Excitation and damping of the Chandler Wobble

The spinning Earth is driven into what is called its free Eulerian nutation or, geophysically, its Chandler wobble by redistributions of mass within the Earth and/or by externally applied shocks. Walter Monk in the 1960s suggested that as earthquakes redistribute lithospheric mass, it may be that one could recognize the moments of earthquakes in the record of the Chandler wobble. Mansinha and Smylie in 1968 suggested that as great earthquakes probably accumulate a redistribution of mass even prior to fracture, it might be possible to predict the times of megathrust events through careful examination of the rotation pole path. Smylie, Clarke and Mansinha[1] (1970) sought to recognize perturbations in the rotation pole path which might be ascribed to earthquakes. They used a classical time-series technique called "deconvolution" that had been so successfully applied in seismic petroleum surveying in the previous decade. While there is no real doubt that earthquakes could and should contribute to episodic excitation of the Chandler wobble and so to variations in the momentary rotation pole path, most geodesists doubt that any clearly demonstrated excitation by earthquakes has been seen. Now, as pole-path measurements are good to within a millimetre or two, one might expect that if a significant contribution to the wobble and pole path is due to earthquakes, we should be able to recognize it. Doug Smylie has continued to research this effect until today. In 1987, Lalu Mansinha and I[2] looked into the problem by employing what we argued was a better statistical model for the excitation and hence claimed a better deconvolution method for the recognition of earthquake-driven pole-path steps. I abandoned that reasearch by 1990 but, now, as the pole-path measurements have so significantly improved, I suggest that it might be worthwhile pursuing the story again and further.

When looking for a small but important physical effect, one normally employs a "forward" theoretical parametric model that efficiently incorporates all the known physics that might affect the data measurements. One, then, "inverts" the data to determine the best-fitting parameters that define the model. "Best-fitting" is a subjective measure. Trivially, we pretend an "objective function" like, for example, a least-squares fit of our discovered parameters to the data. Signal and time-series analysis is a very "philosophical" field of physics and geophysics. Our 1987 "better statistical model" was based on the expectation that the excitations of the wobble would be temporally correlated. For our "objective function" we argued for a "minimum power, flicker-noise

1 Smylie, D. E.; Clarke, G. K. C.; Mansinha, L., Deconvolution of the Pole-Path, 1970, in Earthquake Displacement Field and the Rotation of the Earth, edited by L. Mansinha, D.E. Smylie, and A.E. Beck. ISBN 90-277-0159-8, 1970. Astrophysics and Space Science Library, Vol. 15, p.99

2 *Jensen, O.G.* and Mansinha, L.; Excitation of geophysical systems with fractal, flicker noise, in ***Time Series and Econometric Modelling***, Vol. III, MacNeill, I.B. and Umphrey, G.J. *eds.* , 165-188, Reidel, Dordrecht, 1987.

excitation". Minimum power is an indirect invocation of Fermat's principle in physics -- loosely, nature does what nature does most efficiently, hence with minimum forcings. Flicker noise was invoked as an argument that what is happening today is not entirely disconnected from what happened yesterday or tomorrow. A Gaussian noise model dis-correlates past, present and future excitations.

As a time-series analyst, I normally think in terms of physically consistent "*data models*". For the Chandler wobble, the physics determines the period and rate of damping (energy loss with time) of the wobble. An excitation that might be due to an earthquake event is included in the data model as a so-called "*innovation*". The excitation "surprises" the Earth every now and then. The data model used in inversion in our 1987 paper described this innovation as a minimum power flicker-noise. We find the best-fitting parameterization of our physical model (equivalently, the period and damping time constant of the Chandler wobble), deconvolve the data to correspond with this now-determined physics and expect that the left-over innovation corresponds to wobble excitation which might be due, at least partially, to earthquakes.

The data model:

The Chandler wobble (the free Eulerian nutation) should be essentially modelled as a damped, complex-valued resonance. Damped resonant phenomena are best modelled in time-series analysis by what are called "*autoregressive data models*". For discrete data (and here there are many important details that one must consider responsibly -- for example, Shannon's sampling theorem) the autoregressive data model has the following form:

$$\mathbf{z_n = z_{n-1}\, b_1 + z_{n-2}\, b_2 + z_{n-3}\, b_3 + \ldots}$$

The $\mathbf{z_n}$ represents the measurement of the current pole position in complex notation where the real component represents deviation along the 0 meridian and the imaginary component along the 90 E meridian. $\mathbf{z_{n-1}}$ represents the previous pole position, etc.. The coefficients $\mathbf{b_i}$ represent the "*forecasting operator*". Without excitations, the pole positions evolve in a complete predictable way. With excitation, the current pole position might be pushed by an innovation $\mathbf{p_n}$

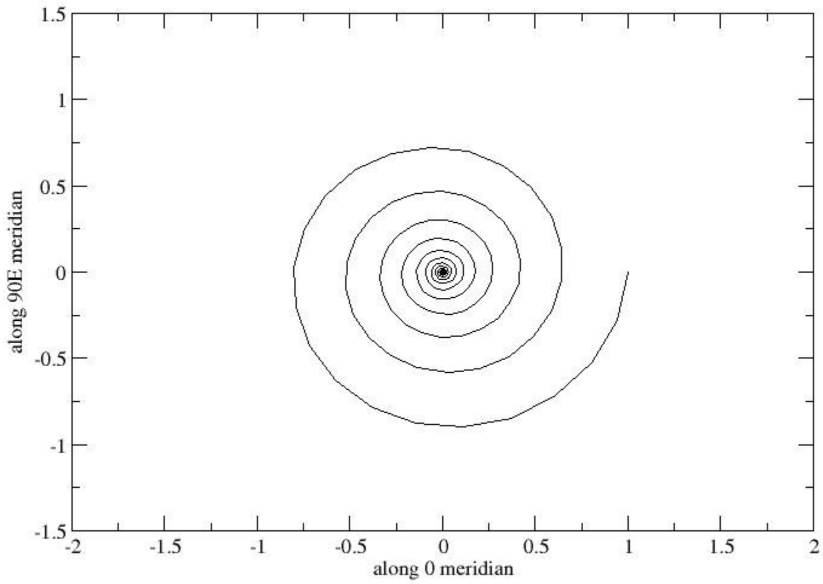
$$\mathbf{z_n = z_{n-1}\, b_1 + z_{n-2}\, b_2 + z_{n-3}\, b_3 + \ldots + p_n}$$

Ideally, the excited Chandler wobble record can be so-constructed with but one complex-valued coefficient:

$$\mathbf{b_1} = e^{i\mathbf{u}\Delta\mathbf{t}}$$

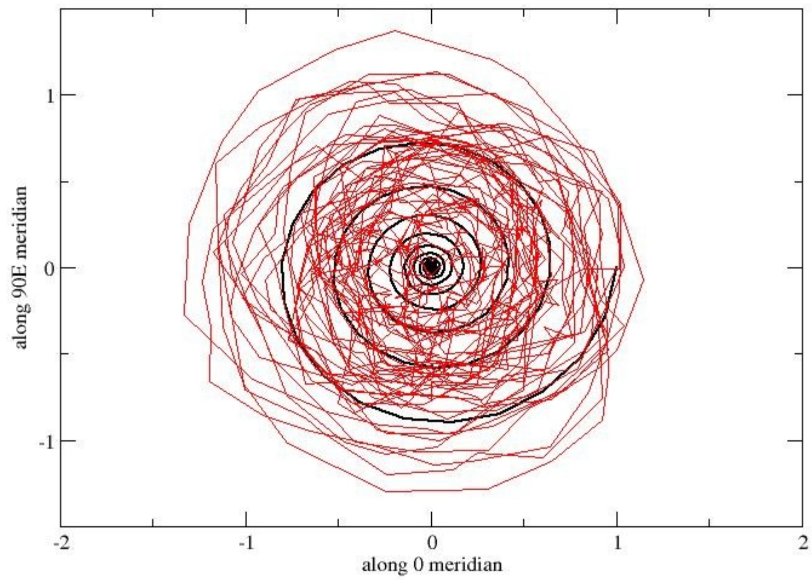
where $\mathbf{u} = \boldsymbol{\omega} + \boldsymbol{i}/\boldsymbol{\tau}$ and where $\boldsymbol{\omega} = \mathbf{2\pi/P}$, the Chandler frequency, and $\boldsymbol{\tau}$ is the damping time constant, that time in which the freely decaying wobble would lose $\mathbf{1}/\boldsymbol{e}$ of its energy. $\boldsymbol{\Delta}\mathbf{t}$ is the interval of "*responsible*" (an oft-violated condition) sampling.

A synthetic data model:

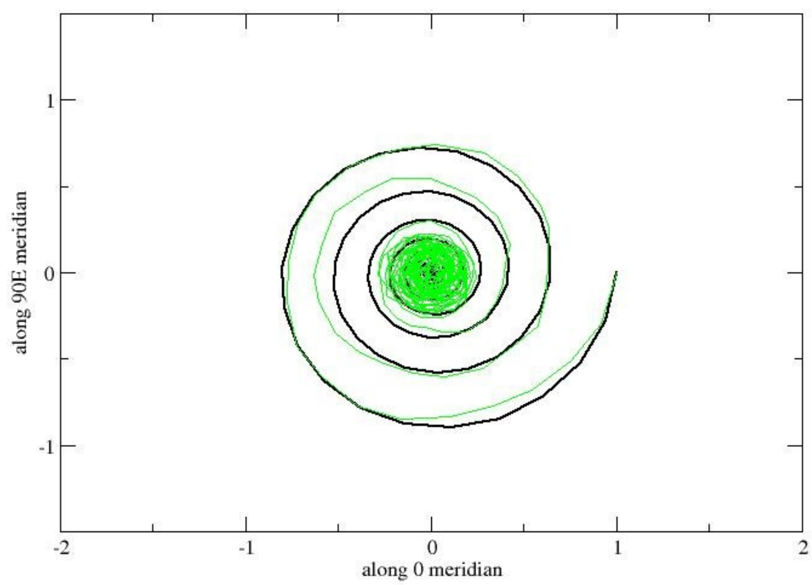


With $\mathbf{P = 435}$ days, $\boldsymbol{\tau} = \mathbf{1000}$ days and a single excitation (innovation) $\mathbf{p_1 = 1.0}$, real-valued and hence a "*push*" from equilibrium along the **0** meridian, we recurse a synthetic data series. In this synthetic, the wobble is only once innovated.

Now excited, we add continuing innovations at every **20-day** data step. In the first case, red path, the innovation is a Gaussian random step with a standard deviation of **0.1** units.



[3]This level of continuing innovation (excitations every **20 days**) produces a wobble that is sustained for the full **20000-day** synthetic record. A lower level of excitation, with innovations of standard deviation **0.02**, eventually collapses down to about **1/5** the amplitude. This amplitude would continue, on average, forever with this continuing level of innovation.



If we were to explore, this latter sequence through a deconvolution and Burg analysis[3], we would obtain a spectrum that recognizes the **435-day** period.

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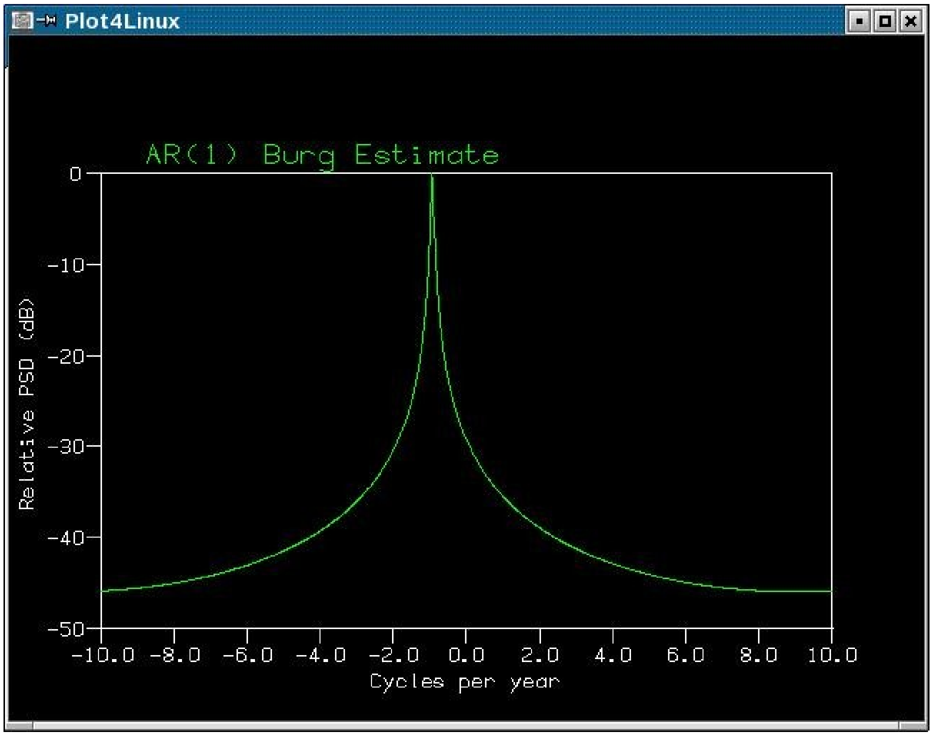
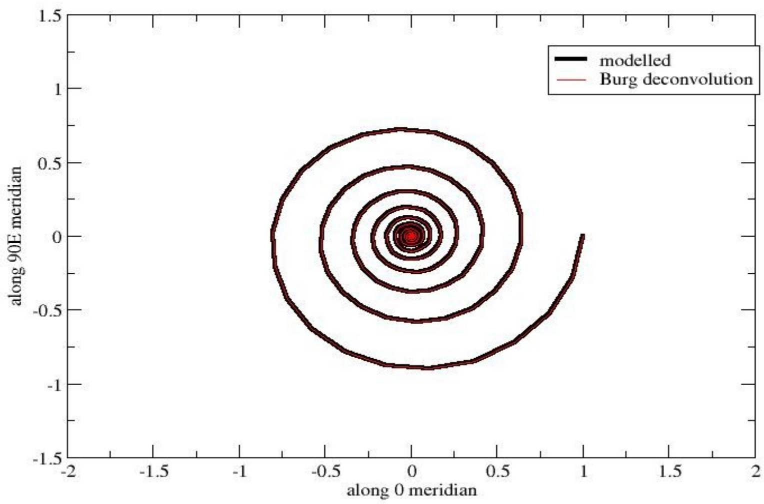
3 The Burg algorithm for AR modelling via MatLab



Here we sought a model with but one forecasting coefficient to find **$\langle b_1 \rangle$ = 0.94752 - 0.28707*i*** . Our original model was developed with **b_1 = 0.93958 - 0.27924*i*** . We find a correct period but with somewhat less damping than we modelled. We find the physical model.

We compare the modelled Chandler resonance to that we found through our Burg inversion process.



Deconvolution of pole-path record to determine excitation:

With the deconvolution process, we can also determine the excitation sequence. Given our original data model, we form:

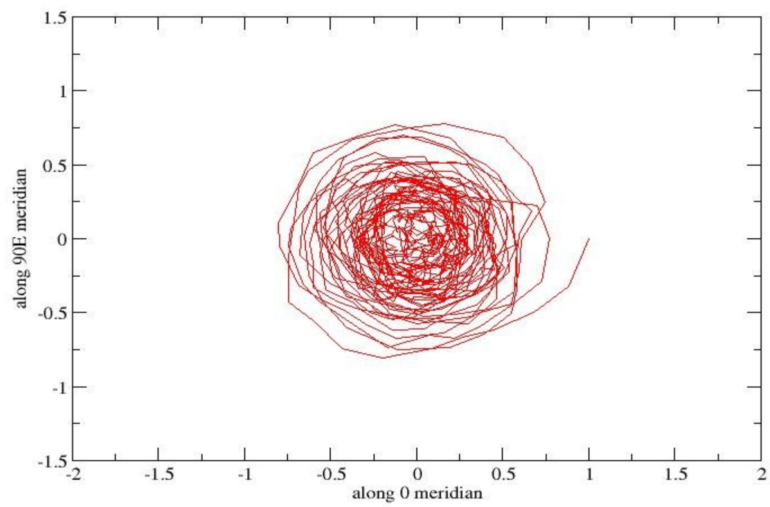
$$\mathbf{p_n = z_n - (z_{n-1} b_1 + z_{n-2} b_2 + z_{n-3} b_3 + \dots).}$$

Given our found value for $\mathbf{b_1}$, we can compute (deconvolve for) the exitation sequence, $\mathbf{<p_n>}$**.** My early interest in this explicit problem (with real data) derived from my early graduate-student years when I read a classical monograph by Munk and McDonald (1960)[4]. Recognizing that variations in the Earth's moment of inertia would have the effect of stimulating variations in the rotation pole path, they suggested that one cause might be due to very large earthquakes which, we well know, can produce major relative movements of mass. Smylie and Mansinha[5] (1967) convened a conference to explore all possible excitations of the pole path. Their suggestion was that a run-up of most mass motion in large earthquakes actually precedes the fracture and so by carefully monitoring the rotation pole path, one might be able to predict large events. While consensus among seismologists and geodesists is no substitute for an empirical demonstration, I suggest that most of us do not believe that we can clearly see evidence of pole-path variations, even now as we can measure a 5-day average pole position to about 1mm, show earthquakes. Following a couple of synthetic examples, I shall look into the last 20 years of the rotation data sets as offered by the IERS to look for earthquakes!

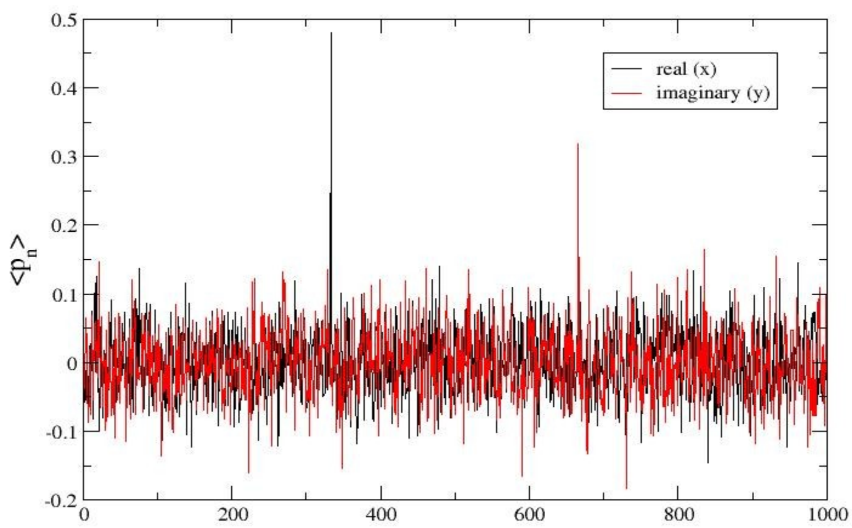
Let us modify the first excited model (that with the red path) by adding in two additional pole-centre jumps, one in the x-direction at time 333 with value +0.5 units and another in the y-direction at 666 with value 0.3 units. These are to represent powerful earthquake excitations. Recall that the background excitation level's standard deviation in that model was 0.1 units. So we are looking for “earthquakes” that exceed the 5σ and 3σ continuing background excitation levels.

4 W. Munk and G.J.F. MacDonald, *The Rotation of the Earth: A Geophysical Discussion*, Cambridge University Press, 1960, revised 1975. ISBN 0-521-20778-9

5 Mansinha, L. and D. E. Smylie, Effect of Earthquakes on the Chandler Wobble and the Secular Polar Shift, 1967 JGR 72/18 4731-4743. doi:10.1029/JZ072i018p04731

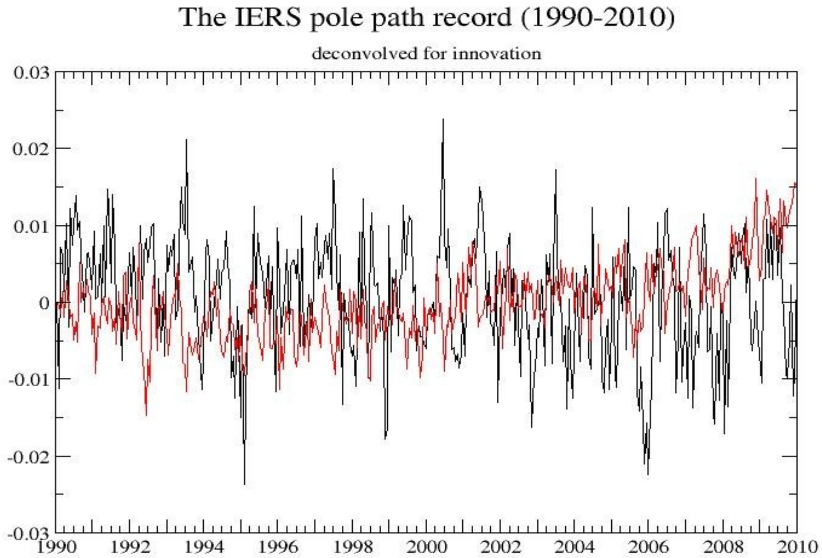


Burg deconvolution obtains **$\langle b_1 \rangle$ = 0.94010 - 0.28798*i*** for the forecasting operator. When the above pole-path record is recursively forecast using that operator, we are left with the forecasting or prediction errors, those that are not foreseen or expected in the data based on its immediate past. The following diagram shows the record of the **$\langle p_n \rangle$, n = 2, 1000**. The black line shows the x-direction excitations and the red, the y-direction.

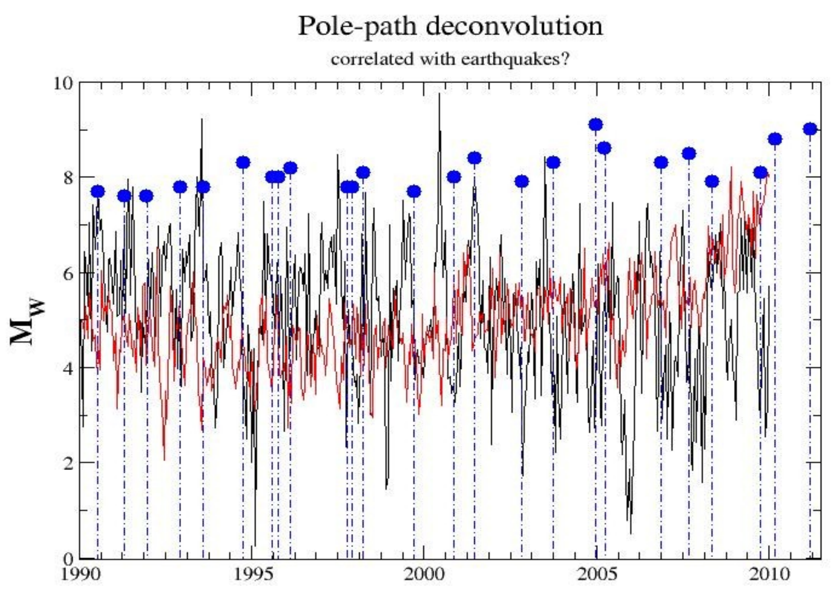


As promised, above, I now look to deconvolve the IERS rotation pole-path data set that I introduced in the lecture of September 18: ***Geophysical Gravity***. A Burg deconvolution of this record (1990-2010, 1/20 year samples) obtains the innovation time series below.

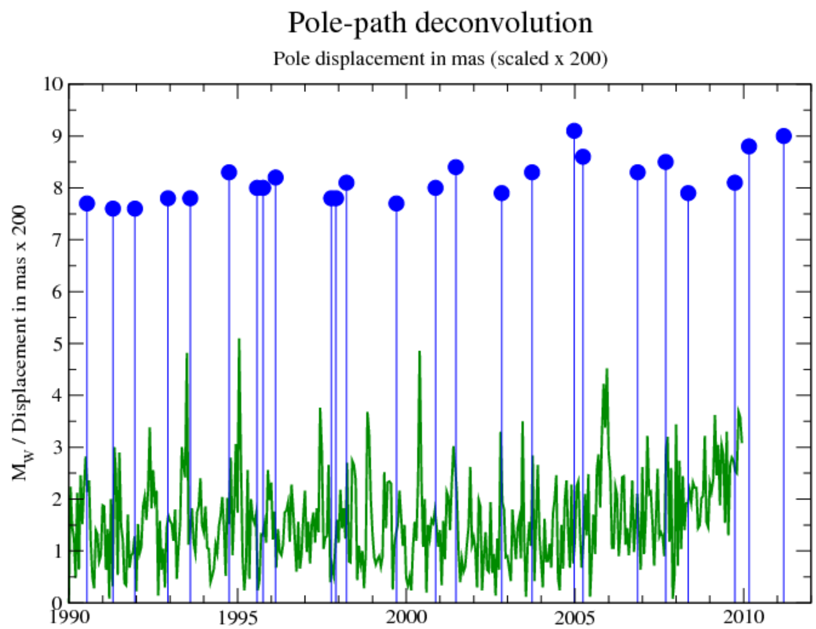
The black line represents pole displacements along the 0 meridian, red along the 90°E meridian. We do see some large excursions that might well exceed a **2σ** level and so might be seen as "significant". Do they correlate with known large earthquakes?



Earthquake correlations?



I offer another comparitive graph, with the amplitude of the pole-path deviations, now shown by the green line.



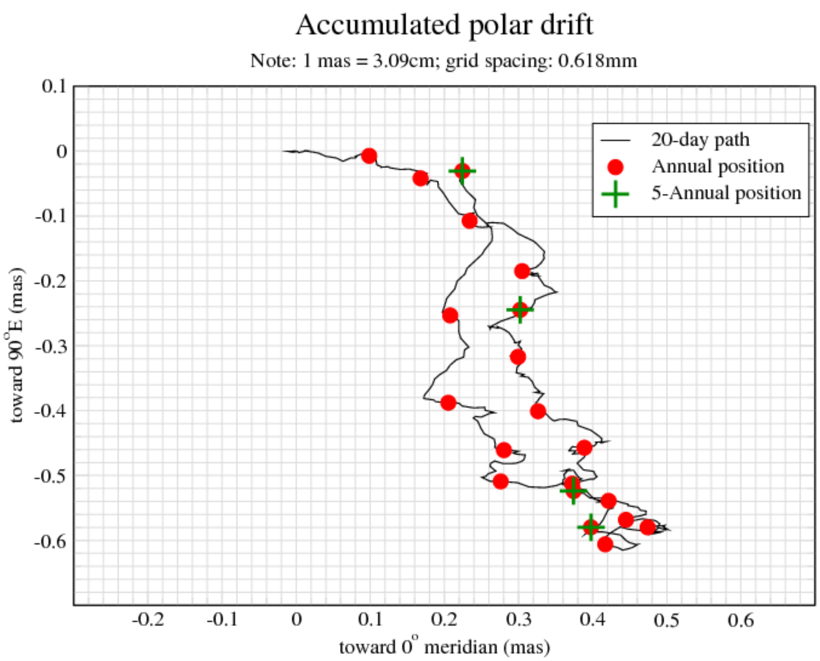
I leave it to you as an exercise to argue for possible correlations.

As you may not find clear and unambiguous correlations with these large events, you might ask yourself ***"Why not?"***. That is, I leave you with this question: ***"Why is our inversion model insufficient for decomposing this data record?"***

Polar Drift?

Our continuing series of innovations represents the additional displacement of the pole at each time step. We can accumulate these incremental displacements to obtain a record of the pole path itself. That is, each innovating excitation of the Chandler wobble that we have found through our deconvolution actually represents a “push” of the shell of the Earth (whatever the cause) that moves the geographical coordinates relative to the rotation axis.

In the diagram below, we plot this accumulated polar drift relative to the January 1, 1990 pole position. These are not absolute positions as measured in geographical coordintates.



Separated X- and Y- innovations are shown in the next 2 diagrams.