# Math-physics of loaded plates

The general equation that describes the vertical deflection, $w(x)$, of a 2-dimensional plate[1] is shown by Turcotte and Schubert[2] to have the form:

$$D\frac{d^4w}{dx^4} = q(x) - P\frac{d^2w}{dx^2}$$

where $q(x)$ is the distributed vertical load, $P$ is the x-directed horizontal force on the plate and $D$ is the flexural rigidity of the plate. For an elastically uniform plate of constant thickness, $h$,

$$D = \frac{Eh^3}{12(1-\sigma^2)}.$$

$E$ represents ***Young's modulus*** and $\sigma$ is ***Poisson's ratio***. Rewritten in terms of rigidity, $\mu$,

$$D = \frac{\mu h^3}{6(1-\sigma)}.$$

This linear differential equation is easily(?) solved for many cases of loaded and forced plates. As in most physical problems, the basic differential system can appear as very simple mathematics; the art of the physicist is in applying physical constraints for the solution through boundary and/or initial conditions. A general solution of this particular system is essentially quite a simple sequential integration. As it is a fourth order system, though, the geophysicist is faced with recognizing four boundary conditions that establish the four constants of sequential integration. Therein is the difficulty. Note that the system has no time dependence and so can only describe "static" conditions.

Turcotte and Schubert artfully deal with several examples concerning the ***Bending of Plates under Applied Moments and Vertical Loads***[3].

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[1]The suppressed $y$-coordinate means that there is no variation in the $y$-direction
[2]Geodynamics, section 3.9
[3]Geodynamics, ed. 2, section 3.10 ff.