Can we see evidence of post-glacial geoidal adjustment in the current slowing rate of rotation of the Earth?

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In this simple analysis, we compare the historical record of the change in length of day to a value calculated from our own model (based on the retreat of the Moon) as well as the ones computed by other authors to determine the role that the post-glacial geoidal adjustment plays in the slowing down of the rotation of the Earth. We conclude that the isostatic rebound is not negligible in this phenomenon and proceed with a brief explanation of how it affects it.

Historically, the observed change in the length of day during the past 2700 years has been recorded in several places by ancient civilizations like the Babylonians, Chinese and Greeks through their studies of eclipses. However, the first suggestion of the decrease of the Earth's period was only hypothesized in the early eighteenth century based on those precise ancient data. The record suggests an average value over this period of 1.7 milliseconds per century (Stephenson, 1997). Our objective was to determine the role of the post-geoidal glacial adjustement in the change in the length of day by comparing historically observed values to values calculated with our model and other authors.

Since the American Moon landing in 1969, when the Apollo 11 mission astronauts left laser-ranging retroreflectors on the surface of the Moon, scientists around the world have been able to calculate to high precisions the distance between the Earth and the Moon. Those measurements showed that the Moon is retreating from the Earth at a current rate of approximately 3.85 cm/yr. This suggests through the loss of angular momentum of the Moon an increase in the length of the terrestrial day of 2.3 ± 0.1 milliseconds per century from lunar tides (Stephenson, 1997).

The rotation of the Earth is influenced by the Moon revolving around it in an elliptical orbit, from Kepler's first law. According to Noether's first theorem, the Earth-Moon system shows conservation of angular momentum. Therefore, any change in the angular momentum of the Moon (which depends on its distance from the Earth) has repercussions on the Earth's angular momentum because they are part of the same system. Hence, any gain in the angular momentum of the Moon is lost from the Earth's.

Our model is based on the assumption that the orbit of the Earth and the spin of the

Moon are both of negligible importance in the calculation of the change in angular momentum that results from the slow retreat of the Moon. From Kepler's third law, the period of the moon changes according to its distance from the Earth, d:

$$P_m = \frac{2\pi}{\sqrt{GM_{e+m}}} d^{3/2}$$

Thus, from the conservation of angular momentum:

$$\Delta L_m = \Delta L_e$$

$$\left| M_m \, \Delta d^2 \frac{2\pi}{\Delta P_m} \right| = \left| I_e \; M_e \; r_e^2 \frac{2\pi}{P_e} \right|$$

$$\left| M_m \sqrt{\Delta d \; G \; M_{e+m}} \right| = \left| \frac{I_e \; M_e \; d^2 \; 2\pi}{\Delta P_e} \right|$$

$$\Delta P_e = \left| \frac{I_e \; M_e \; d^2 \; 2\pi}{M_m \sqrt{\Delta d \; G \; M_{e+m}}} \right|$$

Where,
ΔL_m is the variation in the angular momentum.
M_m is the mass.
Δd is the variation in the distance between the Earth and the Moon.
P is the period.
G is the universal gravitational constant
I is the moment of inertia
r is the equatorial radius
X_e means "X" of the Earth
X_m denotes "X" of the Moon

One can then find that the Earth's rotational period (directly related to the length of day) increases by 2.15×10^{-5} second every year, or 2.15 milliseconds per century. The difference between our model and Stephenson's value (2.3 ± 0.1 milliseconds per century) is probably due to what we neglected from it in the first place.

The simplicity of our model that is, our neglecting of the spin of the Moon and the orbit of the Earth, more than likely accounts for some of that difference. Even though both of those factors affect the angular momentum of the Earth-Moon system, they were not taken into consideration because their effect on the change of the period of the Earth is not as pronounced as the retreat of the Moon is.

The major discrepancy is between our model and the observed changes with data that does not only rely on tidal acceleration. That is, data that is not only from the forces from the Moon that effect the Earth, but also other forces from other sources. The historically observed value of 1.7 ms/cy contrasts our modelled value of 2.15 ms/cy and the lunar laser ranging computed value of 2.3 ms/cy. The difference between the observed value and the one computed from our model is significant enough that other factors might be contributing to the change in the period of rotation of the Earth. This means that the spin of the Earth is not slowing down as much as what would be expected from only the relationship of the Earth and the Moon would suggest. The post-glacial geoidal adjustment certainly plays an important role in this anomaly, by modifying the shape of the Earth, thus its moment of inertia as well.

The few-kilometres thick ice sheets present at the poles during the last ice age, in the Pleistocene, caused the Earth's crust to sink into the plastic mantle, deforming it (as can be seen in figure 1 and 3). Since then, the ice sheets covering the poles have been retreating. This retreat reduced the load pushing down on the viscous mantle which leads to the adjustment of the mantle back towards its equilibrium position. This process is known as the isostatic rebound. The post-glacial rebound changes the shape of the Earth by decreasing its centrifugal flattening over centuries.

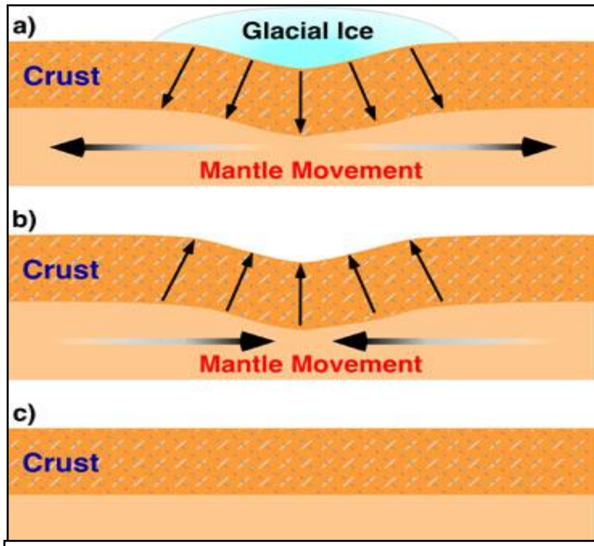

Figure 1
(a) The weight of the ice makes the crust sink down into the viscous mantle.
(b) After deglaciation, the crust rises to towards its equilibrium position, pushed back by the mantle.
(c) Mantle at its equilibrium state.
(http://www.physicalgeography.net/fundamentals/10h.html)

The process of glaciation brings water from the ocean basin to high latitudes, in the polar regions, closer to the rotational axis. This movement of mass along the surface of the Earth causes a decrease in its moment of inertia. This is just like when a figure skater raises his hands above his head and rotates faster. On the other hand, deglaciation does the opposite and causes the moment of inertia of the Earth to increase, which increases its rotation and thus decreases the length of the day (Lambeck, 1977).

Isostatic rebound has been mainly observed in high latitudes where the major ice sheets used to sit, such as over Hudson Bay in North America and Fennoscandia (see figure 2), and varies on the order of a few centimetres per year. The rate at which the rebound happens depends on the past ice load and the proximity to the edge of the past ice cap. The adjustment at the equatorial level is compensated by the movement of ice cap’s melt water from the poles to this region. The rise of the paleo-glaciated continents can be observed from the decrease in the sea level relative to coast lines, and the study of gravity anomalies. (Walcott, 1973)

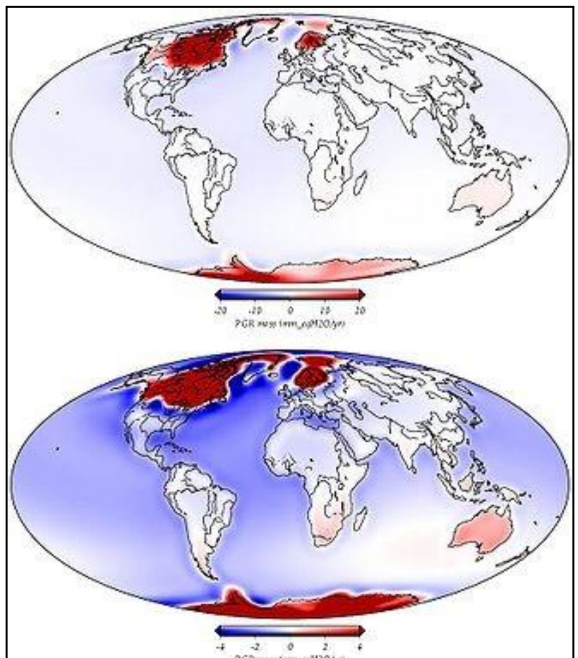

Figure 2: This map is a measure of post-glacial rebound. The zones in red are rising. The blue zones represent the bulge of the mantle adjusting from the ice caps’ weight. The water from the melting ice caps is filling the oceans resulting in the pushing of the ocean crust downwards.
(http://grace.jpl.nasa.gov/data/pgr/)

There exists a relationship between the adjustment of the mantle back to its equilibrium state and the dynamic ellipticity of the Earth; this is called J_2 (see figure 3 and 4). That number represents a measure of the flattening of the Earth at its equatorial radius using the Earth's moments of inertia about the polar axis, C, and the average of the equatorial axes, A (Lambeck, 1980; Stacey, 1977; Wu & Peltier, 1984).

We can find J_2:

$$J_2 = \frac{C - A}{M_e \, r_e{}^2}$$

And its derivative, the change of the centrifugal of the Earth flattening over the Earth's period (a day), $\frac{dJ_2}{dt}$ (Burša, 1987):

$$\frac{dJ_2}{dt} = \frac{\left(\frac{dC}{dt} - \frac{dA}{dt}\right)}{M_e \, r_e{}^2}$$

Which gives us the change in angular velocity, $\frac{d\omega}{dt}$, using the speed of the rotation of the Earth ω, also known as the *spin down deficit*:

$$\frac{d\omega}{dt} = \frac{dJ_2}{dt} \frac{\omega}{2\,J_2}$$

This value gives us the change in the length of day per day, which is easily convertible into seconds per century. This calculated value is a decrease in the length of day at a rate of 6.04×10^{-6} s/yr, or 0.604 ms/cy due to non-tidal effects. This is slightly larger than the difference of the calculated value from laser ranging and the observed historical value (which should be 0.6), but considering the error of 0.1 ms/cy, our $\frac{dJ_2}{dt}$ estimate combined with the tidal acceleration gives us a reasonable value.

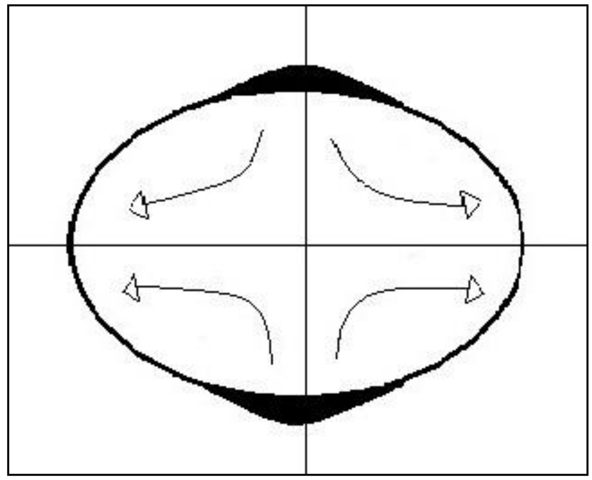

Figure 3: Polar ice loads flatten the body of the Earth, causing the mantle to flow towards the equator.

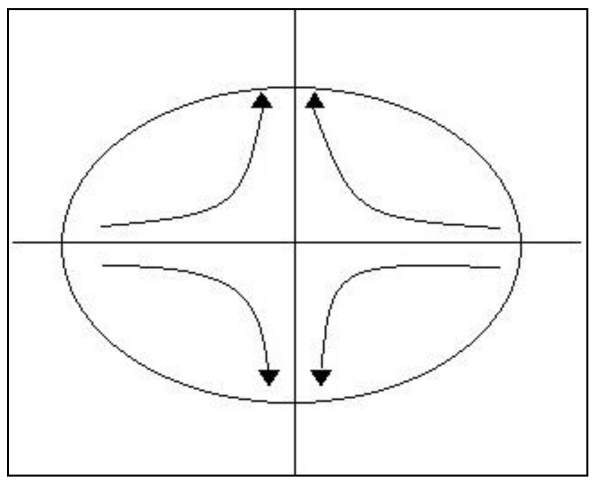

Figure 4*:* After rapid melting, the Earth takes time to recover its equilibrium shape. The mantle flows back from the equator to the poles, this process can be calculated via J_2.

These values would suggest that the rate of change of the spin of the Earth is only dependent of the effect of the Moon, and the isostatic rebound, but that is not the case; there are other factors to take into consideration. For example, the gravitational effect of other planets on the Earth and changes in the effective mass of the sun caused by solar winds. Those effects should be subject to further research, but their effects on the Earth are less pronounced than tidal drag (Moon's rotation, isostatic rebound).

To summarize, in our paper we have demonstrated that the post-glacial geoidal adjustment has a significant effect on the change in length of day. We first modeled the retreat of the Moon away from the Earth, and its effects on the slowing down of our planet's rotation, which was determined to be 2.3 ms/cy. But general observations show a slowing down of "only" 1.7 ms/cy. We demonstrated through calculations of the isostatic rebound and its effect on angular momentum that post-glacial geoidal adjustment was making the Earth spin at a rate about 0.6 ms/cy faster than what it should. This corresponds to the difference observed from the calculated value and the observed value. Hence, a proof that the isostatic rebound has a significant role to play in the change in length of day.

CONSTANTS USED:

$\mathbf{M_e} = 5.9736 \times 10^{24}$ kg	$\mathbf{M_m} = 7.3477 \times 10^{22}$ kg	$\mathbf{A} = 8.010 \times 10^{37}$ kg m^2
$\mathbf{r_e} = 6371000$ m	$\mathbf{d} = 389798000$ m	$\mathbf{C} = 8.036 \times 10^{37}$ kg m^2
$\mathbf{P_e} = 86400$ s	$\boldsymbol{\omega} = 7.292115 \times 10^{-5}$ rad s^{-1}	
$\mathbf{I_e} = 0.3308$ kg m^2	$\mathbf{J_2} = 1.07231 \times 10^{-3}$	

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