. Baker** and then **Alfred Wegener** described theories of "***Continental Drift***[24]". Geophysicists were slow to the accept the possibility because, as **Lord Rayleigh** argued, "Solid rock can't move through solid rock!". Some geophysicists held vainly to an alternate theory, the "Expanding Earth Hypothesis" which argued that Earth's volume had increased over geological time and that the earlier surface was no longer large enough in area to cover the surface of the greater volume. The Atlantic Ocean was seen to be an ever-widening crack on the surface of a growing Earth. In 1963, **J. Tuzo Wilson**, a physicist at the University of Toronto, wrote a famous article entitled "***Continental Drift***" which was published in the April issue of ***Scientific American*** magazine. He argued a mechanism which quickly found acceptance among geophysicists who finally joined their geological colleagues in this view of the tectonic process on Earth.

1.7.1 Earth's mantle is fluid!

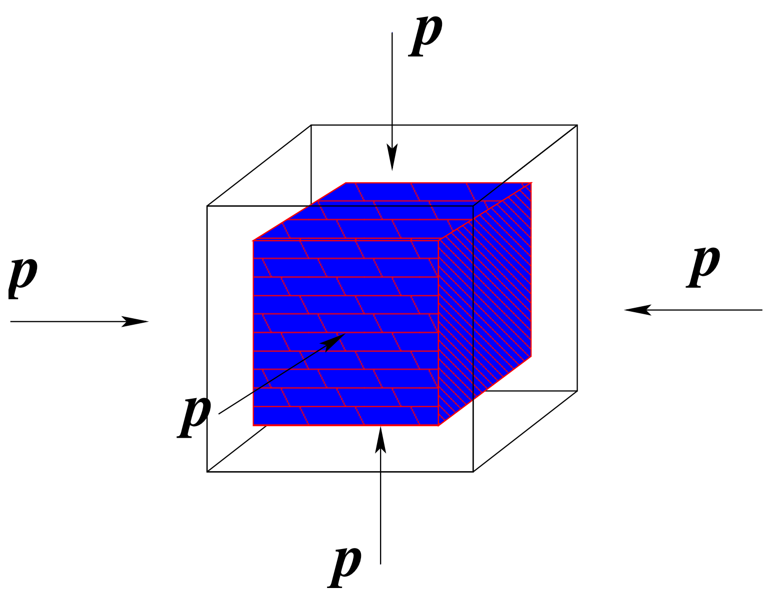
If the Earth's mantle is fluid, it can be brought into circulation if the Earth's interior temperature is sufficiently hot.

- Post-glacial rebound suggests that the mantle is fluid on long timescales.
- If the mantle is a fluid and if there is a sufficiently steep temperature gradient from a hot base to a cool surface, it can efficiently transport heat from depth by a process called ***convection***. Convection drives the drift.

1.7.2 The adiabatic gradient

Suppose we have a small volume of material, say in the form of a cube. Suppose now that we apply a pressure to this cube. Recall that a pressure acts evenly in all directions and so each face of the cube feels the same squeezing force. The dimensions of the cube shorten; it responds to the pressure ***stress*** with a deforming ***strain***. If no energy is allowed to escape from the volume, the energy in the volume is also compressed. It becomes hotter!

[24] On the history of the concept of Continental Drift
http://www.bbm.me.uk/portsdown/PH_061_History_a.htm



How much hotter?

If we compress our cube very quickly, it doesn't have time to come to thermal equilibrium; if we insulate it from its environment so that heat energy can neither flow in nor out of it, it cannot come to equilibrium. Such a compression is called ***adiabatic compression***; what is preserved in adiabatic compression is the ***entropy*** of the cube of material.

- From the physics of ***thermodynamics*** we know that

$$\boldsymbol{\Delta T \propto \Delta P,}$$

$$\boldsymbol{\propto\ T,}$$

$$\boldsymbol{\propto 1/\rho.}$$

  where $\boldsymbol{\rho}$ is the density of the material of our cube.

What is the constant of proportionality?

If we were to heat the gas under constant pressure, it would want to expand as

$$\boldsymbol{\frac{\Delta V'}{V} = \alpha_P \cdot \Delta T',}$$

but it has to expand against the confining pressure and so, effectively, it is being compressed by the confining pressure. It heats up just a little more than if there were no confining pressure by an amount:

$$\boldsymbol{\Delta T \propto \alpha_P}$$

where $\boldsymbol{\alpha_P}$ is the ***volumetric expansion ratio*** per $\boldsymbol{K}$.

Depending on its capacity to hold heat, its ***heat capacity at constant pressure***, $\boldsymbol{C_{H_P}}$, which determines what temperature increase would be caused by an inflow of heat, the material will more or less expand. For a material of very high $\boldsymbol{C_{H_P}}$, it warms less and expands less. So when we compress the gas under some increase of pressure, $\boldsymbol{\Delta P}$, its temperature rise, $\boldsymbol{\Delta T}$, is greater according to its $\boldsymbol{\alpha_P}$ and lesser according to $\boldsymbol{1/C_{H_P}}$.

$$\boldsymbol{\Delta T \propto \frac{\alpha_P}{C_{H_P}}.}$$

The constant in the equation relating an adiabatic temperature increase due to an increased pressure obtains as

$$\boldsymbol{\Delta T = \frac{\alpha_P}{C_{H_P}} \frac{T \Delta P}{\rho}.}$$

The material of the Earth's mantle behaves accordingly.

Pressure within the Earth's mantle increases with depth as $\boldsymbol{\Delta P = |\vec{g}| \rho \Delta z}$, where $\boldsymbol{\Delta z}$ is an increment of depth and $\boldsymbol{\vec{g}}$ is the downwards oriented gravitational force on the material. As it turns out, for our Earth, $\boldsymbol{|\vec{g}| = g \approx 10 m \cdot s^{-2}}$ throughout the mantle. It actually slowly increases from about $\boldsymbol{9.8 m \cdot s^{-2}}$ at the surface to about $\boldsymbol{10.2 m \cdot s^{-2}}$ at the base of the mantle. Within the mantle, then, the adiabatic temperature due to the varying pressure is

$$\boldsymbol{\Delta T = g \frac{\alpha_P}{C_{H_P}} T \Delta z,}$$

and the ***adiabatic temperature gradient*** is

$$\boldsymbol{\frac{\Delta T}{\Delta z} = g \frac{\alpha_P}{C_{H_P}} T.}$$

If the temperature profile in the mantle is adiabatic and we know the temperature at the top of the mantle, say $\boldsymbol{T_{top}}$, we can calculate the temperature throughout. The previous equation determines a *differential equation* which can be solved given a known temperature somewhere in the profile. Letting the $\boldsymbol{\Delta T \rightarrow dT}$ and $\boldsymbol{\Delta z \rightarrow dz}$ become infinitesmal,

$$\boldsymbol{\frac{dT}{T} = g \frac{\alpha_P}{C_{H_P}} dz,}$$

and if we *integrate* both sides of this equation, here shown step-by-step, we obtain

$$\boldsymbol{\int_{T_{top}}^{T(z)} \frac{dT}{T} = \int_{z_{top}}^{z} g \frac{\alpha_P}{C_{H_P}} dz,}$$

$$\boldsymbol{ln\,(T(z)/T_{top}) = g\frac{\alpha_P}{C_{H_P}}(z - z_{top}),}$$
$$\boldsymbol{T(z)/T_{top} = exp\,(g\frac{\alpha_P}{C_{H_P}}(z - z_{top})),}$$
$$\boldsymbol{T(z) = T_{top}\,exp\,(g\frac{\alpha_P}{C_{H_P}}(z - z_{top})).}$$

If we know the temperature at the top of the mantle and we know the thermodynamic constants appropriate to mantle material, we can determine the exponential adiabatic temperature profile to the base of the mantle. A reasonable temperature for the top of the mantle is suggested by the temperature of magmas which issue from great shield volcanos like that of Kilauea[25] in Hawaìi, about $\boldsymbol{1200^oC}$. For typical rock materials, $\boldsymbol{\alpha_P \approx 1.4 \times 10^{-5}\,K^{-1}}$ and $\boldsymbol{C_{H_P} \approx 1.3 \times 10^3 J \cdot kg^{-1} \cdot\ K^{-1}}$.

> **Calculate the adiabatic temperature at the core-mantle boundary at a depth of $\boldsymbol{2900\,km}$, starting from a temperature of $\boldsymbol{1500\,K}$ at the top of the mantle, say at $\boldsymbol{z_{top} = 50\,km}$.[26] Note: $\boldsymbol{1J = 1kg \cdot m^2 \cdot s^{-2}}$; $\boldsymbol{g \approx 10m \cdot s^{-2}}$.**

We shall argue that because the mantle is fluid-like, the mantle is in constant convection and this brings the mantle temperature profile toward the adiabatic temperature profile. While fluids can be driven into convection if the temperature gradients through them are sufficiently high, solids cannot. Heat can be carried through fluids by convection but can only be carried through solids by conduction. The outer ***elastic*** shell of the Earth, its ***lithosphere*** which is typically about $\boldsymbol{50\,km}$ thick, acts as a heat-conductive layer with a very poor heat conductivity. It acts like an insulation between the cold exterior of the Earth and its hot interior. The temperature gradient through this lithospheric shell is very steep, rising from about $\boldsymbol{300\,K}$ on the surface to about $\boldsymbol{1500\,K}$ at its typically $\boldsymbol{50\,km}$-base; the gradient near the surface is about $\boldsymbol{25\,K/km}$. If one were to drill just $\boldsymbol{4\,km}$ into the Earth – and one does drill this deep for natural gas in the Alberta basin – one finds that the temperature is already well above the boiling point of water.

1.7.3 The adiabatic gradient and convection

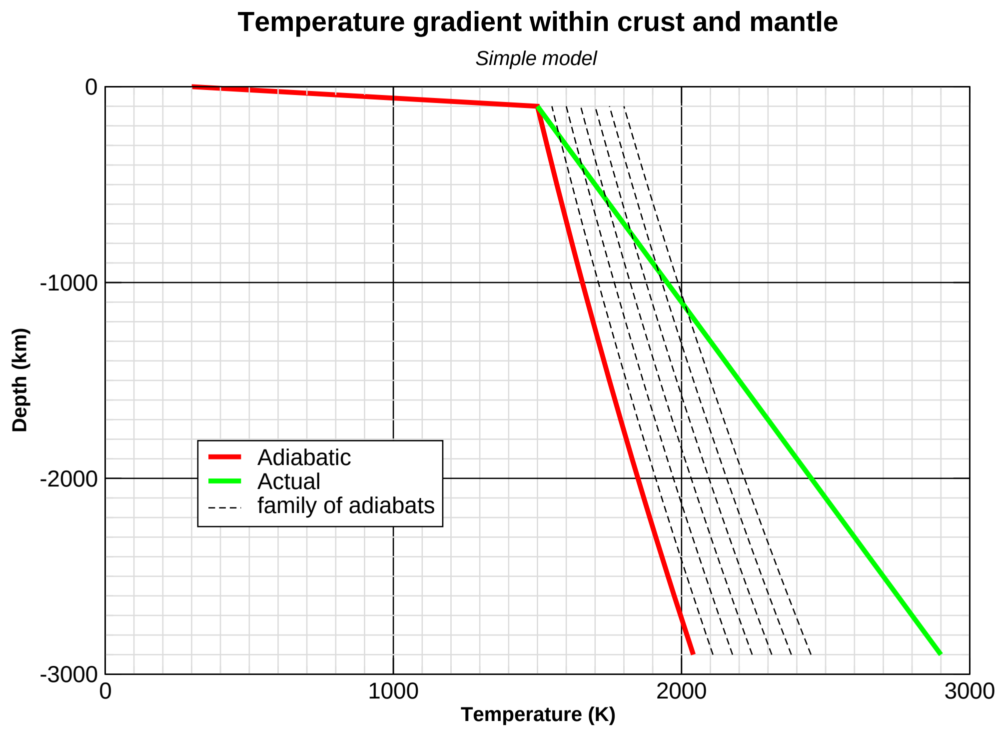
Let us suppose the entire mantle follows the adiabatic temperature profile. What that really tells us is that the pressure and temperature within the mantle are related

[25] Eruptions on Kilauea

[26] Answer: $\boldsymbol{2039\,K}$. Because Earth's mantle is in such *vigorous convection*, we know that the temperature at the base of the mantle must be quite a lot higher than that accounted for by only the adiabatic gradient. Common estimates of the temperature at the core-mantle boundary are about $\boldsymbol{3500\,K}$ with a very steep rise in the 200km just above the boundary. The temperature at the centre of Earth's core is at least $\boldsymbol{5100\,K}$.

in a particular way.

If a volume of mantle material is moved along the adiabatic temperature profile, say we move a volume of material upwards to lower pressure, its temperature decreases as it expands to remain in exact temperature equilibrium with the lower local pressure. The mantle temperature follows the adiabatic gradient (in the figure, below, the red line)[27].

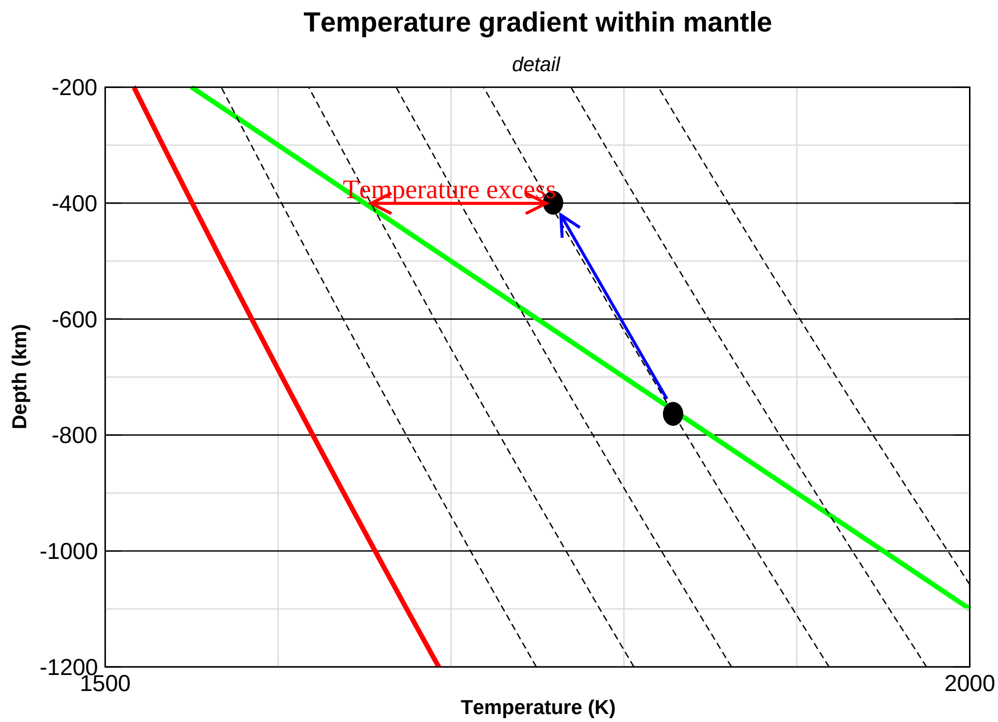


Suppose now, that the actual temperature profile is a little steeper than the adiabatic temperature profile (in the figure, the green line). By "steeper", we mean that temperature increases more rapidly towards depth than would be predicted by the adiabatic gradient.

When a volume of material (black spot at temperature according to the green line) is moved upwards, it cools following the adiabatic gradient – it follows an ***adiabat***. But, in so cooling, it finds itself warmer than other material (its temperature on the green line) at its new lesser depth and lower pressure. Being warmer, it is somewhat expanded in comparison with local material and its density, therefore, is lower; it

[27]In the crust and that part of the upper mantle that comprises the *lithosphere*, the temperature gradient is much steeper than the adiabatic. Through this *insulating* zone, *conduction* transports heat.

is relatively ***bouyant***. It would like to continue to rise even further under its own bouyancy. That is, if we even infinitesmally move a volume of material upwards through a temperature gradient which is steeper than the adiabatic, it wants to move even further upwards under bouyancy. The reverse is true for a volume of material displaced along a temperature gradient steeper than the adiabatic towards greater depth. It wants to continue sinking.



So, if there is any perturbation of material either upwards or downwards, it wants to accelerate away in the direction of the perturbation.

- This is the process of convection.
- The whole material of the mantle is set into continuing motion as heat is carried from depth towards the surface by the convecting fluid.
- The Earth's mantle is and has been convecting for the past **4.5** billion years. Core formation would seem to have already occurred by this time.
- The motion would eventually stop if the actual temperature profile cooled to settle down to the adiabatic. That is, when there is no more excess heat in the deep interior maintaining the steep temperature gradient, convection will cease.

- Much of the heat emanating from the core to heat the base of the mantle is being released by the freezing of iron onto the inner core. The rest is the residual heat from earlier decay of radionuclei and from the gravitational potential energy released as heat during the density stratification (differentiation) of the Earth.

- When the whole core is frozen, convection of the mantle will stop.

  We are probably billions of years from this eventuality on Earth.

It is probably the case that the mantles of Mercury, Mars and the Moon are no longer convecting though as we shall learn later, we have evidence that Venus' mantle was momentarily, at least, in very rapid convection as recently as **500** million years ago.

1.7.4 The Rayleigh number

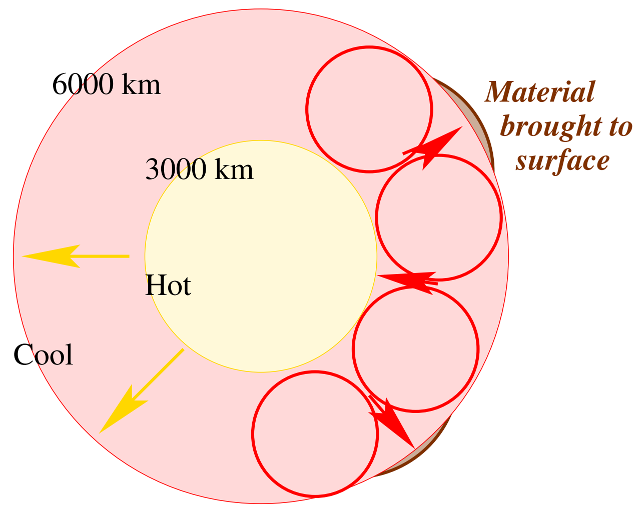
The vigour of the convection is determined by a ratio of forces, the dimensionless ***Rayleigh Number***:

$$\mathbf{R} = \frac{\boldsymbol{bouyant forces}}{\boldsymbol{viscous forces}}.$$

The bouyant forces push the convection while the viscous forces retard it. Through fluid mechanical analysis, we can estimate just what this number must be for the Earth's mantle to remain convective.

In 1964, **Leon Knopoff** determined that for a spherical shell bounded below by a rigid surface contact with a spherical hot core and above by a rigid surface, with the inner radius being just $\mathbf{1/2}$ the outer, convection is maintained when $\mathbf{R > 2380}$. This model approximates the geometry of Earth's mantle. It seems that the relatively vigourous convection of the Earth's mantle would require $\mathbf{R > 10^5}$. The evidence for this is the very rapid motion of ***tectonic plates*** across the surface of the Earth that are being rafted about by the convection process. Because the motion is rapid, **R** must be very high and the bouyant forces must substantially dominate the viscous forces resisting convection. The bouyancy forces are high when the temperature gradient is high, that is, if the deep interior of the Earth is at temperatures much higher than could be accounted for by adiabatic compression. Whereas the temperature of the core-mantle boundary need only be about $\mathbf{2100}\,\boldsymbol{K}$ under a purely adaiabatic gradient, we believe the actual temperature to be much higher than this, $\sim \mathbf{2500-3200}\,\boldsymbol{K}$. The Earth will surely convect for a very long time into the future.

Knopoff's geometry



Convection requires R >2380

Mantle convection within Earth.

1.7.5 Tectonic and boundaries

Seismology, the geophysical science that is concerned with earthquakes and the radiating wave fields that earthquakes produce, and ***tectonics***, the geological science that is concerned with the large scale and continuing slow motions of the mantle and lithospheric plates, divide the Earth according to different layering structure.

- **The tectonic boundaries**
  - ***Lithosphere:*** This is the outer ***elastic*** shell of the Earth; it appears to be quite solid, even when watched over very long times. The lithospheric plates, typically between $\mathbf{0}km$ and $\approx \mathbf{100}km$ are rafted around on the surface by the convective engine.

    This layer is elastic even when viewed on time scales of millions of years; the underlying ***asthenosphere*** is ***plastic*** on time scales as short as months.
  - ***Asthenosphere:*** Viewed during long periods of time, this layer is the softest or most easily deformable and flowing region of the upper mantle. When viewed for very short periods of time, though, as with seismic waves, it appears to be extremely hard.
  - ***Mesosphere:*** Below about $\mathbf{700}km$, the mantle becomes much more viscous and less mobile – by a factor of about $\mathbf{100}$.

- ***Core-mantle boundary zone:*** A mixed zone of varying thickness, $10-200km$ thick, lies at the base of the mantle. It may well be lithospheric materials that have been ***subducted*** to depth by convection. This is the much studied seismic D'' layer.

- **The seismic boundaries**
  - ***Crust:*** This outer skin, typically about 30km thick, shows a relatively low velocity for seismic sound waves, the (***P-waves***). Its base is characterized by a sharp increase in the P-wave velocity to about $8km/s$.
  - ***Mantle:*** Most of the volume ($\approx 87\%$) of the Earth comprises the silicate, rocky mantle. The mantle has a layered structure that affects seismic wave velocities. The structure is caused by ***phase changes*** of the mantle minerals as they are compressed at depth into higher density forms.

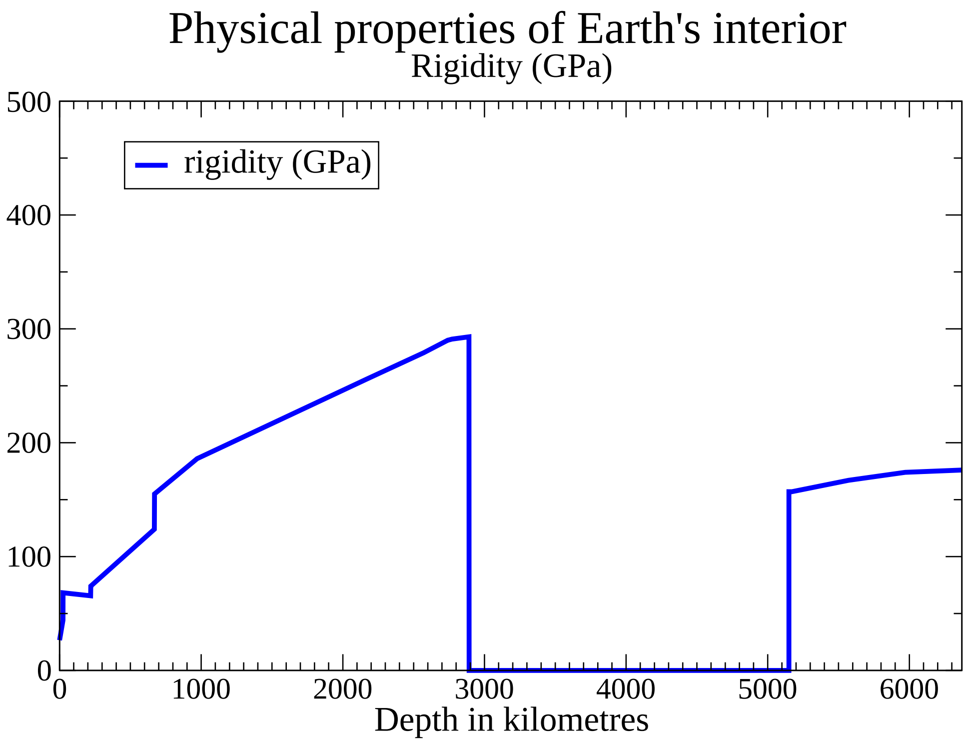
    The most distinct boundaries are at $\approx 440km$ where ***olivine***, $[Mg, Fe]_2SiO_4$, becomes compressed into the ***spinel*** structure and then at $\approx 670km$ where the spinel becomes further compressed into a mix of ***perovskite***, $[Mg, Fe]SiO_3$, and ***periclase***, MgO. The lower mantle is thought to maintain this *compositional* mix to the core-mantle boundary at $2970km$ depth.
  - ***D′′ layer:*** Rather than a subduction graveyard as sometimes argued in tectonics, the D'' discontinuity is now more commonly argued to be evidence of a phase change from perovskite to ***post-perovskite***. This is interesting because for a pure $MgSiO_3$ perovskite, this transition is seen in laboratory experiments at pressure of $120\,GPa$ (just that of this depth in the Earth) at temperature of $2500\,K$. **We may have a temperature tie-point in the deep mantle just above the core boundary.**[28]
  - ***Outer core:*** The outer core is essentially a liquid mix of iron, Fe, and nickel, Ni, with some alloying lighter elements, probably, sulfur, S, and possibly oxygen, O, and even carbon, C. Theories relating to the geophysics of the **self-exciting geodynamo** seem to show the core fluid to be of very low viscosity and convecting vigorously.
  - ***Inner core:*** At the centre of this liquid core is the frozen, probably almost pure iron, inner core. Its radius is $1222km$.

    The outer core and inner core contain about 35% of the Earth's total mass in less than $1/8$ of its volume.

[28] Questioning The Faith (slide 6), O. Jensen

1.7.6 Recalling the elastic properties of Earth's interior

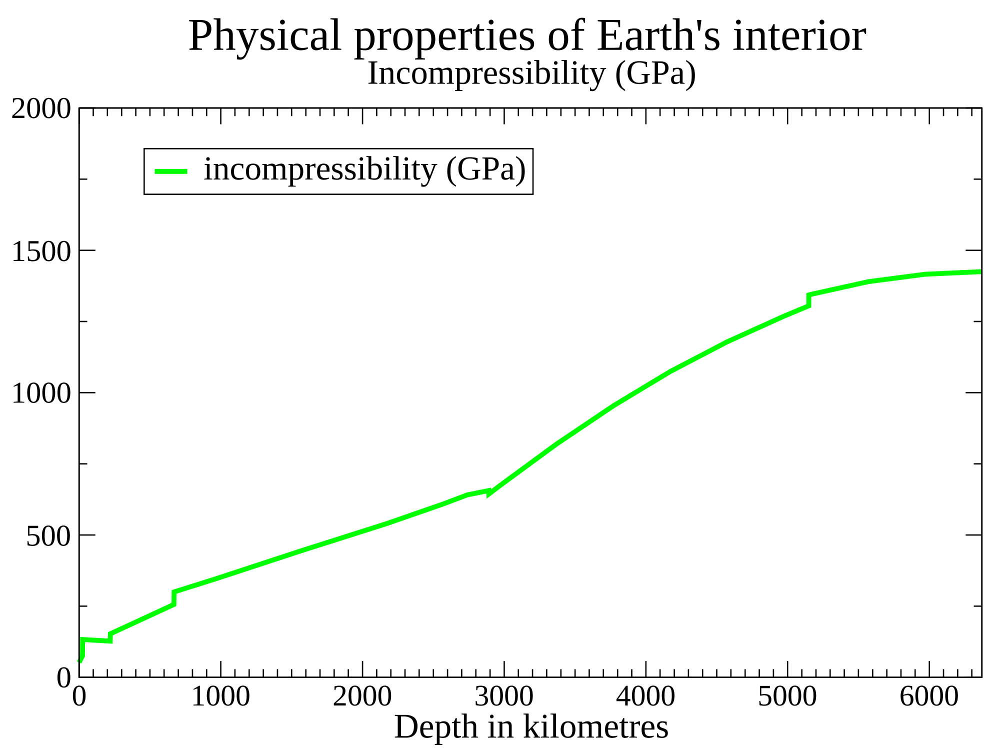
- Rigidity ($\boldsymbol{\mu}$) or shear modulus: Rocks are solid and clearly resistant to shearing stresses. Rigidity is properly a measure of ***hardness***. Liquids have no resistance to shearing stresses which are sufficiently slowly imposed. The extremely high viscosity of the mantle brings the mantle to respond with the nature of a solid to seismic shear waves which pass through it. That is, the mantle does show real rigidity to seismic shear waves for which the stress variations are characterized by time scales of only a few seconds. Stress variations imposed with time scales of hundreds of years would see the mantle as a fluid.



  - The hardest of rocks and minerals we know on the surface show rigidities (or ***shear moduli***) of $\boldsymbol{\mu \approx 80\,GPa}$; for diamond, the hardest of all minerals. $\boldsymbol{\mu \approx 100\,GPa}$.
  - As we move from the base of the lithosphere into the asthenosphere at about **100** km depth, the rigidity seems to decrease marginally (*soften*) from $\boldsymbol{\mu = 75\,GPa}$ to $\boldsymbol{\mu = 70\,GPa}$. Even here, in this most "fluid" region of the mantle, the rigidity is almost as great as that of diamond.

- Rigidity rises relatively smoothly through to the asthenosphere to about $\mu = 120\,GPa$ at $670\,km$ depth where it jumps to $\mu = 150\,GPa$ before rising smoothly again to $\mu \approx 290\,GPa$ at the base of the mantle.
- Rock deep in the mantle is very much more rigid than is rock we know on the surface – and, still, when observed over long times, it appears to be fluid.
- As we move across the core-mantle boundary into the iron outer core, the rigidity drops to $\mu = 0!$. The outer core is such a low viscosity fluid that even short period seismic shear waves see it to possess no rigidity at all. It is because shear waves cannot travel through a fluid that we know that the outer core is fluid.
- The inner core is solid. We know that seismic shear waves, those which impose only shearing stresses in travelling through a solid material, do have velocity in the inner core. That proves its solidity. The inner core is not very rigid, however, with $\mu \approx 1.8 - 2.1 \times 10^{11}\,Pa$ throughout.

• Incompressibility (k) or bulk modulus: Incompressibility is a measure of a material's resistance to changing its volume under pressure.



– Gases are very compressible ($\boldsymbol{k}$ small) while minerals such as diamond have high incompressibility, ($\boldsymbol{k = 443\,GPa}$). Strangely, it has just been discovered that the soft metal, osmium, has an even higher incompressibility than diamond: $\boldsymbol{k_{Os} = 462\,GPa}$. Osmium has much lower rigidity and does not appear as "hard"; diamond scratches or indents osmium easily.

– The hardest rocks of our experience on the surface show a bulk modulus of incompressibility of about $\boldsymbol{k = 100\,GPa}$. The rapid increase of P-wave velocity with depth in the upper regions of the mantle takes this to about $\boldsymbol{k = 300\,GPa}$ at the transition boundary at 670 $\boldsymbol{km}$ depth and then linearly to about $\boldsymbol{k \approx 1500\,GPa}$ in the central inner core.

- Pressure: Pressure increases from $\boldsymbol{1\,bar = 101.3\,kPa}$ at the base of the atmosphere on the Earth's solid surface to $\boldsymbol{360\,GPa}$, or $\boldsymbol{3.6 \times 10^6\,bar}$, at Earth's centre.